

GATE 2018

Electronics Engineering

Solutions



Solutions

1. Ans. D.

This is an infinite GP.

Sum of infinite G.P is given by,

$$S = \frac{a}{1-r}, \text{ where first term } (a) = 1 \text{ and common ratio } (r) = \frac{1}{4}$$

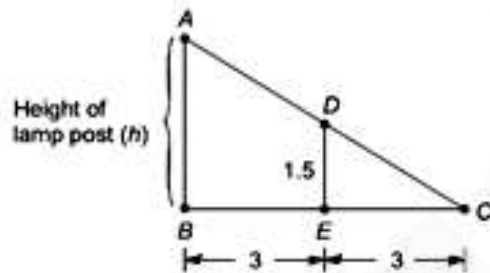
$$\frac{1}{1-\frac{1}{4}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

2. Ans. B.

Piece means 'slice' and Peace means 'silence'.

3. Ans. B.

If we make a figure using the information provided in the question,



Here, $\angle(ACB) = \angle(DCE)$, thus

$$\tan \angle(ACB) = \tan \angle(DCE)$$

$$\frac{AB}{BC} = \frac{DE}{EC}$$

$$\frac{h}{6} = \frac{1.5}{3}$$

$$h = 3 \text{ meters}$$

4. Ans. A.

Even though there is a vast scope for its *improvement*, tourism has remained a *neglected* area. Only these set of words give a meaning to the given sentence.

5. Ans. B.

A number is divisible by 3 if the sum of all digits is be divisible by 3.

$$7 + 1 + 5 + ? + 4 + 2 + 3 = 22 + ?$$

Next numbers after 22 which are divisible by 3 are 24, 27, 30 etc.

Minimum value of ? that would make the given number divisible by 3 is 2 as 24 is divisible by 3.

6. Ans. B.

Alloy A contains Gold and Copper in ratio 2 : 3.

Let there be 10x mass of alloy A, so that we have Gold and Copper as 4x : 6x.

Alloy B contains Gold and Copper in ratio 3 : 7.

Let there be 10x mass of Alloy B, so that we have Gold and Copper as 3x : 7x.

As masses of Alloy A is equal to Alloy of mass B,

Resultant ratio of Gold to Copper when equal masses of Alloy A and Alloy B are mixed would be

$$4x + 3x : 6x + 7x$$

$$7x : 13x$$

$$7 : 13.$$

7. Ans. B.

We have the formula for compound interest as follows,

$$A = P * \left(1 + \frac{R}{100}\right)^n$$

Where, A is final amount, P is initial amount, R is rate of interest and n is the number of years the interest is compounded.

We have A = 1000000, R = 10 %, n = 5 years. Then P can be found out,

P = 6,21,000.

$$1000000 = P * \left(1 + \frac{10}{100}\right)^5$$

$$P = \frac{1000000}{\left(1 + \frac{10}{100}\right)^5} = 620921.32 \approx 621000$$

8. Ans. C.

Probability that accident was caused by blue cab,

$$P(\text{Blue Cab}) = (P(\text{Blue}) * P(\text{Correct})) + (P(\text{Green}) * P(\text{Not Correct}))$$

This gives total number of accidents being identified caused by a Blue cab.

$$P(\text{correct}) = 0.8$$

$$P(\text{not correct}) = 1 - 0.8 = 0.2$$

Actual probability that accident is caused by blue cab,

$$P(\text{Actually Blue}) = (P(\text{Blue}) * P(\text{Correct})) / P(\text{Blue Cab})$$

$$P(\text{Actually Blue}) = (0.15 * 0.8) / (0.15 * 0.8 + 0.85 * 0.2)$$

$$P(\text{Actually Blue}) = 0.4137$$

Thus, 41.37% is the probability.

9. Ans. D.

(A) No such information given that supports this option.

Option (B) suggests only one part of the paragraph.

Similarly (C) can also be discarded because of no such information is given in paragraph.

While (D) option suggests helicopter as the connectivity means which is the crux of the paragraph.

10. Ans. C.

Statement (i) is not true as nowhere it is mentioned that John being a Captain at Junior level.
 Statement (ii) can be concluded from the paragraph as the last line suggests.
 Statement (iii) cannot be concluded from the given information as qualities sought by selectors can be concluded but similar cannot be said about opinion of fans and viewers.
 Statement (iv) can be concluded from the part in paragraph mentioning about last 3 seasons of John.

11. Ans. D.

(A) is true.
 (B) is true as in a minimum-phase system, Bode magnitude plot is enough to obtain a general approximation of its Nyquist plot.
 (C) Routh criterion can be applied to any system to check the stability of a system but a transport lag controller can only be explained using Nyquist Criterion.
 (D) We can obtain closed-loop frequency response for Unity Feedback system easily by substituting $s = j\omega$, and draw the plot for different values of ω . Usually this is not done as it is not necessary as OLTF is enough to comment on the stability. Thus, (D) is false.

12. Ans. C.

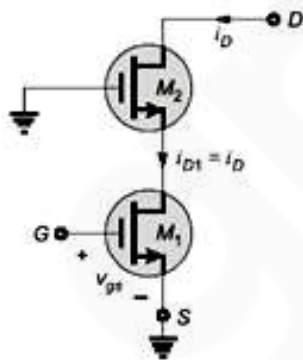
For Short circuit, $z = \text{Re}(z) = \text{Im}(z) = 0$; Point "P"
 For Open circuit, $z = \infty$; Point "R"
 For Matched load, $z = \text{Re}(z) = 1$ and $\text{Im}(z) = 0$; Point "Q"

P: Short Circuit, **Q**: Matched Load, **R**: Open Circuit

13. Ans. B.

Here, $p(s)$ has 3 roots whereas $p'(s)$ has 2 roots and none of which are real. Thus, $p(s)$ has to have 1 real and 2 complex roots.

