Mechanical JE 2019

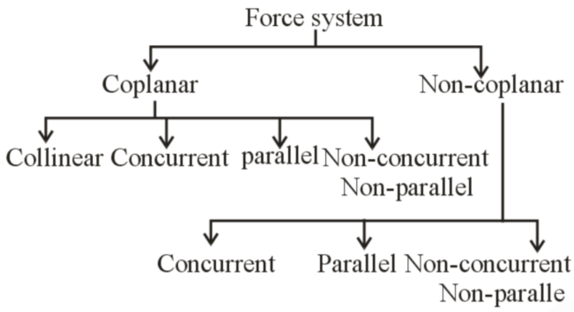
Mechanics

Important Formulas

ENGINEERING MECHANICS IMPORTANT FORMULA

FORCE

1. SYSTEM OF FORCES



1.1. Coplanar forces

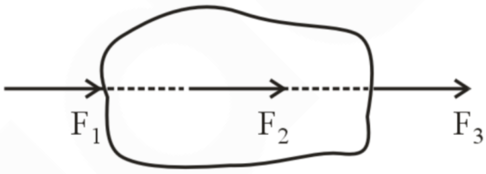
Forces whose lines of action lies on the same plane

1.2. Non-coplanar forces

Forces whose lines of action do not lie on the same plane.

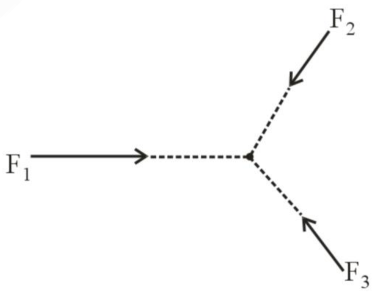
1.3. Collinear forces

Forces whose lines of action lie on the same line.



1.4. Concurrent forces

Forces, whose lines of action meet at one point They may or may not be collinear & coplanar



1.5. Parallel forces

Forces, whose lines of action are parallel to each other. They may or may not be coplanar.

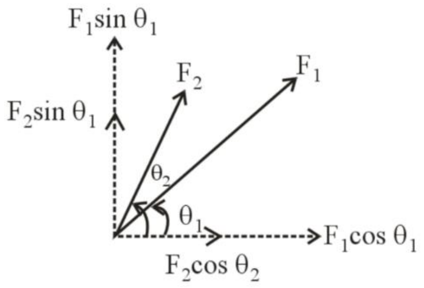
1.6. Non-concurrent & Non-parallel forces

Forces, whose lines of action do not meet or tend to meet at same point. They are also not parallel to each other.

They may or may not be coplanar.

2. RESOLUTION OF FORCES

The splitting up the given force into number of components, without changing its effect on the body is called resolution of a force.



$\Sigma H = F_1 \cos\theta_1 + F_2 \cos\theta_2$

$\Sigma V = F_1 \sin\theta_1 + F_2 \sin\theta_2$

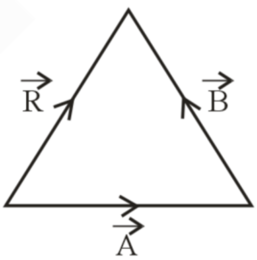
Resultant force = $\sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

3. LAWS OF RESULTANT FORCE

3.1. Triangle law of forces

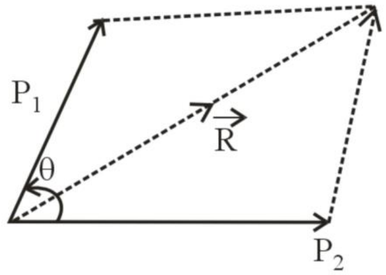
If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle, taken in order; their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order.



$\vec{R}$ is the resultant of $\vec{A} \& \vec{B}$

3.2. Parallelogram law of forces

If two forces, acting simultaneously on a particle, are represented in magnitude & direction by the two adjacent sides of a parallelogram; their resultant may be represented in magnitude & direction by the diagonal of the parallelogram, passing through their point of intersection.



