Set Theory, Relation & Function

Set

A set is defined as a combination of a certain number of objects or attributes together as a single entity.

Clearly, we can understand 'set' as a group of some allowed objects stored in between curly brackets ({}).

Anything stored in between curly brackets is treated as a 'set' in mathematics (other than algebra when they can be used as second brackets {}.

For example:

Let A be a set of natural numbers from one to 10. Then A can be represented as

A = {1,2,3,4,5,6,7,8,9,10}.

Types of Set

Finite set: A set which contains a finite or countable number of elements is called a finite set. For example: A = {2,7,9,10}. The number of elements in A is 4 which is a countable number of a finite number. Hence, A is a finite set.

Infinite set: A set which contains an infinite or uncountable number of elements is called an infinite set. For example, B is a set of all-natural numbers. Thus, B can be written as

B = {1,2,3, 4….}. As the upper end of the set, B is not known hence it cannot be concluded where the set must end. Thus, it is an infinite set.

Null set: A set containing no element or having zero number of elements is called the Null set. A null set is denoted by {} or

Equal sets: Two sets A and B are said to be equal sets if they fulfil the following criteria

Both A and B must have the same number of elements.

Both A and B must have exactly the same elements.

For example, A is a set of all-natural numbers

B is a set of all positive integers

Equivalent sets: Two sets A and B are said to be equal sets if both A and B must have the same number of elements.

For example: A = {1,2,3,4}

B = {a, b, c, d}

NOTE: All equal sets are also equivalent sets but all equivalent sets are not equal sets.

Subset and Superset: A set S is said to be a subset of T if T contains all the elements of S and also the number of elements in T is greater than or equal to S. Consequently, T is known as the superset of S.

For example:

A = {1,2,3,4,5,6,7,8,9,10}

B = {2,4,6,8,10}

Now here B is a subset of A

And A is the superset of B

Subset is denoted by ⊂ .

A superset is denoted by ⊃ .

Thus, from the above example, B ⊂ A and A ⊃ B

NOTE: An equal set is a type of subset.

Proper Subset: Just like subset we have a proper subset. A set S is said to be a proper subset of T if all the elements of S are also there in T but T must contain a number of elements greater than S.

Note: The above example shows a proper subset.

A proper subset is denoted by

Thus, in the above example, we can say B is a proper subset of A or

Overlapping set: Two sets A and B is said to overlapping set if at least one element is common in both. For example A={1,2,4} and B{1,2,3} are overlapping set

Disjoint set. Two sets are said to be disjoint id no element is common between the two sets. For example A={1,2,4} and B={a,b,c} are disjoint sets.

Universal set: The total set of which the given subsets are mere subsets is called universal set. For example: A={1,2,3,4,5,6,7,8,9}. Then we agree that A is a subset of Natural numbers set. Hence, A is merely a subset of the natural number set working space. Thus, the natural number becomes the universal set of A. It is denoted by U(<name of the set>)

Cardinality OF A SET: Cardinality of a set A is defined as the number of elements present in that set. It is denoted by ‘n(“set name”)’. Thus, the cardinality of A is denoted as n(A)

1. If A is a set representing the first 10 multiples of 8 and B represents the first 10 multiples of 5. (i) Represent A and B (ii) Find cardinal of A and cardinal of B (iii) Is A and B equal set or equivalent set? (iv) Are they subsets or proper-subsets or nothing? (v) Are they finite sets or infinite sets? (vi) Are A and B disjoint set or overlapping set?

Ans.

A={8, 16, 24, 32, 40, 48, 56, 64, 72, 80}

B={5, 10, 15, 20, 25, 30, 35, 40, 45, 50}

The cardinality of A=n(A)=10

The cardinality of B=n(B)=10

As A and B has same cardinality but elements are different so they are equivalent sets and not equal sets.

A and B has only one element in common. No set (A or B) has all the elements of the corresponding set i.e. A does not have all elements of B simultaneously B does not have all elements of B, hence they are not subsets.