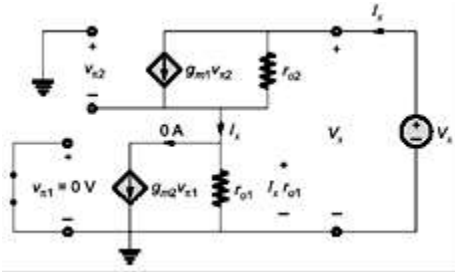
14. Ans. C.



Overall g_m can be calculated as follows,

$$g_m = \frac{\Delta I_D}{\Delta V_{in}} = \frac{i_D}{v_{gs}} = \frac{i_{D1}}{v_{gs}} = g_{m1}$$

To calculate r_o : we need to draw the small signal equivalent of the arrangement,



$$V_{x2} = -I_x r_{o1}$$

$$I_x = g_{m2} V_{x2} + \frac{(V_x - I_x r_{o1})}{r_{o2}}$$

$$I_x = -g_{m2} r_{o1} I_x + \frac{V_x}{r_{o2}} - I_x \frac{r_{o1}}{r_{o2}}$$

$$V_x = r_{o2} \left[1 + r_{o1} g_{m2} + \frac{r_{o1}}{r_{o2}} \right] I_x$$

$$r_o = \frac{V_x}{I_x} = r_{o1} + r_{o2} + r_{o1} r_{o2} g_{m2} \approx r_{o1} r_{o2} g_{m2}$$

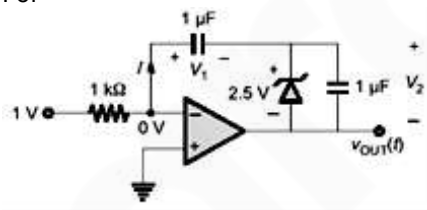
Thus, overall $g_m = g_{m1}$ and overall $r_o = r_{o1} r_{o2} g_{m2}$.

15. Ans. D.

A good transimpedance amplifier should have low input impedance and low output impedance as it is generally used as a current to voltage convertor.

16. Ans. C.

For $t > 0$,



$$I = \frac{1V}{1k\Omega} = 1mA$$

from $t=0$; both the capacitor charges and the zener is off (open circuited)

the capacitor across zener will charge upto 2.5V after that zener will behave as voltage regulator since it will go in breakdown region.

$$v(t) = \frac{1}{C} \int_0^t i dt$$

for $v(t) = 2.5V$ $t = 2.5msec$

Till $t = 2.5 msec$, both V_1 and V_2 will increase as C_1 and C_2 will get charged and

after $t = 2.5 \text{ msec}$, $V_1 = 2.5 \text{ V}$ and V_2 increases with time as C_1 is completely charged whereas C_2 is not completely charged.

So, when $v_{\text{out}}(t) = -10 \text{ V}$, $V_1 = 7.5 \text{ V}$

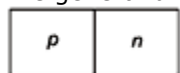
$$\text{So, } \frac{1}{1\mu\text{F}} \int_0^t (1\text{mA}) dt = 7.5 \text{ V}$$

$$10^3 t = 7.5$$

$$t = 7.5 \text{ msec}$$

17. Ans. D.

The general direction of carrier movement can be denoted as follows for pn junction,

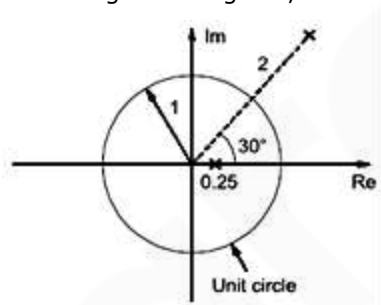


- \rightarrow hole diffusion (\rightarrow hole diffusion current direction)
- \leftarrow electron diffusion (\rightarrow electron diffusion current direction)
- \rightarrow hole drift direction (\rightarrow hole drift current direction)
- \rightarrow electron drift direction (\leftarrow electron drift current direction)

so only option D is incorrect

18. Ans. B.

According to data given, we can draw the poles in z-domain as follows,



For the system to be stable, ROC should include the unit circle. From the given pole pattern, it is clear that to make the system stable, the ROC should be two-sided and hence the impulse response of the system should be also two-sided.

19. Ans. C.

We know that SOP and POS contain exhaustive terms in their expression. Any expression that is present in the SOP would not be present in the POS form. Thus,

$$F(A, B, C, D) = \overline{A}BC + \overline{A}B\overline{C} + A\overline{B}C$$

$$F(A, B, C, D) = \text{IIm}(1, 3, 5, 6, 7) = \overline{C} \cdot (\overline{A} + C) = \overline{A} \cdot \overline{C}$$

20. Ans. D.

Truth table of the logic circuit,

X	Y	P1	P2	P3	P4	N1	N2	N3	N4	f(X,Y)
0	0	OFF	ON	ON	OFF	ON	ON	OFF	OFF	0
0	1	OFF	OFF	ON	ON	ON	OFF	OFF	ON	1
1	0	ON	ON	OFF	OFF	OFF	ON	ON	OFF	1
1	1	ON	OFF	OFF	ON	OFF	OFF	ON	ON	0

This denotes the XOR function.