Resultant R is given by R = $\sqrt{P_1^2 + P_2^2 + 2P_1P_2\cos\theta}$

The angle (α) which the resultant makes with P_2

$= \tan\alpha = \frac{P_1 \sin\theta}{P_2 + P_1 \cos\theta}$

Special cases:

(i) When θ = 0°, R = $P_1 + P_2$

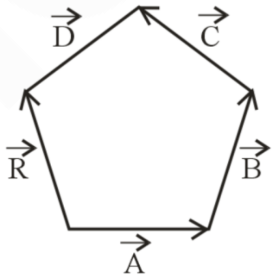
(ii) When θ = 90°, R = $\sqrt{P_1^2 + P_2^2}$

(iii) When θ = 180°, R = $P_1 - P_2$

(iv) When $P_1 = P_2$, R = $2P\cos\left(\frac{\theta}{2}\right)$

3.3. Polygon law of forces

If number of forces acting simultaneously on a particle, be represented in magnitude & direction, by the sides of the polygon taken in order, then the resultant of all these forces is represented, in magnitude & direction by the closing side of the polygon, taken in opposite order.



$\vec{R}$ is the resultant of $\vec{A}, \vec{B}, \vec{C}$ & $\vec{D}$ vectors.

FRICTION

The friction is a force distribution at the surfaces of contact and acts tangential to the surface of contact.

1. LIMITING FRICTION

The maximum value of frictional force, which comes into play, when a body just begins to slide over the surface of the other body.

2. LAWS OF FRICTION

2.1. Laws of static friction

1. Force of friction always acts in a direction, opposite to that in which the body tends to move.
2. The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces.

$$\frac{F}{R} = Constant$$

Where,

F = limiting friction

R = normal reaction

3. The force of friction is independent of the area of contact between the two surfaces.
4. The force of friction depends upon the roughness of the surfaces.

2.2. Laws of kinetic or dynamic friction

1. The force of friction always acts in a direction opposite to that in which the body is moving.
2. The magnitude of kinetic friction bears a constant ratio ratio to the normal reaction between two surfaces.

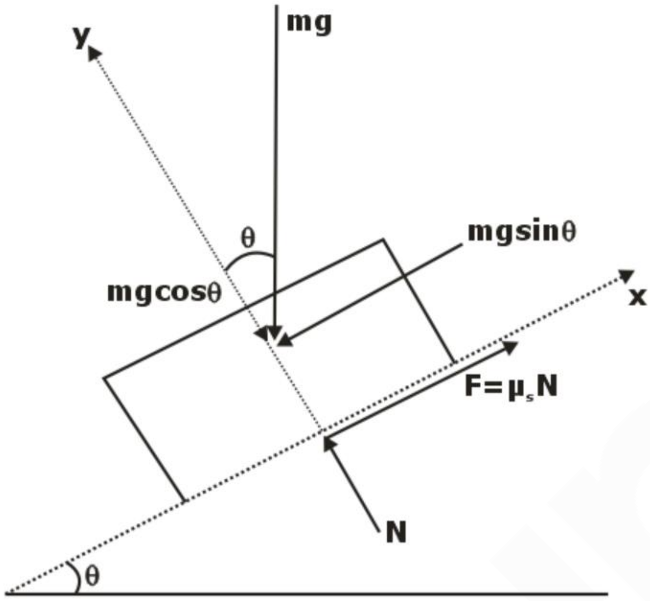
This ratio is slightly less than that in case of limiting friction.

$$\frac{F}{R} = \mu$$

$$\boxed{\mu_s > \mu_k}$$

2.3. Angle of Repose (φ)

The value of the angle of inclination θ corresponding to impending motion is called the angle of repose.



Since the block is still in equilibrium, it follows from the free body diagram that

