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JEE Main Physics

Short Notes

Unit, Dimensions, and Errors in Measurement

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Unit, Dimensions, and Errors in Measurement is an important topic from JEE Main / JEE Advanced Exam Point of view. Some questions can be asked directly. Most importantly, the whole Physics includes this topic. Thus, it is very important to have a clear cut on this topic. This short notes on Unit, Dimensions and Errors in Measurement will help you in revising the topic before the JEE Main & IIT JEE Advanced Exam. You can also download Unit, Dimensions and Errors in Measurement notes PDF at end of the post.

Unit, Dimensions, and Errors in Measurement

- To measure or express a physical quantity we need a standard of measurement so that different measurements of one physical quantity can be related with respect to each other. This standard is called the **unit of the specific physical quantity.**
- To measure any physical quantity, we need two parts = Numerical value (n) × Unit (u)
- Numerical value gives how many times the physical quantity is measured with respect to the standard unit. The second part gives the name of the unit.
- **Base Units:** The units for the fundamental or base quantities are called fundamental or **base units**.
- **Derived Units:** The units of all other physical quantities can be expressed as combinations of the base units. Such units obtained for the derived quantities are called **derived units.**

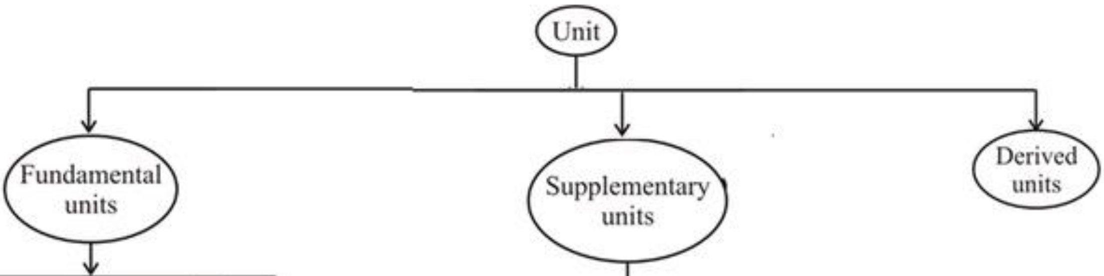
1. The International System of Units

Following are a few measurement systems that are used.

- **CGS System-** This system of the unit is based on the centimeter as the unit of length, gram as the unit of the mass, and second as the unit of the time.
- **FPS System-** This system of the unit is based on the foot as the unit of length, pound as the unit of the mass, and second as the unit of the time.
- **MKS System-** This system of the unit is based on the meter as the unit of length, kilogram as the unit of the mass, and second as the unit of the time.

The system of units which is at present internationally accepted for measurement is the (SI System - International System of Units). It is based on the MKS system.

SI Quantity and Units



Length	metre (m)
Mass	kilogram (kg)
Time	second (s)
Current	Ampere (A)
Temperataure	Kelvin (K)
Luminous Intensity	Candela (Cd)
Amount of Substance	Mole (mol)

Plane Angle	Radian (rad)
Solid Angle	Steradian (Sr)

2. Accuracy, Precision, and Errors in Measurement

- **Accuracy -** The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. It depends on the number of significant figures in it. The larger the significant digit the higher the accuracy.
- **Precision-** Precision is the degree of exactness. It depends on the least count of measuring instrument. The smaller the least count, the more precise will be measurement.

e.g. Suppose the exact (true) value of a certain mass is 45.2646 kg. Let it measure by an instrument as 45.2 by an instrument of least count 0.1 and 45.17 by other instruments of least count 0.01.

