A complex number is a number that can be expressed in the form p + iq, where p and q are real numbers, and i is a solution of the equation x2 =−1. √1 =i or i2=−1. Examples of complex numbers: 8 – 2i, 2 +31i, 2+ 45i, etc. Complex numbers are denoted by ‘z’.

General form of Complex Number: z = p + iq

Where,

- p is known as the real part, denoted by Re z
- q is known as the imaginary part, denoted by Im z

If z = 12 + 35i, then Re z = 12 and Im z = 35. If z1 and z2 are two complex numbers such that z1 = p + iq and z2 = r + is. z1 and z2 are equal if p = r and q = s.

Algebra of Complex Numbers

Addition of complex numbers

Let z1 = m + ni and z2 = o + ip be two complex numbers. Then, z1 + z2 = z = (m + o) + (n + p)i, where z = resultant complex number. For example, (12 + 13i) + (-16 +15i) = (12 – 16) + (13 + 15)i = -4 + 28i.

- The sum of complex numbers is always a complex number (closure law)
- For complex numbers z1 and z2: z2 + z1= z1 + z2 (commutative law) For complex numbers z1, z2, z3: (z1 + z2) + z3 = z1 + (z2 + z3) [associative law].
- For every complex number z, z + 0 = z [additive identity]
- To every complex number z = p + qi, we have the complex number -z = -p + i(-q), called the negative or additive inverse of z. [z + (–z) = 0]

Difference of complex numbers

Let z1 = m + ni and z2 = o + ip be two complex numbers, then z1 – z2 = z1 + (-z2). For example, (16 + 13i) – (12 – 1i) = (16 + 13i) + (-12 + 1i) = 4 + 14i and (12 – 1i) – (16 + 13i) = (12 – 1i) + (-16 – 13i) = -4 – 14i

Multiplication of complex numbers

Let z1 = m + ni and z2 = o + ip be two complex numbers then, z1 × z2 = (mo – np) + i(no + pm). For example, (2 + 4i) (1 + 5i) = (2 × 1 – 4 × 5) + i(2 × 5 + 4 × 1) = -22 + 14i The product of two complex numbers is a complex number (closure law)

- For complex numbers z1 and z2, z1 × z2 = z2 × z1 (commutative law).
- For complex numbers z1, z2, z3, (z1 × z2) × z3 = z1 × (z2 × z3) [associative law].

Let z1 = m + in and z2 = o + ip. Then,

- z1 + z2 = (m + o) + i (n + p)
- z1 z2 = (mo – np) + i(mp + on)
- The conjugate of the complex number z = m + in, denoted by z, is given by z = m – in.

The Modulus and Conjugate of Complex Numbers

Let z = m + in be a complex number. Then, the modulus of z, denoted by |z| = √m2−n2 and the conjugate of z, denoted by z is the complex number m – ni.In the Argand plane, the modulus of the complex number m + in = √m2−n2 is the distance between the point (m, n) and the origin (0, 0). The x-axis is termed as the real axis and the y-axis is termed as the imaginary axis.

Complex Numbers and Quadratic Equations Practice Questions

1. Find the modulus and argument of the complex number $\frac{1+i}{1-i}$
2. Convert the complex number in the polar form $\frac{i-1}{cos\ \frac{\pi}{3} - sin\ \frac{\pi}{3}i}$
3. Solve the following equation: $x^3 - 3x^2 + 2x - 1$
4. Represent the given complex number in the polar form $z = 1 + \sqrt{3}i$
5. Solve $\sqrt{5}x^2 + x + \sqrt{5}$