$\Sigma F_x = \mu_s N - mg\sin\theta = 0 \Rightarrow \mu_s N = mg\sin\theta$

$\Sigma F_y = N - mg\cos\theta = 0 \Rightarrow N = mg\cos\theta$

Equating above two equations, we get

$\mu_s = \tan\theta$

and since angle of static friction

$\mu_s = \tan\varphi_s$

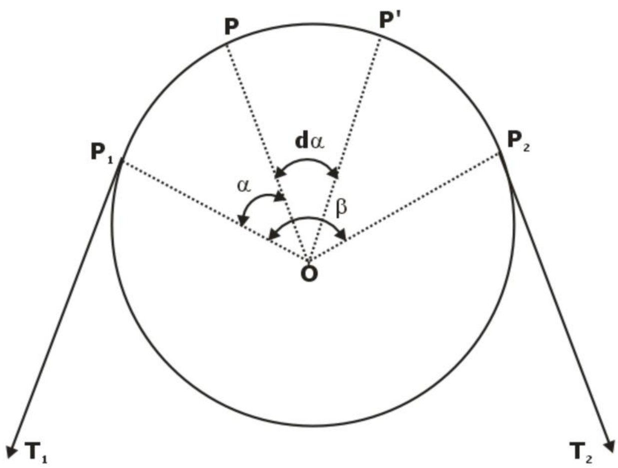
therefore $\theta = \varphi_s$

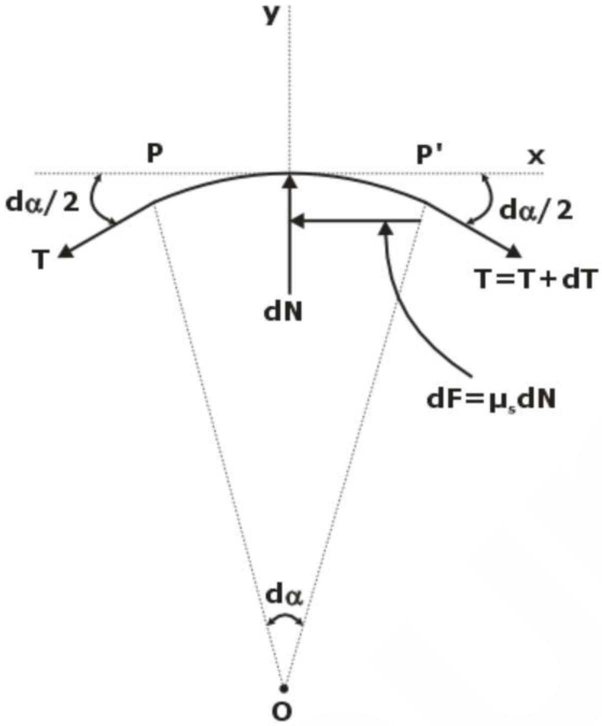
i.e., value of angle of repose has the same value as that of angle of static friction.

2.4. Belt friction

Belts are used to transfer the energy from one axis to another by winding over pulley or drum.

A flat belt passing over a drum where T_1 and T_2 ($T_2 > T_1$) are the tensions in the belt when belt is about to slide to right.





Equilibrium equations in x and y directions are

$$T_2 = T_1 e^{\mu_s \beta}$$

KINETICS

1. D'ALEMBERT'S PRINCIPLE

Equation of motion of the particle P

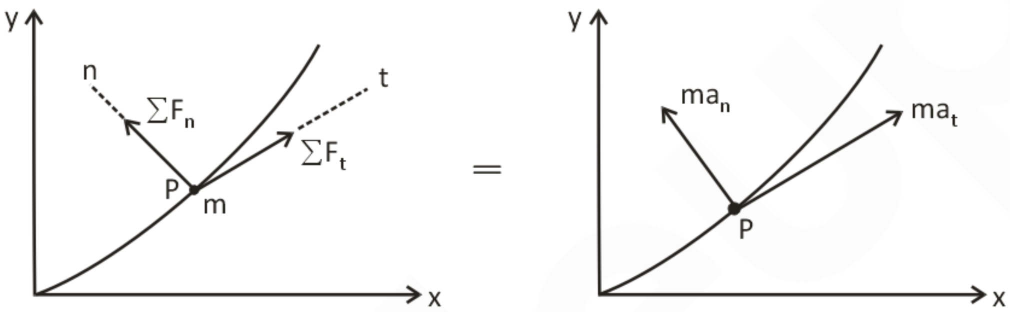
$$\Sigma F = ma$$

Can be written as

$$\Sigma F - ma = 0$$

Resultant of the external force (ΣF) and inertia force (–ma) is zero.

2. EQUATIONS OF MOTION IN TANGENTIAL AND NORMAL COMPONENTS



$$\Sigma F_t = ma_t \quad ; \quad \Sigma F_n = ma_n$$

If the particle is moving in a curved path of radius of curvature ρ and having the velocity v at any instant,

$$a_t = \frac{dv}{dt} \qquad a_n = \frac{v^2}{\rho}$$

$$\Sigma F_t = m\frac{dv}{dt} \qquad \Sigma F_n = m\frac{v^2}{\rho}$$

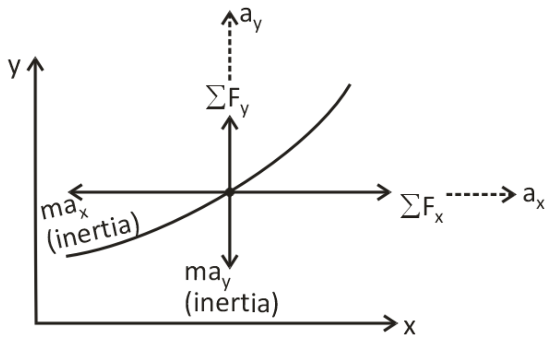
If the particle is moving along circular path, then ρ = radius of the circle

