##  Kinematics

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Kinematics is an important topic from JEE Main / JEE Advanced Exam Point of view. Some questions can be asked directly. Most importantly, the whole Physics includes this topic. Thus, it is very important to have a clear cut on this topic. This short notes on kinematics will help you in revising the topic before the JEE Main \& IIT JEE Advanced Exam. You can also download Kinematics notes PDF at end of the post.

## Kinematics

## 1. Frame of Reference

Frame of reference - Frame of reference is the set of axes which is used to specify the position of the object in a space. The set of axes is rectangular coordinate system which consist three mutually perpendicular axis $X, Y$, and $Z$. The point of intersection of these axes is known as the origin or reference point.


## 2. Motion in a Straight Line

Position- Position of any object is essential to describe the motion of the object. The position of object is the set of axes from a reference point.
e.g. In above image the position of point A from the reference point is, $\vec{r}=x \widehat{i}+y \widehat{j}+z \widehat{k}$

Motion- An object is said to be in motion if it changes its position with time, with respect to its surroundings.
Motion of the object can be represented by the position-time graph. The position-time graph helps to analyze the motion of an object.

## - Uniform Motion

If an object moving along the straight line covers equal distances in equal interval of time this type of motion is known as uniform motion.


## - Non- Uniform Motion

If an object covers unequal distances in equal interval or equal distance in unequal time interval this type of motion is known as non-uniform motion.


Distance- The length of the actual path between initial and terminal position of a particle in an interval of time is called distance covered by the particle. Distance is also known as the path length.

1. Distance is a scalar quantity.
2. It never reduces with time.
3. Distance of the object can't be negative.
4. SI unit of distance is metre (m).
5. Dimension of the distance is $\left[\mathrm{M}^{\circ} \mathrm{L}^{1} \mathrm{~T}^{\circ}\right]$

Distance time graph- The gradient of distance time graph represents the speed of the object.
Displacement- The difference between the final and initial position is called displacement.

1. Displacement is a vector quantity.
2. Displacement of the object is changes with time.
3. Displacement of the object can be negative, positive or zero.
4. SI unit of displacement is metre (m).
5. Dimension of the distance is [ $\mathrm{M}^{\circ} \mathrm{L}^{1} \mathrm{~T}^{\circ}$ ]

Displacement time graph- The gradient of displacement time graph represents the velocity.
Speed- Speed of an object is the ratio of distance travelled by the object to the time taken.

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Speed $=\frac{\text { Distance travelled }}{\text { Time taken }}$

- Speed is scalar quantity.
- SI unit of speed is $\mathrm{m} / \mathrm{s}$.
- Dimension of the speed is $\left[\mathrm{M}^{\circ} \mathrm{LT}^{-1}\right]$
- Speed of an object can't be negative.

Types of Speed


- Uniform speed- An object is said to be moving with a uniform speed, if it covers equal distance in equal intervals of time.
- Non-Uniform speed- An object is said to be non-uniform speed if it covers equal distance in unequal time interval or unequal distance in equal time interval.
- Average speed- The ratio of total path length travelled divided by the total time interval during the motion is known as the average speed of the object.

Average speed $=\frac{x_{1}+x_{2}+x_{3}+\ldots}{t_{1}+t_{2}+t_{3}+\ldots .}=\frac{\sum_{i=1}^{n} x_{i}}{\sum_{i=1}^{n} t_{i}}$

- Instantaneous speed- The speed of the body at any instant of time or at a position is called instantaneous speed.

Instantaneous speed $=\lim _{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}=\frac{d x}{d t}$
Velocity- Velocity of an object is the ratio of displacement to the total time taken by object.
Velocity $=\frac{\text { Displacement }}{\text { Time }}$

- Velocity is vector quantity.
- SI unit of velocity is $\mathrm{m} / \mathrm{s}$.
- Dimension of the velocity is [ $\mathrm{M}^{\circ} \mathrm{LT}{ }^{-1}$ ]
- Velocity of an object can be zero, negative, or positive.


## Types of Velocity



- Uniform velocity- An object is said to be moving with a uniform velocity, if it covers equal distance in equal intervals of time.
- Non-Uniform velocity- An object is said to be non-uniform velocity if it covers equal distance in unequal time interval or unequal distance in equal time interval.
- Average velocity- The ratio of total path length travelled divided by the total time interval during the motion is known as the average velocity of the object.