21. Ans. D.

Given in question,

$$f(x, y) = \frac{ax^2 + by^2}{xy} = a\left(\frac{x}{y}\right) + b\left(\frac{y}{x}\right)$$

$$\frac{\partial f}{\partial x} \Big|_{(1,2)} = \left[\frac{a}{y} - \frac{by}{x^2} \right] \Big|_{(1,2)} = \frac{a}{2} - 2b$$

$$\frac{\partial f}{\partial y} \Big|_{(1,2)} = \left[-\frac{ax}{y^2} + \frac{b}{x} \right] \Big|_{(1,2)} = -\frac{a}{4} + b$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$$

$$\frac{a}{2} - 2b = -\frac{a}{4} + b$$

So,

$$\frac{3a}{4} = 3b$$

$$a = 4b$$

22. Ans. C.

Eigen vectors corresponding to distinct eigen values are linearly independent.
So, "S2 implies S1".

23. Ans. C.

Presence of a constant term introduces non-linearity.

Thus, $y = au + b, b \neq 0$ is a non-linear system.

24. Ans. C.

There is no change in the value of Fourier series coefficients if there is a change in time period or frequency.

So, $b_k = a_k$

$$\sum_{k=-\infty}^{\infty} |b_k| = \sum_{k=-\infty}^{\infty} |a_k| = 16$$

25. Ans.

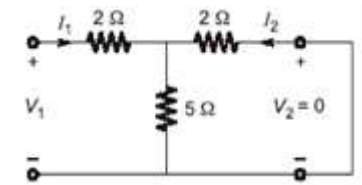
For ABCD Parameters, the general equations are,

$$V_1 = AV_2 - BI_2 \dots\dots(A)$$

$$I_1 = CV_2 - DI_2 \dots\dots(B)$$

$$B = -\left. \frac{V_1}{I_2} \right|_{V_2=0}$$

When $V_2 = 0$ (i.e., when port-2 is short circuited),



Writing KVL equations in loops with I_1 and I_2 separately,

$$V_1 = 2I_1 + 5(I_1 + I_2) = 7I_1 + 5I_2 \dots\dots(1)$$

$$V_2 = 2I_2 + 5(I_2 + I_1) = 5I_1 + 7I_2 \dots\dots(2)$$

$$I_1 = \frac{-7}{5}I_2 + \frac{1}{5}V_2 \dots\dots(3)$$

by equation (2)

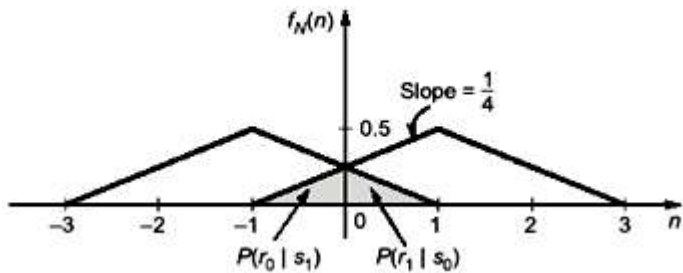
by (1) & (3)

$$V_1 = \frac{7}{5}V_2 - \frac{24}{5}I_2 \dots\dots(4)$$

So, BY (A) and (4)
B=4.8

26. Ans.

Let s_0 and s_1 be the transmitted symbols representing the transmitted value $\{-1, 1\}$ respectively and let r_0 and r_1 be the received symbols.



Probability of error is given as,

$$P_e = P(s_1) * P(r_0 | s_1) + P(s_0) * P(r_1 | s_0)$$

Where $P(r_0 | s_1)$ is probability of receiving r_0 when s_1 is transmitted and $P(r_1 | s_0)$ is probability of receiving r_1 when s_0 is transmitted.

$$P(r_0 | s_1) = P(r_1 | s_0) = 0.5 * 1 * 0.25 = 0.125$$

Given that, $P(s_0) = P(s_1) = \frac{1}{2}$ (Probability of transmitting a signal)

So,
$$P_e = \frac{1}{2} \left(\frac{1}{8} + \frac{1}{8} \right) = \frac{1}{8} = 0.125$$

27. Ans.

According to the given data,

- GREEN → 70 seconds**
- YELLOW → 5 seconds**
- RED → 75 seconds**
- Clock period → 5 seconds**

$$= \frac{70 + 5 + 75}{5} = 30$$

Total number of unique states required

Minimum number of flip-flops required is,

$$n = \lceil \log_2(30) \rceil = \lceil 4.91 \rceil = 5$$

flip flops are required for the stable output to make transition error zero and false triggering of output.

28. Ans.

Given signal,

$$s(t) = \cos(2000\pi t) + 4 \cos(2400\pi t) + \cos(2800\pi t)$$

It can be compared with the standard form of the AM signal,

$$s(t) = \frac{\mu A_c}{2} \cos[2\pi(f_c - f_m)t] + A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2} \cos[2\pi(f_c + f_m)t]$$

By comparison, we get, $A_c = 4$ and with further manipulation,

$$\therefore \mu = \frac{A_m}{A_c}$$

& comparing standard equation

$$\frac{\mu A_c}{2} = 1$$

$$\mu = \frac{2}{A_c}$$

$$\frac{A_m}{A_c} = \frac{2}{A_c} \Rightarrow A_m = 2$$

$$\frac{P_m}{P_c} = \frac{\frac{1}{2} A_m^2}{\frac{1}{2} A_c^2} = \frac{A_m^2}{A_c^2} = \frac{(2)^2}{(4)^2} = \frac{1}{4} = 0.25$$