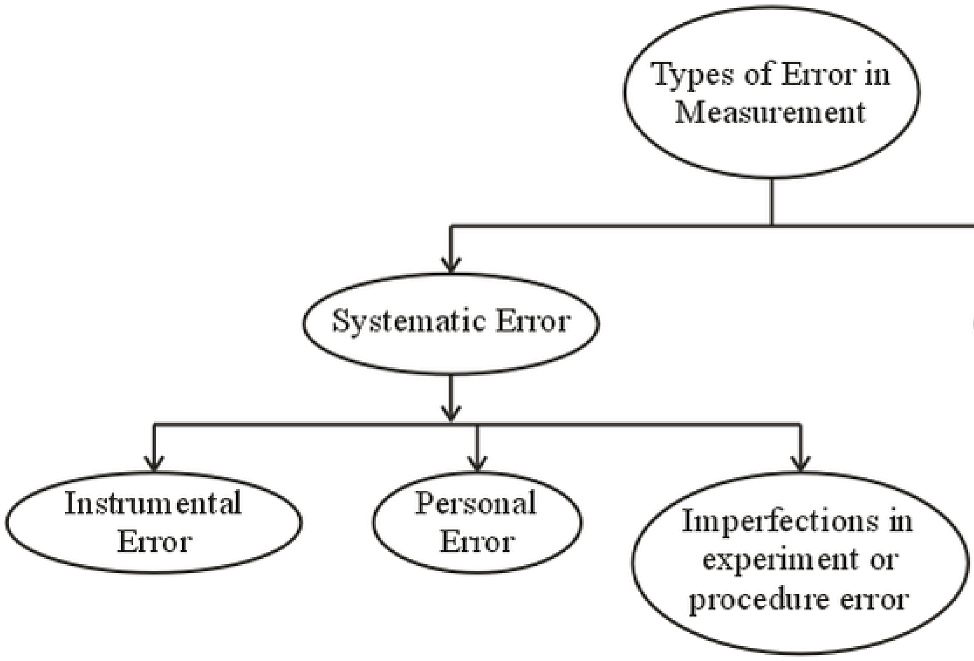
The first reading is more accurate because it is closer to the true value but less precise because its resolution is 0.1. The second reading is more precise because its resolution is 0.01 but less accurate.

- **Error**

The uncertainty in a measurement is called **error.** Every calculated quantity which is based on measured values also has an error.

Error in measurement can be broadly classified as (1) Systematic error and (2) Random error.

1. **Systematic Error:** Systematic error is caused due to the fault of the measuring device, design of the experiment, or imperfect method of observation. These errors can be reduced by improving experimental conditions, repeating measurement using a different method or different equipment.
2. **Random Error:** The random error is those errors, which occur due to random and unpredictable fluctuations in experimental conditions personal error by the observer. These errors can be reduced by conducting repeat trials, using precise apparatus.



Error in measuring the value of a physical quantity can be expressed by

- **Absolute error:** The magnitude of the difference between the measured and the true value of the quantity is called the absolute error of the measurement.

e.g. let us assume that true value or exact of a physical quantity is A_{True} and the measured value is $A_{mea.}$, then

the absolute error in Physical quantity A is = $|A_{True} - A_{mea.}|$

- **Relative error:** The ratio of absolute error to the true value in measuring a physical quantity is known as relative error.

e.g. let us assume that true value or exact of a physical quantity is A_{True} and the measured value is $A_{mea.}$, then

the relative error in Physical quantity A is = $\frac{|A_{true} - A_{mea}|}{A_{true}}$

- **Percentage error:** When the relative error is expressed in percent, it is called the percentage error.

e.g. let us assume that true value or exact of a physical quantity is A_{True} and the measured value is $A_{mea.}$, then the percentage error in Physical quantity A is = $\left(\frac{|A_{true}-A_{mea}|}{A_{true}}\right)\times 100\%$

Rules of arithmetic operation for error

- **In case of Addition and Subtraction**

Suppose a physical quantity(Z) depends on the other quantity A and B, Z = A ± B

Error in quantity A is ΔA and in B is ΔB then for both addition and subtraction the absolute error is added up.

Absolute error in Z is ΔZ = ΔA + ΔB, And

Percentage error in the value of Z is $\left(\frac{\Delta A+\Delta B}{A\pm B}\right)\times 100\%$

- **In case of Multiplication and Divisions**

Suppose a physical quantity(Z) depends on the other quantity A and B such a way that

Z = AB or $Z=\frac{A}{B}$

Then for both multiplication and division, the percentage error or relative are added up. Then

Relative error in the value of Z is $\left(\frac{\Delta Z}{Z}\right)=\left(\frac{\Delta A}{A}\right)+\left(\frac{\Delta B}{B}\right)$

Percentage error in the value of Z is $\left(\frac{\Delta Z}{Z}\times 100\%\right)=\left(\frac{\Delta A}{A}\times 100\%\right)+\left(\frac{\Delta B}{B}\times 100\%\right)$

- **In the case of the Power function**

Suppose a physical quantity(Z) depends on the other quantity A, C, and B such a way that $Z=\frac{A^pC^r}{B^q}$

In the condition of measuring error, the power of each physical quantity multiplied by respectively. Then

Relative error in value of Z is $\left(\frac{\Delta Z}{Z}\right)=p\left(\frac{\Delta A}{A}\right)+q\left(\frac{\Delta B}{B}\right)+r\left(\frac{\Delta C}{C}\right)$

Percentage error in value of Z is

$$\left(\frac{\Delta Z}{Z}\times100\%\right)=p\left(\frac{\Delta A}{A}\times100\%\right)+q\left(\frac{\Delta B}{B}\times100\%\right)+r\left(\frac{\Delta C}{C}\times100\%\right)$$

3. Significant Figures

Significant figures indicate the precision of measurement which depends on the least count of the measuring instrument.

