

**Gravitation** is an important topic for **CDS**, **AFCAT**, **Air Force Group X & Y Exam**. Every year there are 1-2 questions asked from this topic. It is very interesting and easy topic, therefore, one can score good marks from this topic.

# **Physics: Important Notes on Gravitation**

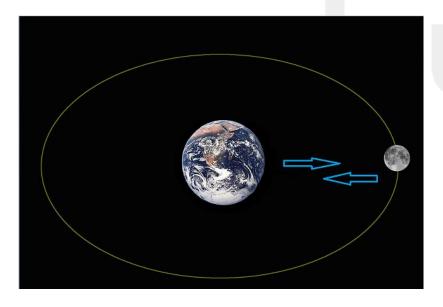
#### The Universal Law of Gravitation and Gravitational Constant

In the universe, everybody attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. This attraction force is known as the gravitational force.

$$\vec{F} \propto \frac{M_1 M_2}{R^2}$$

$$\vec{F} = \frac{GM_1M_2}{R^2}$$

where  $M_1$  and  $M_2$  are the masses of two bodies, R is the distance between and G is the Gravitational Constant.  $G = 6.67 \times 10^{-11} \text{ Nm}^2 (\text{kg})^{-2}$ 



- 1. Gravitational force is always attractive in nature.
- 2. Gravitational force is independent of the nature of the intervening medium.
- 3. Gravitational force is conservative in nature.
- 4. It is a central force so it acts along the line joining the center of the two interacting bodies and it obeys inverse square law.

Acceleration due to Gravity of the Earth and its Variation



The gravitational pull exerted by the earth is called gravity. The acceleration produced in a body due to the force of gravity is called as acceleration due to gravity(g).

$$\mathbf{g} = \frac{GM_e}{R_e^2}$$
 , where  $M_e$  is the mass of earth, and  $R_e$  is the radius of the earth.

Let the density of the earth is  $\rho$ . Then the acceleration due to gravity of the earth is

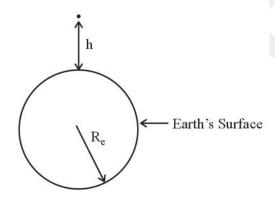
$$g = \frac{GM_e}{R_e^2} = \frac{G\frac{4}{3}\pi R_e^3 \rho}{R_e^2} = \frac{4}{3}\pi GR_e \rho$$

## Variation of Acceleration due to gravity

#### Due to altitude (h)

$$g_h = g \left( 1 - \frac{2h}{R_e} \right)$$

The acceleration due to gravity at a height h above the earth's surface is



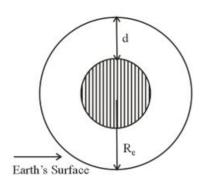
Thus the value of acceleration due to gravity decreases with the increase in height h.

#### Due to depth (d)

$$g_{d} = g \left( 1 - \frac{d}{R_{e}} \right)$$

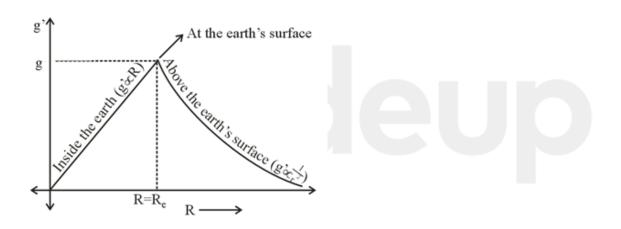
The acceleration due to gravity at depth d below the earth's surface is





Thus the value of acceleration due to gravity decreases with the increase in depth d and becomes zero at the center of the earth.

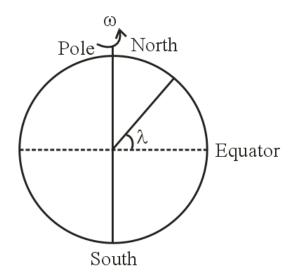
Variation of acceleration due to gravity (g') with distance from the center of the earth (R) is as shown below



#### Due to rotation of the earth about its axis

The acceleration due to gravity at latitude  $\lambda$  is  $\mathbf{g}_{\lambda} = \mathbf{g} - \mathbf{R}_{e} \omega^{2} \cos^{2} \lambda$  where  $\omega$  is the angular speed of rotation of the earth about its axis.





At the equator,  $\lambda$  = 0°, value acceleration due to gravity  $g_{\text{equator}} = g - R_{\text{e}} \omega^2$ 

At the pole,  $\lambda = 90^{\circ}$ , value acceleration due to gravity  $g_{pole} = g$ 

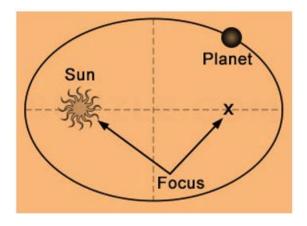
Thus, the value of acceleration due to gravity increases from the equator to the pole due to the rotation of the earth.

If the earth stops rotating about its axis,  $\omega = 0$ , the value of g will increase everywhere, except at the poles. But if there is increase in the angular speed of the earth, then except at the poles the value of g will decrease at all places.

## **Kepler's Laws of Planetary Motion**

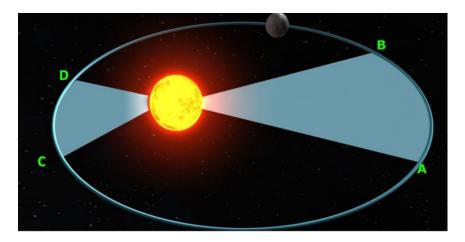
To explain the motion of planets, Kepler formulated the following three laws.