29. Ans.

We know the dependence of feature size on wavelength as follows,

$$L_{\min} \propto \lambda$$

$$\frac{L_{\min 1}}{L_{\min 2}} = \frac{\lambda_1}{\lambda_2} = \frac{156 \text{ nm}}{325 \text{ nm}} = 0.48$$

30. Ans.

Given function, a_2 would appear alongwith 2nd differential of $f(x)$. We can calculate a_2 as follows,

$$f(x) = \int_0^x e^{-\left(\frac{t^2}{2}\right)} dt$$

$$f'(x) = e^{-x^2/2} - 1$$

$$\text{and } f''(x) = e^{-x^2/2}(-x)$$

$$f''(0) = 0$$

$$a_2 = \frac{f''(0)}{2!} = 0$$

31. Ans.

Probability of X_4 being smallest is given as follows,

$$P(X_4 \text{ is smallest}) = \frac{3!}{4!} = \frac{1}{4} = 0.25$$

Note that here all four are similar random variables, so the probability of any one of them being smallest is same.

32. Ans.

Given that, $n = 5$ and $d_{\min} = 2$

Without any constraint, $2^5 = 32$ codewords can be formed.

By maintaining $d_{\min} = 2$, the codewords can be formed as follows :

0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

⇒ Total 16 codewords are possible

Thus, 16 codewords are possible.

33. Ans.

The following condition is true for a distortionless transmission line,

$$\frac{L}{R} = \frac{C}{G}$$

Propagation constant is given by,

$$\begin{aligned}\gamma &= \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \\ &= \sqrt{RG} \left(1 + j\omega \frac{L}{R} \right)\end{aligned}$$

And the attenuation constant, which is real part of the propagation constant,

$$\alpha = \sqrt{RG}$$

Characteristic impedance,

$$\begin{aligned}Z_o &= \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} = \sqrt{\frac{R}{G}} \\ \sqrt{G} &= \frac{\sqrt{R}}{Z_o}\end{aligned}$$

$$\begin{aligned}\alpha &= \sqrt{R} \cdot \frac{\sqrt{R}}{Z_o} = \frac{R}{Z_o} = \frac{0.05}{50} = \frac{0.01}{10} \\ \text{So, } &= 0.001 \text{ Np/m}\end{aligned}$$

34. Ans.

We have the formula for Width of depletion region in a pn junction, and from that relation between width and voltage can be directly applied to get the answer as follows,

$$\begin{aligned}W_{\text{dep}} &= \sqrt{\frac{2\epsilon}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) (V_{bi} - V_{AK})} \\ \frac{\sqrt{(0.65 - V_{AK})}}{\sqrt{0.65}} &= \frac{0.6 \mu\text{m}}{1 \mu\text{m}} = 0.6 \\ 1 - \frac{V_{AK}}{0.65} &= 0.36 \\ V_{AK} &= 0.65(1 - 0.36) \\ &= 0.65 \times 0.64 = 0.416 \text{ V}\end{aligned}$$

35. Ans.

If $AX = 0$ has infinitely many solutions and X is non-zero, then $|A| = 0$

$$\begin{vmatrix} k & 2k \\ k^2 - k & k^2 \end{vmatrix} = 0$$

$$k^3 - 2k^3 + 2k^2 = 0$$

$$k^2(2 - k) = 0$$

$k = 0, 2 \Rightarrow$ "two" distinct values of k

36. Ans.

Hilbert transform does not alter the amplitude spectrum of the signal and using CTFT to determine the amplitude,

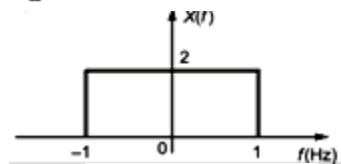
$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

So,

$$\text{sinc}(t) \xleftrightarrow{\text{CTFT}} \text{rect}(f)$$

$$4 \text{sinc}(2t) \xleftrightarrow{\text{CTFT}} \frac{4}{2} \text{rect}\left(\frac{f}{2}\right) = 2 \text{rect}\left(\frac{f}{2}\right)$$

$$\int_{-\infty}^{\infty} |X(f)|^2 dt = 2 \times (2)^2 = 8$$



$$\int_{-\infty}^{\infty} |y(t)|^2 dt = 8$$

Then,

37. Ans.

Maximum resonant peak is given as (In terms of damping factor),

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 2$$

$$2\zeta\sqrt{1-\zeta^2} = \frac{1}{2}$$

$$\zeta^2(1-\zeta^2) = \frac{1}{16}$$

$$\zeta^4 - \zeta^2 + \frac{1}{16} = 0$$

$$\zeta^2 = \frac{1}{2} \pm \sqrt{\frac{1-\frac{1}{4}}{4}} = \frac{1}{2} \pm \frac{\sqrt{3}}{4}$$

But we know, $M_r = 2 > 1, \xi < \frac{1}{\sqrt{2}}$ and $\xi^2 < \frac{1}{2}$

Then,
$$\xi^2 = \frac{1}{2} - \frac{\sqrt{3}}{4}$$

$$G(s) = \frac{K}{s(s+2)} = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

Also given in question,

So, $\omega_n = \sqrt{K}$

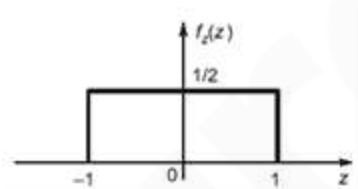
$$2\xi\sqrt{K} = 2$$

$$\sqrt{K} = \frac{1}{\xi}$$

$$K = \frac{1}{\xi^2} = \frac{1}{\left(\frac{1}{2} - \frac{\sqrt{3}}{4}\right)} = \frac{4}{2 - \sqrt{3}} = 14.928$$