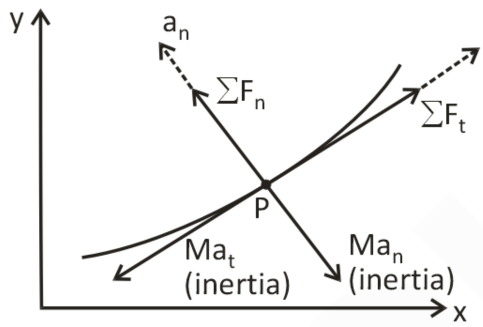
If moving with constant velocity v, then $a_t = \frac{dv}{dt} = 0$

3. D'ALEMBERT'S PRINCIPLE

$$\Sigma F_x = ma_x \quad \& \quad \Sigma F_y = ma_y$$

$$\Sigma F_x + \underbrace{(-ma_x)}_{\text{Inertia Force}} = 0 \quad \& \quad \Sigma F_y + \underbrace{(-ma_y)}_{\text{Inertia Force}} = 0$$





4. WORK OF A PARTICULAR FORCE

dW = Fds cosα

where α is the angle between the force and the displacement vector.

The work done by a force during a finite displacement from position P_1 to P_2 can be obtained by integrating the above equation,

$$W_{1-2} = \int_{S_1}^{S_2} (F \cos\alpha) dS$$

Where S_1 and S_2 are displacement measured along the path as shown.

Unit of work is (Nm) or Joule.

5. WORK OF VARIOUS FORCES

- **Work of Force of gravity**

$$\boxed{W_{1-2} = -mgy}$$

- **Work of the force of spring**

Consider a spring of stiffness (or spring constant) K which is stretched a distance x from its unperformed position as shown in figure.

$$W_{1-2} = -\frac{1}{2}(F_1 + F_2)(x_2 - x_1)$$

Where,

$$F_1 = kx,$$

$$F_2 = kx_2$$

$$W_{1-2} = -(\text{average force})\ (\text{Displacement})$$

6. WORK-ENERGY PRINCIPLE

$$W_{1-2} = m\left(\frac{v^2}{2} - \frac{v_1^2}{2}\right)$$

$$\downarrow$$

$$W_{1-2} = \underset{\text{K.E. at }(2)}{\frac{mv^2}{2}} - \underset{\text{K.E. at }(1)}{\frac{mv_1^2}{2}}$$

Work done by a force acting on a particle during its displacement is equal to the change in the kinetic energy of the particle during that displacement.

7. PRINCIPLE OF CONSERVATION OF ENERGY

Work done = change in kinetic energy

$W_{1-2} = (K.E.)_2 - (K.E.)_1$

If particle moves under the action of conservative force, work done in stored as potential energy

$W_{1-2} = (P.E.)_1 - (P.E.)_2$

$(K.E.)_2 - (K.E.)_1 = (K.E.)_2 + (P.E.)_2$

$(K.E.)_1 + (P.E.)_1 = (K.E.)_2 + (P.E.)_2$

Sum of potential and kinetic energy of a particle remains constant during the motion under the action of conservative forces.

8. POWER

Rate of change of work done

9. AVERAGE POWER

If ΔW is the work done during an interval of time Δt, then

$$\text{Average power} = \frac{\Delta W}{\Delta t}$$

$$\text{Instantaneous power} = \underset{\Delta t \to 0}{LT} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

Power = F.v. (Dot product)

Unit of power ; (Joule/sec) or watt(W)

1W = 1J/s = 1 Nm/s

1 Metric H.P. = 735.5 watt

1 Mechanical H.P. = 745.7 watt

CENTROID

The plane figures (like triangle, quadrilateral, circle etc.) have only areas, but no mass. The centre of area of such figures is known as centroid.