The 2 sets A and B have a countable number of elements hence, they are finite sets.

A and B has one element common {40} hence they are overlapping set.

The total number of subsets of a set is given by the formula where n is the number of elements in the set itself.

Power Set

It is defined as a set containing all the subsets of a given set. It is denoted by P(<name of set>)

For example:

2. If set A={1,2,3} find the number of subsets and the power set.

Ans.

As A={1,2,3}

Thus, n(A)=3… n(A) is cardinality

Or n=3…where n is the number of elements

Thus, number of subsets =

Power set is

P(A)={{},{1},{2},{3},{1,2},{1,3},{2,3},{1,2,3}}

NOTE: (i) Every set is a subset of itself

(ii) A null set is a subset of every set

Set Representation (Notations of writing a SET)

Roster form: The general notation in which we write the elements of a set without repetition of an element such that each element is separated from each other by a comma (,).

For example: A={1,2,3} is in roster form

Set-Builder notation: The descriptive style of writing a set in small phrases is called set-builder notation

For example {x|x is a set of all-natural numbers less than 12}

Or {x|x<13

It is read as x is to x is a set where x is less than 13 and x belongs to the set of natural numbers.

Some predefined sets

N – Set of natural numbers

Z or I – Set of integers

Q – Set of complex numbers

R – Set of real numbers

{}- null set

Arithmetic Operations on Set

UNION: Union of 2 sets represents all the elements present in the two sets combined under a single set. (Repeated elements are represented only once). It is denoted by U for example union of set A and set B is represented as AUB

Intersection: Intersection of 2 sets is the representation of all the elements common in the 2 sets under a single set. It is denoted by. For example, the intersection of A and B is represented by

The complement of a set: Complement of a set is defined as the elements of all the set present in the universal set but not in the given set represented as a single set. It is represented by <name of set>^{c} or <name of set>'

Subtraction: The subtraction operator means the representation of those elements present in the first set(set before "-" sign) but not in the second set(set after "-" sign). It is denoted by A-B where A and B are set.

3. If A={1,2,3} and B={2,4}. Find (i) AUB (ii) A B (iii) (iv) A-B

Ans.

AUB={1,2,3,4}

A B={2}

Since the universal set is not mentioned it is assumed that AUB is the universal set.

A-B={1,3}

NOTE: (i) n(AUB) = n(A) + n(B) – n(A B)

ii) A' = U - A

Properties of the operators of sets

Commutative: (AUB)=(BUA) or (A∩B)=(B∩A)

Distributive: A∩(BUC)=(A∩B)U(A∩C) or AU(B∩C)=(AUB)∩(AUC)

Associative: AU(BUC)=(AUB)UC or A∩(B∩C)=(A∩B)∩C

Idempotent property=$(A^c)^c$

De-morgan property=$(AUC)^c= (A^c \cap B^c)$

Only A= A-B

Only B= B-A

AUA^c =U

$A\cap A^c$={}

AU{}=A

A {]={}

Relation

A relation is defined as the common link in between two set of elements or in other words relation is a linear operation that describes the relationship between two sets based on certain rules.

A relation between A and B is represented as R: A->B and the 2 elements linked is represented as

The above figure shows a relation between 2 sets A and B

According to the above diagram, the elements of the relation are as follows

R={(a,2),(b,1),(b,4),(c,3)}

Cartesian Product

The cross product between every element of the first set to every element of the second set is called the Cartesian product. It is denoted by x

For example: if A={1,2,3} and B={a,b} Find A x B

Ans.

A x B={(1,a),(1,b),(2,a),(2,b),(3,a),(3,b)}

In Cartesian product, the number of elements in the final set formed is (m)(n) where m is the number of elements in the first set and n is the number of elements in the second set.