38. Ans.

The noise profile can be drawn as follows,

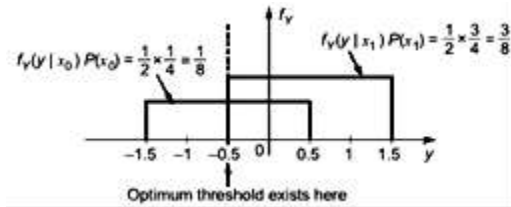


And we know the probabilities of two signals X_0 and X_1 ,

$$P(x_0) = \frac{1}{4}$$

$$P(x_1) = \frac{3}{4}$$

MAP criteria, $f_Y(y | x_0)P(x_0)f_Y(y | x_1)P(x_1)$



So, $\alpha = -0.50$

39. Ans.

The circuit given here has no initial conditions as no energy is stored in inductor prior to switching.

$$i(t) = \frac{1}{1+1}(1 - e^{-t/\tau}) A ; t > 0$$

Loop current,

$$\tau = \frac{L}{R_{eq}} = \frac{1}{1+1} = \frac{1}{2} \text{ sec}$$

$$i(t) = \frac{1}{2}(1 - e^{-2t}) A ; t > 0$$

So current at $t = 0.5 \text{ sec}$,

$$i(t) = \frac{1}{2}(1 - e^{-1}) A = 0.316 A$$

40. Ans.

At phase crossover frequency $G(s)$,

$$M_{dB}(\omega_{pc}) = 20 \text{ dB}$$

When cascaded with k ,

$$GM_{dB} = -20 \text{ dB} - 20 \log_{10}(k) > 0 \text{ dB}$$

$$20 + 20 \log_{10}(k) < 0$$

$$20 \log_{10}(k) < -20$$

$$k < 10^{-1} = 0.10$$

$$k_0 = 0.10$$

Thus, $k_0 = 0.10$

41. Ans.

We have $x(k)$ given and we need to find the sum of downsampled version of $x(n)$. So,

$$x(k) = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$\begin{aligned} \sum_{n=0}^3 x[2n] &= x[0] + x[2] + x[4] + x[6] \\ &= 4.5 - 0.5 - 0.5j - 0.5 - 0.5 + 0.5j \\ &= 4.5 - 1.5 = 3 \end{aligned}$$

42. Ans.

From the given equation of the electric field, we can get the following phase relation,

$$\vec{K}_i = 2\pi(\hat{x} + \sqrt{2}\hat{z}) = 2\pi\sqrt{3}\left(\frac{1}{\sqrt{3}}\hat{x} + \sqrt{\frac{2}{3}}\hat{z}\right)$$

$$\cos\theta_x = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan\theta_x = \sqrt{2}$$

Since there is no reflected wave,

$$\theta_x = \theta_B = \text{Brewster angle}$$

And as the wave is parallel polarized,

$$\tan\theta_B = \sqrt{\frac{\epsilon_r \epsilon_0}{\epsilon_0}} = \sqrt{\epsilon_r} = \sqrt{2}$$

$$\text{So, } \epsilon_r = 2$$

43. Ans.

Given that x and y are independent variables. From the relations given in the problem statement,

$$r = x^2 + y - z \quad \dots(i)$$

$$z^3 - xy + yz + y^3 = 1 \quad \dots(ii)$$

$$\frac{\partial r}{\partial x} = 2x - \frac{\partial z}{\partial x} \quad \dots(iii)$$

$$3z^2 \frac{\partial z}{\partial x} - y + y \frac{\partial z}{\partial x} = 0$$

$$\frac{\partial z}{\partial x} = \frac{y}{3z^2 + y}$$

By substituting $\frac{\partial z}{\partial x}$ in equation (iii), we get,

$$\frac{\partial r}{\partial x} = 2x - \frac{y}{3z^2 + y}$$

At given point (2, -1, 1),

$$\frac{\partial r}{\partial x} = 2(2) - \frac{(-1)}{3(1)^2 + (-1)} = 4 + \frac{1}{2} = 4.50$$

44. Ans.

The standard formula for built-in potential is,

$$V_0 = \frac{kT}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

since in the given semiconductor two doping are there so overall built in potential will be equal to difference of V_{01} and V_{02}

$$V_{01} = \frac{kT}{q} \left(\frac{N_{A1} \times N_{D1}}{n_{i1}^2} \right)$$

$$\text{and } V_{02} = \frac{kT}{q} \left(\frac{N_{A2} \times N_{D2}}{n_{i2}^2} \right)$$

since same material is doped $n_{i1} = n_{i2}$ and $N_{D1} = N_{D2}$

$$\text{so } V_{bi} = V_{02} - V_{01}$$

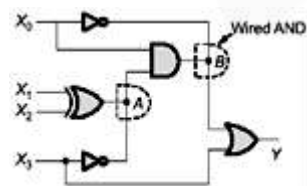
$$= \frac{kT}{q} \left(\frac{N_{A2} \times N_{D2}}{n_{i2}^2} \times \frac{n_{i1}^2}{N_{A1} \times N_{D1}} \right)$$

$$V_{bi} = \frac{kT}{q} \ln \left(\frac{N_{A2}}{N_{A1}} \right) = \frac{1.38 \times 3}{1.6 \times 100} \ln(100) \text{ V}$$

$$= 0.1192 \text{ V}$$

45. Ans.

We can redraw the figure as,



So that following relations can be established clearly,

$$A = (X_1 \oplus X_2) \bar{X}_3$$

$$B = [(X_1 \oplus X_2) \bar{X}_3 X_0] \cdot \bar{X}_0 = 0$$

$$Y = B + X_3 = 0 + X_3 = X_3$$

if X_3 is 1 then $y=1$ so from 0000 to 1111 the value of X_3 is high for 8 values

Out of $2^4 = 16$ possible combinations of X_0, X_1, X_2 and X_3 , the output Y will be high for 8 combinations.