Average velocity $=\frac{x_{1}+x_{2}+x_{3}+\ldots}{t_{1}+t_{2}+t_{3}+\ldots .}=\frac{\sum_{i=1}^{n} x_{i}}{\sum_{i=1}^{n} t_{i}}$

- Instantaneous velocity- The velocity of the body at any instant of time or at a position is called instantaneous velocity.

Instantaneous velocity $=\lim _{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}=\frac{d x}{d t}$

Acceleration- The rate of change in velocity of an object is known as the acceleration of the object.
Acceleration $a=\frac{v_{2}-v_{1}}{t_{2}-t_{1}}=\frac{\Delta v}{\Delta t}$

- Acceleration is vector quantity.
- SI unit of acceleration is $\mathrm{m} / \mathrm{s}^{2}$.
- Dimension of the acceleration is [M0 $\mathrm{LT}^{-2}$ ]
- Acceleration of an object can be zero, negative, or positive.


## Types of Acceleration



- Uniform Acceleration- A body is said to have uniform acceleration if magnitude and direction of the acceleration both remains constant during motion.
- Non-Uniform Acceleration- A body is said to have non-uniform acceleration if magnitude and direction of the acceleration both change during motion.



## Equation of motion for a uniformly accelerated motion

- $v=u+a t$, where v is the final velocity, u is initial velocity, a is the acceleration and t is the time taken during the motion.
- $v^{2}=u^{2}+2 a s$, where v is the final velocity, u is initial velocity, a is the acceleration and s is the distance travelled by object during the motion.
- $s=u t+\frac{1}{2} a t^{2}, \mathrm{u}$ is initial velocity, a is the acceleration, t is the time taken and s is the distance travelled by object during the motion.
- $s_{n}=u+\frac{a}{2}(2 n-1), \mathrm{u}$ is initial velocity, a is the acceleration, $s_{n}$ is the distance covered by object in nth second.


## Equation of motion for a free-falling body under gravity

- $v=u+g t$, where v is the final velocity, u is initial velocity, g is the acceleration due to gravity and t is the time taken during the motion.
- $v^{2}=u^{2}+2 g h$, where v is the final velocity, u is initial velocity, g is the acceleration due to gravity and h is the height covered by object.
- $h=u t+\frac{1}{2} g t^{2}, \mathrm{u}$ is initial velocity, g is the acceleration due to gravity, t is the time taken and h is the height covered by object.
- $h_{n}=u+\frac{g}{2}(2 n-1), \mathrm{u}$ is initial velocity, g is the acceleration due to gravity, $h_{n}$ is the height covered by object in nth second.


## Relative Velocity

Consider two object X and Y are moving uniformly with velocities $v_{X}$ and $v_{Y}$ in one dimension.
Velocity of object $Y$ relative to object $X$ is, $\left|v_{Y X}\right|=v_{Y}-v_{X}$
Velocity of object X relative to object Y is, $\left|v_{X Y}\right|=v_{X}-v_{Y}$

## 3.Basic Concept of Vectors

Any Physical quantity is classified as Vector or Scalar.

- Scalar Quantity- Any Physical quantity which can't associated with direction but has magnitude is known as scalar quantity.
- Vector Quantity- Any Physical quantity which has both a direction and a magnitude and obeys triangle law of addition or parallelogram law of addition, is known as vector quantity. A vector quantity is represented in bold or draw an arrow on it.
e.g. A is a vector quantity then A will be represented as $\overrightarrow{\mathrm{A}}$ or $\mathbf{A}$.


## Types of Vector

- Unit Vector- Unit vector is a vector that has unit magnitude and points in a particular direction. Unit vector along the $x, y$, and $z$ axes of a rectangular coordinate system denoted by $\hat{i}, \hat{j}$, and $\hat{\mathrm{k}}$ respectively.


- Zero Vector or null vector- Unit vector is a vector that has zero magnitude. It denoted as $\hat{0}$.
- Equal Vector- If two vector $A$ and $B$ have same direction and magnitude then they are equal vector $\vec{A}=\vec{B}$.
- Collinear Vector- Collinear vector are two or more vector which are parallel to the same line irrespective of their magnitude and direction.