Note: We can never switch positions of the ordered pairs in the final Cartesian product set

For illustration: If suppose (9,b) is a member of the Cartesian product then we cannot represent (9,b) as (b,9). (9,b) (b,9)

Types of Relations

Reflexive relation:- A relation is said to be reflexive if an element is in relation with itself when a relation is defined as follows R: A->A where A is a set on which the relation is defined. i.e A={a,b,c} if on defining R:A->A

Symmetric relation: A relation defined on 2 sets A and B is said to be symmetric if every element of A related to B also contains its inverse elements in the set. i.e. Suppose if A={1,2,a} and its Cartesian product is {(1,1),(2,2),(a,a),(1,a),(2,a),(1,2),(a,2),(2,1),(a,1)}.Thus, as seen (1, a) is a member also its inverse (a,1) is a member. Then it is called a symmetric relation.

Transitive relation:- According to the definition of transitivity it states that if and then . Suppose if A={1,2,a} and its cartesian product is {(1,1),(2,2),(a,a),(1,a),(2,a),(1,2),(a,2),(2,1),(a,1)}.Thus, as seen (1,a) is a member and also (a,2) is member then (1,2) must be present for the relation to be transitive.

Equivalence relation: A relation is said to be equivalent if it is reflexive, symmetric and transitive.

Note: Cartesian product is the largest equivalence relation.

4. A relation R is defined on the set Z by "a R b if a – b is divisible by 5" for a, b ∈ Examine if R is an equivalence relation on Z.

Solution:

(i) Let a ∈ Z. Then a – a is divisible by 5. Therefore aRa holds for all a in Z and R is reflexive.

(ii) Let a, b ∈ Z and aRb hold. Then a – b is divisible by 5 and therefore b – a is divisible by 5.

Thus, aRb ⇒ bRa and therefore R is symmetric.

(iii) Let a, b, c ∈ Z and aRb, bRc both hold. Then a – b and b – c are both divisible by 5.

Therefore a – c = (a – b) + (b – c) is divisible by 5.

Thus, aRb and bRc ⇒ aRc and therefore R is transitive.

Since R is reflexive, symmetric and transitive so, R is an equivalence relation on Z.

Inverse of a Relation

If (a,b) is an element present in a relation R then the inverse of the element is given by (b, a)

For illustration:

Find the inverse of the relation R={(2,1),(8,9),(3,3)}.

Ans.

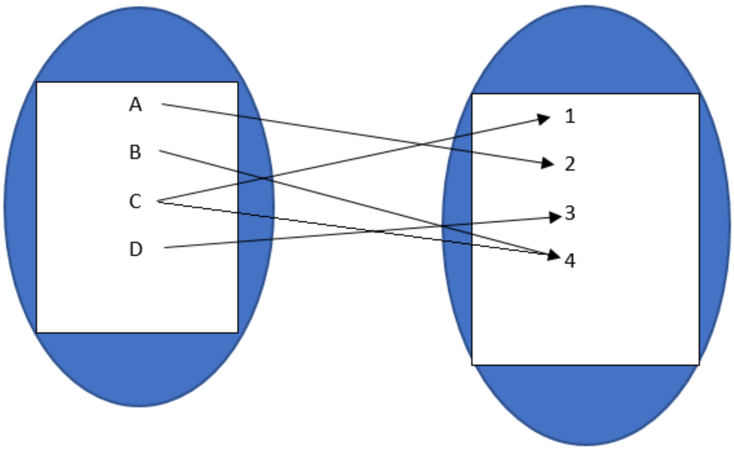
The inverse of the relation R= ={(1,2),(9,8),(3,3)}