So, Y will be high for 8 combinations.

46. Ans.

The device constant K_n ,

$$K_n = \frac{\mu_n C_{ox}}{2} \left(\frac{W}{L} \right)$$

Given that,
$$\left(\frac{W}{L} \right)_2 = 2 \left(\frac{W}{L} \right)_1$$

Then,
$$K_{n2} = 2K_{n1}$$

For M_1

$$V_{GS1} - V_T = 2 - 1 = 1 \text{ V}$$

Now, for M_2

$$V_{GS2} - V_T = 2 - V_x - 1 = 1 \text{ V} - V_x < 1 \text{ V}$$

$$V_{DS2} = (3.3 - V_x) > (V_{GS2} - V_T)$$

Here, clearly M_1 will be in linear region and M_2 will be in saturation region. But current across them would be same,

$$I_{D1} = I_{D2}$$

$$K_{n1} [2(V_{GS1} - V_T)V_{DS1} - V_{DS1}^2] = K_{n2}(V_{GS2} - V_T)^2$$

$$K_{n1} [2(2-1)V_x - V_x^2] = 2K_{n1}(2-V_x-1)^2$$

$$2V_x - V_x^2 = 2(1+V_x^2 - 2V_x) = 2V_x^2 - 4V_x + 2$$

$$3V_x^2 - 6V_x + 2 = 0 ; V_x^2 - 2V_x + \frac{2}{3} = 0$$

$$V_x = 1 \pm \sqrt{\frac{4 - \frac{8}{3}}{43}} = 1 \pm \sqrt{\frac{1}{3}} \text{ V}$$

$$V_{GS2} = (2 - V_x) \geq V_T \Rightarrow (1 - V_x) \geq 0$$

$$V_x = 1 - \sqrt{\frac{1}{3}} = 0.4226 \text{ V}$$

So, the only valid value,

47. Ans.

Given condition,

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + \frac{5y}{4} = 0$$

$$y(0) = 1$$

$$y'(0) = 0$$

This can be solved easily in laplace domain,

$$s^2Y(s) - s(1) + sY(s) - 1 + \frac{5}{4}Y(s) = 0$$

$$Y(s) = \frac{s+1}{s^2 + s + \frac{5}{4}} = \frac{s+1}{\left(s + \frac{1}{2}\right)^2 + 1}$$

$$= \frac{\left(s + \frac{1}{2}\right)}{\left(s + \frac{1}{2}\right)^2 + 1} + \frac{\frac{1}{2}}{\left(s + \frac{1}{2}\right)^2 + 1}$$

By taking inverse Laplace transform we get y(t),

$$y(t) = e^{-t/2} \left[\cos(t) \frac{1}{2} \sin(t) \right] ; t > 0$$

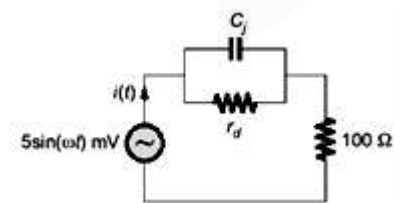
Now its value at $t = \pi$,

$$y(t = \pi) = e^{-\pi/2} [(-1) + (0)] = e^{-\pi/2}$$

$$= -0.2078 \approx -0.21$$

48. Ans.

Small-signal equivalent model of the given circuit needs to be realized as follows,



Given information in question,

$$\omega = 2 \times 10^6 \text{ rad/sec}$$

$$C_j = 0.5 \text{ nF}$$

$$I_{DC} = 26 \mu\text{A}$$

$$V_T = 26 \text{ mV}$$

$$\eta = 1$$

So we can obtain impedances,

$$r_d = \frac{\eta V_T}{I_{DC}} = \frac{26 \text{ mV}}{26 \mu\text{A}} = 1 \text{ k}\Omega$$

$$\frac{1}{\omega C_j} = \frac{1}{2 \times 10^6 \times 0.5 \times 10^{-9}} \Omega = 1 \text{ k}\Omega$$

Now, total impedance of the circuit will be,

$$Z = \left(r_d \parallel \frac{1}{j\omega C_j} \right) + 100 \Omega$$

$$\left(r_d \parallel \frac{1}{j\omega C_j} \right) = \frac{(1000)(-j1000)}{1000 - j1000} \Omega = \frac{-j(1+j)}{2} \text{ k}\Omega$$

$$= \frac{1}{2}(1-j) \text{ k}\Omega = (500 - j500) \Omega$$

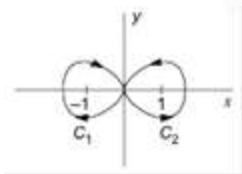
$$Z = 600 - j500 \Omega$$

$$|Z| = 100\sqrt{36 + 25} = 100\sqrt{61} \Omega$$

$$I_m = \frac{V_m}{|Z|} = \frac{5 \text{ mV}}{100\sqrt{61} \Omega} = \frac{50}{\sqrt{61}} \mu\text{A} = 6.40 \mu\text{A}$$