**1.** Law of Orbits (First Law): The planets in the solar system revolve in elliptical orbits around the Sun in elliptical orbits with the Sun located at any one of the foci of the elliptical path set by the respective planet.





**2. Law of areas (Second Law):** The rate of the area swept by the position vector of the revolving planet with respect to the Sun per unit time remains same irrespective of the position of the planets on the set elliptical path. Kepler's second law follows the law of conservation of angular momentum.



According to the Kepler's second law, the areal velocity of the planet is constant.

$$\frac{d\overline{A}}{dt} = constant$$
 that means the planet is closer to the sun on the elliptical path, it moves faster, thus covering more path-area in the given time.

**3.** Law of periods (Third Law): The square of the period of revolution around the sun of a planet is proportional to the cube of the semimajor axis of its orbit-path around the sun.

$$T^2 \propto R^3$$

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

## **Gravitational Field and Potential Energy**

**Gravitational Field (E)** - It is the space around a material body in which its gravitational pull can be experienced by other bodies. The intensity of the gravitational field at a point due to a

$$E = \frac{GM}{r^2}$$

body of mass M, at a distance r from the center of the body, is

**Gravitational Potential (V)** - Gravitational Potential at a point in the gravitational field of a body is defined as the amount of work done in bringing body of unit mass from infinity to

$$V = -\frac{GM}{r}$$

that point.

$$E = -\frac{dV}{dr}$$

Gravitational potential (V) is related with gravitational Field (E) as



**Gravitational Potential Energy-** The Gravitational potential energy of a body at a point in a gravitational field of another body is defined as the amount of work done in bringing the given body from infinity to that point.

The gravitational potential energy of mass m in the gravitational field of mass M at a distance

$$U = -rac{GMm}{r}$$
r from it is

## Satellite and its Velocity

Satellite is a natural or artificial body describing the orbit around a planet under its gravitational attraction.

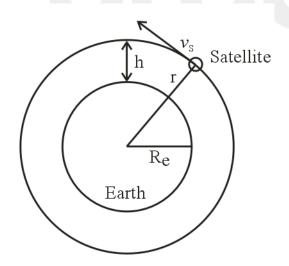
**Escape velocity-** The velocity of the object needed for it to escape the earth's gravitational pull is known as the escape velocity of the earth. The escape velocity of the earth

$$v_e = \sqrt{\frac{2GM}{R}}$$

**Orbital velocity-** The orbital velocity of the satellite revolving around the earth at a height h

$$v_{o} = \sqrt{\frac{2GM_{e}}{r}} = \sqrt{\frac{GM_{e}}{R_{e} + h}} = R_{e}\sqrt{\frac{g}{R_{e} + h}}$$

is



When the satellite is orbiting close to the earth's surface,  $h \ll R_e$ , then the orbital velocity of

$$v_o = \sqrt{gR_e}$$

the satellite is,

For a point close to earth's surface the escape velocity and orbital velocity related

$$v_e = \sqrt{2}v_o$$



#### Time period of a satellite

Time period is the time taken by satellite to complete one revolution around the earth,

$$T = \frac{2\pi r}{v_{o}} = 2\pi \sqrt{\frac{r^{3}}{GM_{e}}} = 2\pi \sqrt{\frac{\left(R_{e} + h\right)^{3}}{GM_{e}}} = \frac{2\pi}{R_{e}} \sqrt{\frac{\left(R_{e} + h\right)^{3}}{g}}$$

$$T = 2\pi \sqrt{\frac{R_e}{g}}$$

When the satellite is orbiting close to the earth's surface, h << R<sub>e</sub>, then

#### Energy of an orbiting satellite

$$K = \frac{1}{2}mv_o^2 = \frac{1}{2}\frac{GM_em}{r} = \frac{1}{2}\frac{GM_em}{(R_e + h)}$$

The kinetic energy of a satellite is,

$$U = -\frac{GM_em}{r} = -\frac{GM_em}{(R_e + h)}$$

The potential energy of a satellite is,

$$E = K + U = -\frac{GM_{e}m}{2r} = -\frac{GM_{e}m}{2(R_{e} + h)}$$

The total energy of the satellite is,

**Geostationary Satellite-** A satellite which revolves around the earth in its equatorial plane with the same angular speed and in the same direction as the earth rotates about its own axis is called a geostationary satellite.

- 1. They have a fixed height of 3600 km from the earth's surface.
- 2. They revolve in an orbit oriented in the equatorial plane of the earth.
- 3. They have their rotation same as that of earth about its own axis i.e., from west to east.
- 4. The period of revolution around the earth is the same as that of the earth about its own axis.

**Polar satellite-** A satellite revolves in a polar orbit is called a polar satellite.



- 1. These satellites have their orbit such that they pass the north and the south pole once every 24 hours. They revolve around the earth along the meridian lines.
- 2. They are situated at an altitude much lower than the geostationary satellites(850 km).
- 3. They are therefore capable of providing more detailed info about the clouds and storms.

**Weightlessness-** When the body is unsupported and no force is working on your body, it is the experience of weightlessness.

When an object is in free fall with an acceleration equal to the acceleration of earth, the object is said to be weightless as there is no force acting on it.