Function

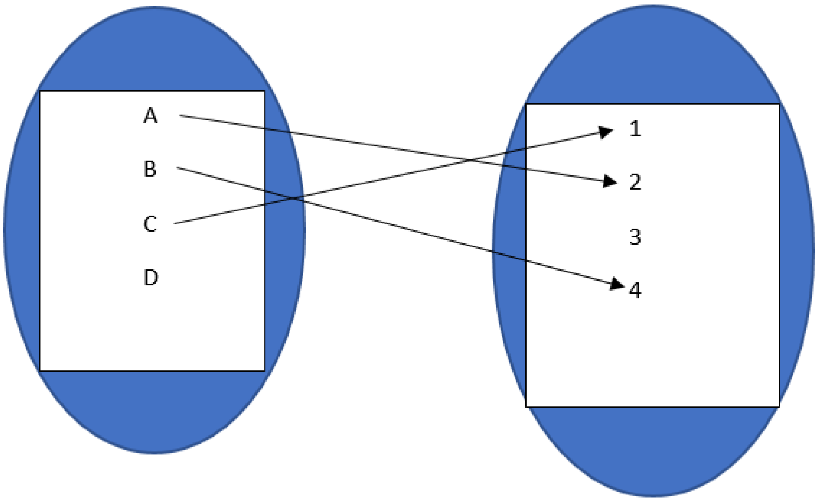
A function is defined as a relation in which every pre-image in the pre-image set must have one and only one image in the image set.

Examples of cases when relations are not functions

CASE 1: When one pre-image has multiple images

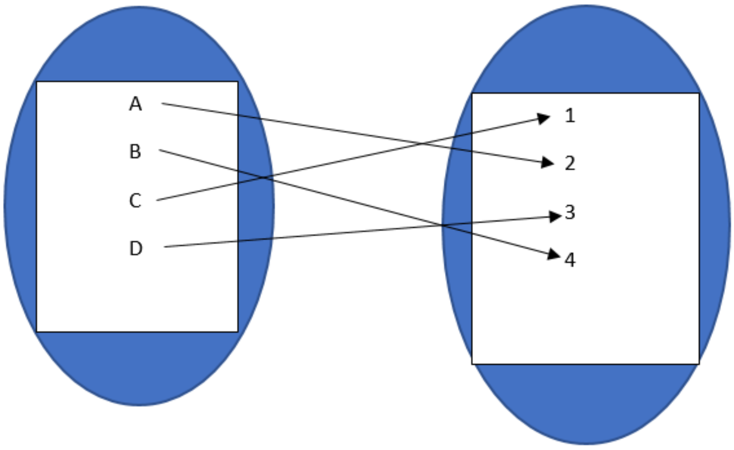


CASE 2: When all pre-image does not have an image

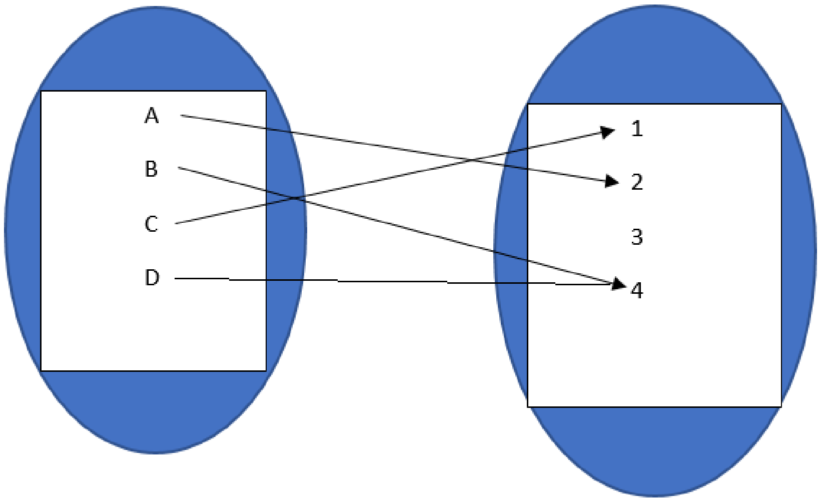


Types of Functions

One-One functions: A function is said to be a one-one function if each pre-image points to a unique image. As illustrated in the following diagram.