49. Ans.

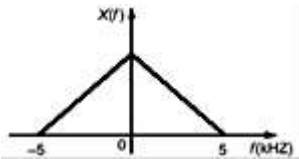
Given complex integral in the question can be solved as follows after denoting the encirclement properly,



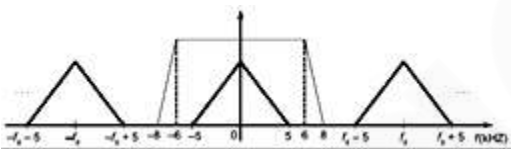
$$\begin{aligned} & \frac{1}{\pi j} \oint_C \frac{dz}{z^2 - 1} \\ &= 2 \left[\frac{1}{\pi j} \oint_{C_1} \frac{dz}{(z+1)(z-1)} + \frac{1}{\pi j} \oint_{C_2} \frac{dz}{(z+1)(z-1)} \right] \\ &= 2 \left[-\left(\frac{1}{z-1} \right) \Big|_{z=-1} + \left(\frac{1}{z+1} \right) \Big|_{z=1} \right] \\ &= 2 \left[-\left(-\frac{1}{2} \right) + \left(\frac{1}{2} \right) \right] = 2 \end{aligned}$$

50. Ans.

We have to assume an arbitrary spectrum for x(t) as shown below :



Then we would obtain spectrum of the sampled signal can be given as,



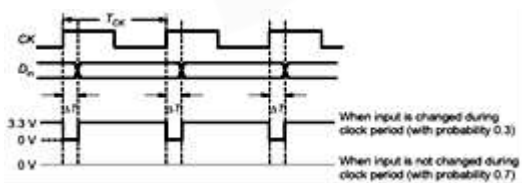
For proper reconstruction of the signal, the next sample must not overlap with previous sample,

$$\begin{aligned} f_s - 5 &\geq 8 \\ f_s &\geq 8 + 5 = 13 \text{ kHz} \end{aligned}$$

So, $f_{s(\min)} = 13 \text{ kHz}$

51. Ans.

The timing diagram for the circuit can be drawn as follows,



Now average voltage at node X can be calculated according to the timing diagram,

$$\begin{aligned}
 V_{x(\text{avg})} &= \left[0.3 \times 3.3 \left(1 - \frac{\Delta T}{T_{CK}} \right) \right] + [0.7 \times 0] V \\
 &= 0.3 \times 3.3 \times (1 - 0.15) V \\
 &= 0.3 \times 3.3 \times 0.85 V = 0.8415 V
 \end{aligned}$$

52. Ans.

We have the relation between the cutoff frequencies of two different modes as follows,

$$\begin{aligned}
 f_{c(01)} &= 2f_{c(10)} = \frac{2c}{2a} = \frac{c}{a} \\
 \frac{c}{2b} = \frac{c}{a} &\Rightarrow a = 2b \Rightarrow b = \frac{a}{2}
 \end{aligned}$$

Given operating frequency,

$$f = 1.25 f_{c(10)}$$

$$f_{c(10)} < 1.25 f_{c(10)} < [f_{c(10)} = 2f_{c(10)}]$$

According to the given frequency, the waveguide will work in TE_{10} mode clearly.

$$\lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{f_{c(10)}}{f} \right)^2}} = \frac{c/f}{\sqrt{1 - \left(\frac{1}{1.25} \right)^2}} = \frac{c/f}{0.6}$$

So,

$$\begin{aligned}
 \frac{c}{(1.25)f_{c(10)(0.6)}} &= \lambda_g = 4 \text{ cm} \\
 \frac{c}{f_{c(10)}} &= 3 \times 10^{-2} = 2a \\
 a &= 1.5 \text{ cm} \\
 b &= \frac{a}{2} = 0.75 \text{ cm}
 \end{aligned}$$

53. Ans.

Since we have an Op-Amp, Applying the concept of virtual ground,

$$V_o = -\frac{R_2}{R_1} \cdot V_{in}$$

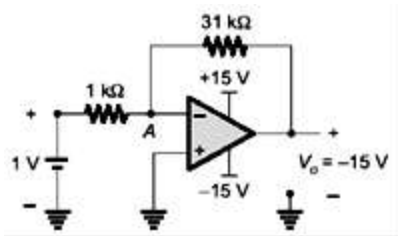
[∴ non-inverting amplifier]

$$\therefore V_o = -\frac{31\text{k}\Omega}{1\text{k}\Omega} \times 1\text{V}$$

$$V_o = -31\text{V} < -15\text{V}$$

Which is not possible.

So, the output voltage of the op-amp is equal to -15V.



Now applying KCL of node 'A', we get,

$$\frac{V_A - (-15)}{31\text{k}\Omega} + \frac{V_A - 1}{1\text{k}\Omega} = 0$$

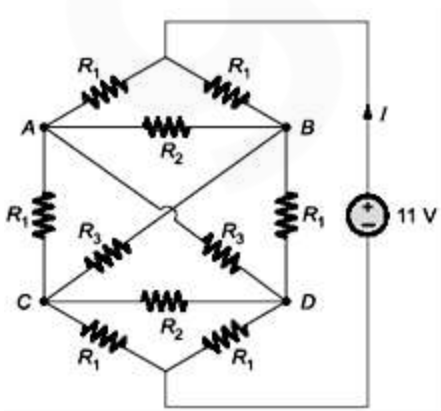
$$\frac{V_A}{31\text{k}\Omega} + \frac{V_A}{1\text{k}\Omega} = \frac{-15}{31\text{k}\Omega} + \frac{1}{1\text{k}\Omega}$$

$$V_A \left[\frac{1}{31} + \frac{1}{1} \right] = \frac{15}{31} + 1$$

$$V_A = 0.5\text{V}$$

54. Ans.

Redrawing the circuit by renaming the nodes as A, B, C and D.