Rules of Significant figures

1. All the non-zero digit are significant.
2. All the zeros between two non-zero digit are significant, no matter where the decimal point is.
3. If the number is less than 1, the zeroes on the right of decimal point but to the left of the first non-zero digit are non-significant. e.g. in 0.00532, zero before digit 5 is non-significant
4. The terminal zeros in a number without a decimal point are not significant.

e.g. 45200 cm = 452 m has three significant figures.

5. The trailing zeros in a number with a decimal point are significant. e.g. 54.500 has five significant figures.

Rules for Arithmetic Operations with Significant Figures

- **In case of Addition and subtraction**

In addition or subtraction, the result should retain as many decimal places as are there in the number with the least decimal places.

e.g. 656.34 m + 73.2463 m + 624.14 m = 1353.726 m, therefore the result be rounded off to 1353.73 m

- **In case of Multiplication and Division**

In multiplication or division, the result should retain as many decimal places as are there in the original number with the least significant figures.r4e

e.g. Force = 13.55 kg × 12.563 m/s^2 = 170.22865 N, therefore the result be rounded off to 170.23 N

Rounding off the Uncertain Digits

- Preceding digit is raised by 1 if it the insignificant digit to be dropped is more than 5.

e.g. 55.686 is rounded off to 55.69

- Preceding digit is unchanged if it the insignificant digit is less than 5.

e.g. 55.681 is rounded off to 55.68

- If the insignificant digit is 5 and the preceding digit is even, the insignificant digit is dropped and if it is odd, the preceding digit is raised by 1.

e.g. 55.685 is rounded off to 55.68

e.g. 55.675 is rounded off to 55.68

4. Dimension of Physical Quantities

The dimension of a physical quantity can be defined as the powers to which the base quantities are raised to represent that quantity. All the physical quantities can be expressed in terms of base quantities.

Let's take an example:

Consider a physical quantity - density

$$\text{Density}(\rho) = \frac{\text{Mass(M)}}{\text{Volume(V)}}$$

$$\text{Density}(\rho) = \frac{\text{Mass(M)}}{[\text{Length(L)}]^3}$$

$$\text{Density}(\rho) = \text{Mass(M)} \times [\text{Length(L)}]^{-3}$$

So, the dimension of density is 1 in mass, and -3 in Length. Thus,

Dimensional formula of the density = $[ML^{-3}]$

5. Dimensional Analysis and its Applications

- **To convert a physical quantity from one system of unit to another**

For any system of unit, Numerical value(n)×unit (u) = constant. So, on changing unit, numerical value will also get changed.

Let n_1u_1 is the value of the physical quantity in one system of unit and n_2u_2 is the value in other systems then,

$$n_1u_1 = n_2u_2$$

$$n_2 = n_1\frac{[u_1]}{[u_2]}$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

Using the above method, we can convert a physical quantity from one system of unit to another.

- **To check the dimensional correctness of physical equation.**

It is based on the principle of homogeneity, which states that a given physical equation is dimensionally correct if the dimension of the various terms on either side of the equation is the same. But if the dimension of either side of the equation is not same then the physical equation is wrong.

e.g. Let us consider an equation, $\frac{1}{2}mx^2 = mgt$, where m is mass, is distance, g is gravity, and t is the time.

The dimension of LHS and RHS are

$[M][L^2] = [M][LT^{-2}][T]$

$[ML^2] = [MLT^{-1}]$

Since both side dimension unequal, so the given equation is dimensionally incorrect.

- **To establish a relationship between different physical quantities**

If we know the dependency of the physical quantity on other quantities, then we can find the relation among different quantity by using the principle of homogeneity.

e.g. Let us consider a physical quantity time t, it depends on length l, mass m, and gravity g. Then we can find the relation of time among other quantity

let time depends on length, mass, and gravity as the power of a, b, and c respectively. Then

$$t \propto l^a m^b g^c$$

$$t = kl^a m^b g^c \quad ....(1) \qquad \text{(k is a proportionality constant)}$$

Equating the dimension of both sides

$$[T] = [L^a][M^b][L^c T^{-2c}]$$

$$[M^o L^o T] = [M^b L^{a+c} T^{-2c}]$$

Comparing dimensions of both side

$$b = 0, c = -\frac{1}{2}, a = \frac{1}{2}$$

Substituting the value of a, b, and c in equation (1)

$$t = k\sqrt{\frac{l}{g}}$$

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