Many-one function: When many pre-images points to a single image it is called as a many-one function. The function can be depicted as

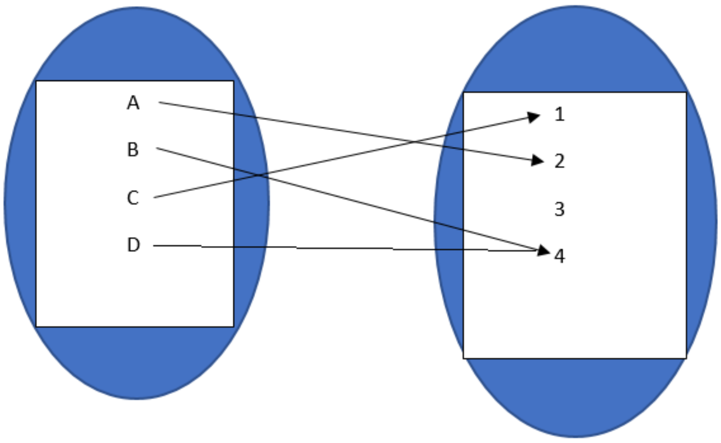


It is also known as an injective function.

Here C and D both have same image '4'

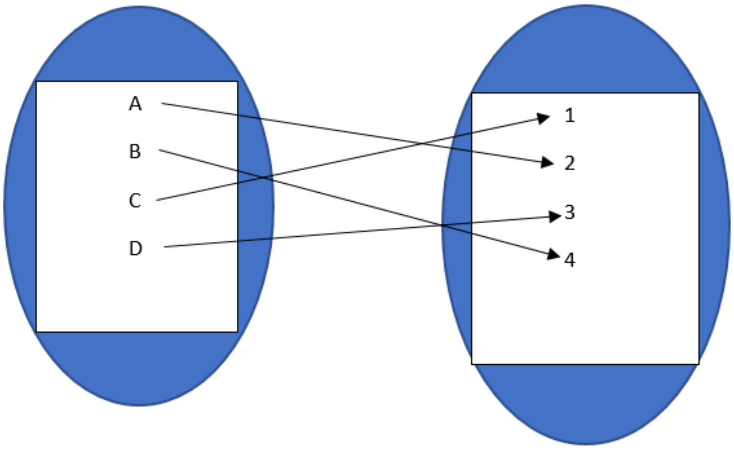
Into function: A function is said to be an into a function if at least one image in the image set has got no pre-image

It can be depicted as

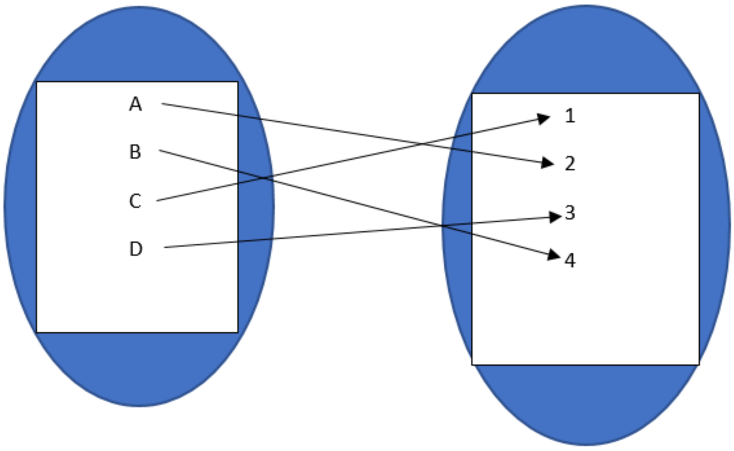


Here 3 doesn't have a pre-image.

Onto function: A function is said to be an onto function if all the images or elements in the image set has got a pre-image.



Bijective function: A function is said to be a bijective function if it is both a one-one function and an onto function.



NOTE: For the inverse of a function to exist, it must necessarily be a bijective function.