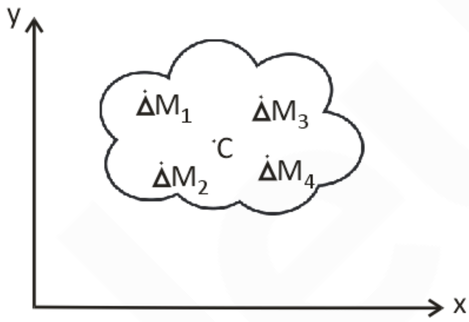
1. CENTRE OF MASS

Point where the entire mass of a body may be assumed to be concentrated.

2. CENTRE OF GRAVITY

Point of a body through which the resultant of the distributed gravity forces acts irrespective of the orientation of the body.

2.1. Determination of centre of gravity by moments method



Consider a body of mass M, composed of 'n' number of masses ΔM_1, ΔM_2 __ _ _ _ ΔM_n, distributed within the body such that

$M = \Delta M_1 + \Delta M_2 + \_ \_ \_ \_ \_ \_ \_ \_ \Delta M_n$

The distance of these masses with respect to the axes be,

$(x_1, y_1), (x_2, y_2) \_ \_ \_ \_ \_ \_ \_ \_ (x_n, y_n)$

Let the centre of gravity of the whole mass M lie at a distance (x_c, y_c) ωrt reference axes.

Gravitational forces acting on the masses will be $\Delta M_1 g$, $\Delta M_2 g$ & so on.

$$Therefore,\ x_c = \frac{\sum(\Delta M_i x_i)}{\sum(\Delta M_i)}$$

$$Similarly,\ yc = \frac{\sum(\Delta M_i y_i)}{\sum(\Delta M_i)}$$

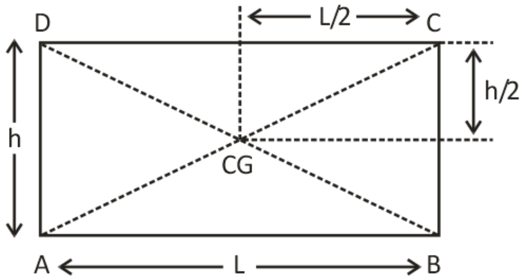
2.2. Centre of Gravity by geometrical considerations

1. Uniform Rod

Centre of gravity lies at its middle point

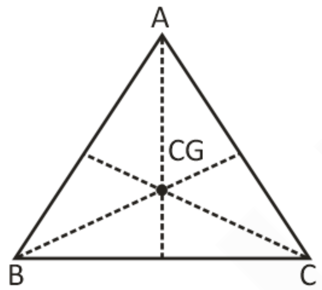
2. Rectangle (or parallelogram)

Centre of gravity lies at the intersection of its diagonals

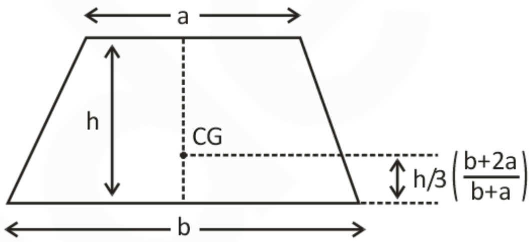


3. Triangle

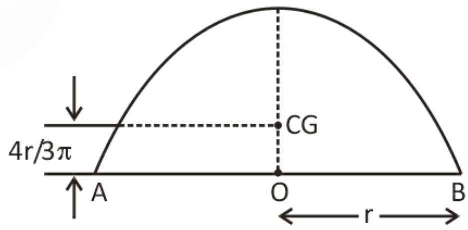
Centre of gravity lies at the intersection of its medians.