## Algebra of Vectors

- Addition- Let two vectors $\vec{A}$ and $\vec{B}$ to be added. To get the resultant vector the tail of $\vec{B}$ coincide with the head of $\vec{A}$. The vector joining the tail of $\vec{A}$ with the head of $\vec{B}$ is the vector sum of $\vec{A}$ and $\vec{B}$.


Magnitude of the resultant is, $\mathbf{R}=\sqrt{A^{2}+B^{2}+2 A B \cos \theta}, \theta$ is the angle between vector $\vec{A}$ and $\vec{B}$.
The vector addition is commutative, $\vec{A}+\vec{B}=\vec{B}+\vec{A}$

- Subtraction- Let two vectors $\vec{A}$ and $\vec{B}$ to be subtracted. Let $\theta$ is the angle between vector $\vec{A}$ and $\vec{B}$.

To subtract $\vec{B}$ from $\vec{A}$, invert the direction of $\vec{B}$ and add to $\vec{A}$.


Magnitude of the resultant is,
$\mathbf{R}=\sqrt{A^{2}+B^{2}+2 A B \cos (\pi-\theta)}$
$\mathbf{R}=\sqrt{A^{2}+B^{2}-2 A B \sin (\theta)}$
If we multiply a vector $\vec{A}$ with a positive number X, it gives a vector whose magnitude is changed by the factor X but the direction is the same as that of $\vec{A}$.
$|\mathrm{XA}|=\mathrm{X}|\mathbf{A}|$, if $\mathrm{X}>0$

## Resolution of vectors

If the vector is not in the $X-Y$ plane, it may have non-zero projections along $X$, and $Y$ axes and we can resolve it into parts

$\vec{A}=A_{x} \cos \theta+A_{y} \sin \theta$
Magnitude of vector is $\mathbf{A}=\sqrt{A_{x}^{2}+A_{y}^{2}}$
Angle between the vector is, $\tan \theta=\frac{A_{x}}{A_{y}}$
Similarly, we can resolve a vector into three components along $X, Y$, and $Z$.

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$\vec{A}=A \cos \alpha+A \cos \beta+A \cos \gamma$


Magnitude of Vector is, $\quad \mathbf{A}=\sqrt{A_{x}^{2}+A_{y}^{2}+A_{z}^{2}}$

## Scalar and Vector Product of Vector

The multiplication of vector is two type one is scalar and other is vector.

## - Scalar Product

The scalar product or dot product of two vectors $\vec{A}$ and $\vec{B}$ is not a vector, but a scalar quantity. Let the vector $A$ is, $\vec{A}=A_{x} \widehat{i}+A_{y} \hat{j}+A_{z} \widehat{k}$ and Vector $B$ is, $\vec{B}=B_{x} \widehat{i}+B_{y} \widehat{j}+B_{z} \widehat{k}$


The scalar or dot product of the vector is
$\overrightarrow{\mathrm{A}} \cdot \overrightarrow{\mathrm{B}}=\left(\mathrm{A}_{\mathrm{x}} \mathrm{B}_{\mathrm{x}}+\mathrm{A}_{\mathrm{y}} \mathrm{B}_{\mathrm{y}}+\mathrm{A}_{\mathrm{z}} \mathrm{B}_{\mathrm{z}}\right)$ or $|\overrightarrow{\mathrm{A}} \cdot \overrightarrow{\mathrm{B}}|=\mathbf{A B} \cos \theta$, where $\theta$ is the angle between vector $\vec{A}$ and $\vec{B}$

## - Vector Product



The vector product or cross product of two vectors $\vec{A}$ and $\vec{B}$ is not a vector, but a vector quantity. Let the vector $A$ is, $\vec{A}=A_{x} \hat{i}+A_{y} \hat{j}+A_{z} \widehat{k}$ and Vector $B$ is, $\vec{B}=B_{x} \hat{i}+B_{y} \hat{j}+B_{z} \widehat{k}$

Then the vector product is, $|\overrightarrow{\mathrm{A}} \times \overrightarrow{\mathrm{B}}|=\mathbf{A B} \sin \theta \widehat{\mathrm{n}}, \theta$ is the angle between vector $\vec{A}$ and $\vec{B}, \widehat{\mathrm{n}}$ at right angles to both $\vec{A}$ and $\vec{B}$.