Inverse of a Function

The necessary condition for a function to have an inverse is that it must be a bijective function

NOTE: If f(x)=g(x)+h(x), if g(x) or h(x) or both are one-one function then f(x) is one-one function…(i)

If f(x)=g(x)+h(x), if and only iff both g(x) and h(x) is onto function then f(x) is onto function…. (ii)

5. Check whether inverse of the following function exists?

Ans.

The given function $f(x)=x^2 + x$

ONE-ONE checking

Definitely, on a set of real numbers for every value of x, we will get an f(x) so it supports the definition of being a function

……..

Now let us consider $f(x)=u(x)+v(x)$

$u(x)=x^2$

$v(x)=x^2$

Now u(x) is not one-one because using the general idea we can determine the value of x=2 and x=-2.

Hence, we need to check for $v(x)=x$

Now v(x) is definitely a one-one function

Thus, from postulate (i) we get f(x) is a one-one function.

……

ONTO function

Definitely u(x) is onto function. Also v(x) is an onto function. Thus, from postulate (ii) f(x) is an onto function.

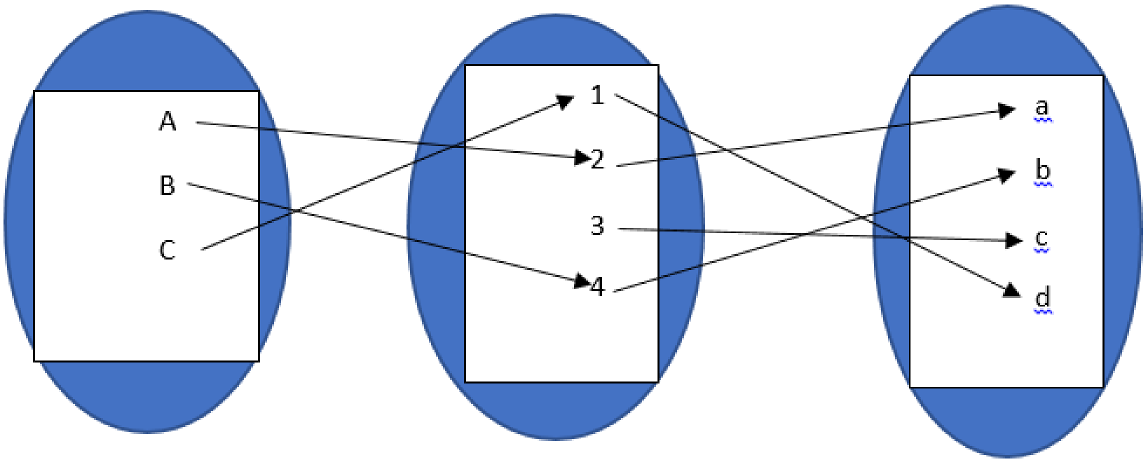
Thus, it satisfies the definition of bijective function hence it has got inverse function.

Composition of Functions

The composition of functions also called as a function of a function is defined as a function which depends on another function to obtain its pre-image set

For illustration Suppose f(g(x))= Then g(x) has an image set. The image set of g(x) becomes the pre-image set of f(g(x)).

Pictorial representation is as shown



The first set of relation diagram is f(g(x)) whereas the last 2 relation set gives g(x) only

NOTE: f(g(x))=fog(x)

6.For $f(x) = 2x + 3$ and $g(x) = -x^2 + 1$, find the composite function defined by (fog)(x) Soln:

Ans.

$(fog)(x) = f(g(x))$
$= 2\ (g(x)) + 3$
$= 2(\ -x^2 + 1\ ) + 3$
$= -\ 2x^2 + 5$

7. Given f(2) = 3, g(3) = 2, f(3) = 4 and g(2) = 5, evaluate (fog)(3)

Ans.

(fog)(3) = f(g(3))

(fog)(3) = f(2) = 3