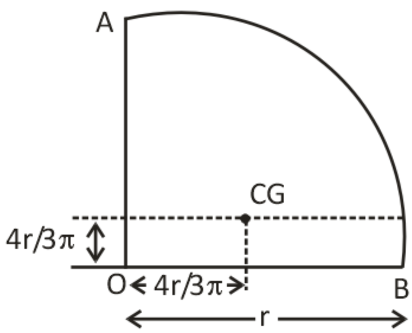
4. Trapezium



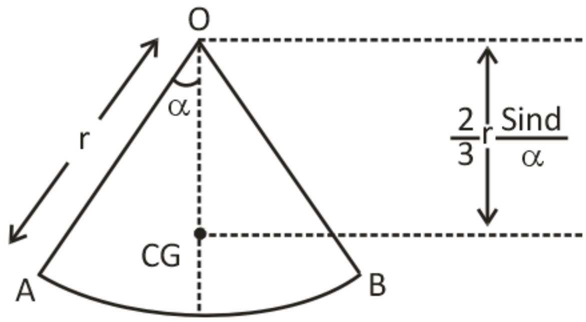
5. Semi–circle



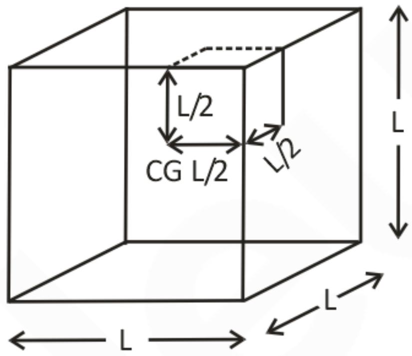
6. Quarter Circle



7. Circle sector making semi – vertical angle α



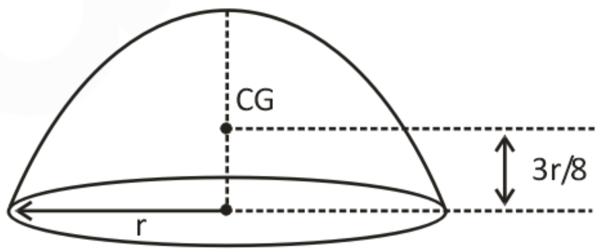
8. Cube



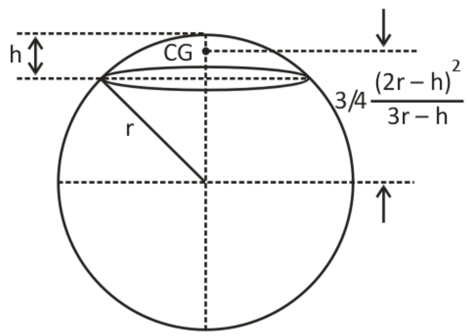
9. Sphere

Centre of gravity of a sphere lies at a distance of d/2 from every point (where d is the diameter of the sphere)

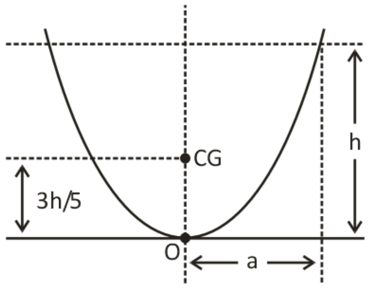
10. Hemisphere



11. Segment of a sphere



12. Parabola

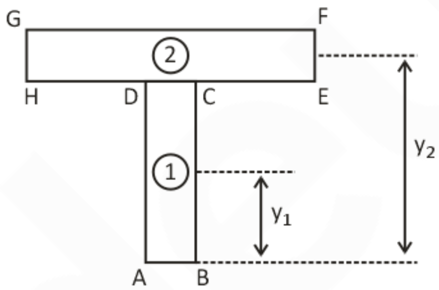


Centre of gravity of symmetrical sections

In such cases, only either x_c or y_c is to be calculated

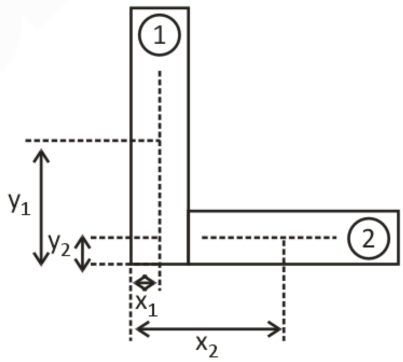
13. T-section

The section is symmetrical about y – y section, so, its centre of gravity will lies on this axis.



$$y_c = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$

14. L-section

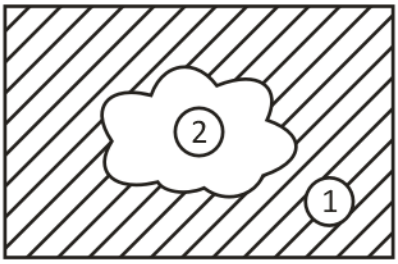


$$x_c = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} \qquad y_c = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$

2.3. CENTRE OF GRAVITY OF SOLID BODIES

Centre of gravity of solid bodies (hemispheres, cylinders, right circular circular cones) is found out in the same way as that of plane figures. The only difference, between the plane figures & solid bodies is that in the case of solid bodies, volume is calculated instead of areas.

2.4. CENTRE OF GRAVITY OF SECTIONS WITH CUT OUT ROLES.



$$x_c = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} \quad y_c = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

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