The direction of resultant vector $\overrightarrow{\mathrm{A}} \times \overrightarrow{\mathrm{B}}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.

## 4. Motion in a Plane

## Motion in a Plane with Constant Acceleration

If an object is moving in a two-dimensional plane, then we can treat two separate simultaneous onedimensional motion with constant acceleration along two perpendicular direction.

Consider an object is moving in a two-dimensional plane with velocity $\vec{v}$ and acceleration $\vec{a}$.


Velocity of the object, $\vec{v}=v_{x} \widehat{\mathrm{i}}+v_{y} \widehat{\mathrm{j}}$
Acceleration of the object, $\vec{a}=a_{x} \widehat{\mathrm{i}}+a_{y} \widehat{\mathrm{j}}$

## x - axis Motion

$v_{x}=u_{x}+a t$
$s_{x}=u_{x} t+\frac{1}{2} a t^{2}$
$v_{x}^{2}=u_{x}^{2}+2 a s_{x}$

## y-axis Motion

$$
\begin{aligned}
v_{y} & =u_{y}+a t \\
s_{y} & =u_{y} t+\frac{1}{2} a t^{2} \\
v_{y}^{2} & =u_{y}^{2}+2 a s_{y}
\end{aligned}
$$

## Relative Velocity in two dimensions

Suppose that two objects P and Q are moving uniformly with velocities $v_{P}$ and $v_{Q}$ in two-dimensional ( $\mathrm{x}-\mathrm{y}$ ) plane. Their velocity $v_{P}=v_{P x} \widehat{\mathrm{i}}+v_{P y} \hat{\mathrm{j}}$ and $v_{Q}=v_{Q x} \hat{\mathrm{i}}+v_{Q y} \widehat{\mathrm{j}}$.

Velocity of object Q relative to object P is,
$\left|v_{Q P}\right|=v_{Q}-v_{P}$
$\left|v_{Q P}\right|=\left(v_{Q x} \hat{\mathrm{i}}+v_{Q y} \hat{\mathrm{j}}\right)-\left(v_{P x} \hat{\mathrm{i}}+v_{P y} \hat{\mathrm{j}}\right)$
$\left|v_{Q P}\right|=\left(v_{Q x}-v_{P x}\right) \hat{\mathrm{i}}+\left(v_{Q y}-v_{P y}\right) \hat{\mathrm{j}}$
Velocity of object P relative to object Q is,
$\left|v_{P Q}\right|=v_{P}-v_{Q}$
$\left|v_{P Q}\right|=\left(v_{P x} \hat{\mathrm{i}}+v_{P y} \hat{\mathrm{j}}\right)-\left(v_{Q x} \hat{\mathrm{i}}+v_{Q y} \hat{\mathrm{j}}\right)$
$\left|v_{P Q}\right|=\left(v_{P x}-v_{Q x}\right) \hat{\mathrm{i}}+\left(v_{P y}-v_{Q y}\right) \hat{\mathrm{j}}$

## Projectile Motion

Projectile motion is a motion in which object is moved in a parabolic path. The motion of the object is the result of two separate components of motions. One component is along a horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to the force of gravity.


Horizontal Velocity of the particle is, $u_{x}=u \cos \theta$

Vertical Velocity of the particle is, $u_{y}=u \sin \theta$

Equation of trajectory is, $y=x \tan \theta-\frac{1}{2} g \frac{x^{2}}{u^{2}} \sec ^{2} \theta$
Range of the particle is, $R=\frac{u^{2} \sin 2 \theta}{g}$
Time of flight is, $T=\frac{2 u \sin \theta}{g}$
Height of the projectile is, $H=\frac{u^{2} \sin ^{2} \theta}{2 g}$

## Projectile motion on an Inclined plane

Let us assume that a particle is projected from an incline plane which is incline at an angle $\phi$ to the horizon.
Particle is moving with a velocity u at angle of elevation $\theta$.


Range of the projectile is, $R=\frac{u^{2}}{g \cos ^{2} \phi}[\sin (2 \theta-\phi)-\sin \phi]$
Time of flight is, $T=\frac{2 u \sin (\theta-\phi)}{g \cos \phi}$
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