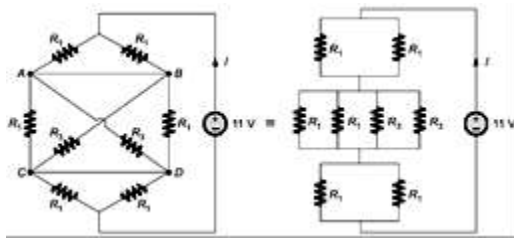


The given network is symmetric,

$$\text{So, } V_A = V_B \text{ and } V_C = V_D$$

Current through resistors R_2 is zero and as $V_A = V_B$ and $V_C = V_D$.

Electrically this circuit can be reduced as,



Total resistance R_T is resultant of following combination,

$$\begin{aligned} R_T &= 2(R_1 \parallel R_1) + (R_1 \parallel R_1 \parallel R_3 \parallel R_3) \\ &= R_1 + \left(\frac{R_1}{2} \parallel \frac{R_3}{2} \right) \end{aligned}$$

We have values for R_1 and R_3 ,

$$R_1 = 1 \Omega \text{ and } R_3 = 3 \Omega$$

$$\text{So, } R_T = 1 + \left(\frac{1}{2} \parallel \frac{3}{2} \right) \Omega = 1 + \frac{3/2}{4} = \frac{11}{8} \Omega$$

Thus, current through 11 V voltage source is,

$$I \frac{11V}{R_T} = \frac{11}{(11/8)} = 8 \text{ A}$$

55. Ans.

For solar cell open circuit voltage is given by,

$$V_{oc} = V_T \ln \left(\frac{I_{sc}}{I_0} \right)$$

Since, the Current through the solar cell is directly proportional to intensity of light,

$$V_{OC2} - V_{OC1} = V_T \ln \left(\frac{I_{SC2}}{I_{SC1}} \right) = V_T \ln \left(\frac{0.20}{1.0} \right)$$

$$V_{OC2} = V_{OC1} - 0.026 \ln(5)$$

$$= 0.65 - 0.041845 = 0.608 \text{ V}$$

$V_{OC2} = 0.608 \text{ V.}$

56. Ans. B.

We know the relation of wavelengths,

$$\lambda_R > \lambda_G > \lambda_B$$

Energy gap is related to wavelength as follows,

$$E_g \propto \frac{1}{\lambda}$$

So, $E_{gR} < E_{gG} < E_{gB}$

Materials with high energy gap will have high built-in voltages, when doping concentrations are same.

So, $V_R < V_G < V_B$

57. Ans. A.

In terms of State Space form, the Transfer function is given as,

$$T(s) = \frac{Y(s)}{U(s)} = C[sI - A]^{-1} B$$

$$A = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$C = [1.5 \quad 0.625]$$

$$[sI - A] = \begin{bmatrix} s+4 & 1.5 \\ -4 & s \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s^2 + 4s + 6)} \begin{bmatrix} s & -1.5 \\ 4 & s+4 \end{bmatrix}$$

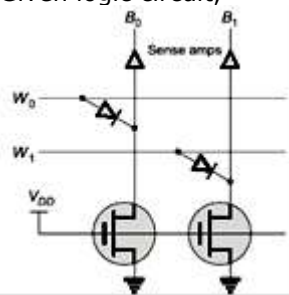
$$[sI - A]^{-1} B = \frac{1}{s^2 + 4s + 6} \begin{bmatrix} 2s \\ 8 \end{bmatrix}$$

$$C[sI - A]^{-1} B = \frac{1}{s^2 + 4s + 6} [1.5 \quad 0.625] \begin{bmatrix} 2s \\ 8 \end{bmatrix}$$

$$T(s) = \frac{3s + 5}{s^2 + 4s + 6}$$

58. Ans. A.

Given logic circuit,



The following can be observed,

When $W_0 = V_{DD}, B_0 = V_{DD}$; otherwise $B_0 = 0$

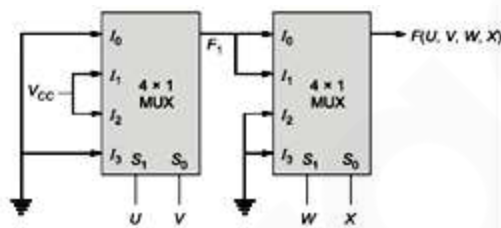
When $W_1 = V_{DD}, B_1 = V_{DD}$; otherwise $B_1 = 0$

So, $B_0 = W_0$ and $B_1 = W_1$

$$B_0 \ B_1 = \begin{matrix} W_0 & [1 & 0] \\ W_1 & [0 & 1] \end{matrix}$$

Hence,

59. Ans. C.



Output of the first 4x1 multiplexer can be expressed as,

$$F_1 = UV + UV$$

Output of the second 4x1 multiplexer can be expressed as,

$$F = \bar{W}XF_1 + \bar{W}XF_1 = \bar{W}F_1 = (UV + UV)\bar{W}$$

60. Ans. B.

Power Spectral Density of noise input,

$$S_N(f) = 0.5 \text{ W/Hz}$$

Power of \$y(t)\$,

$$P_y = \int_{-\infty}^{\infty} S_N(f) |H(f)|^2 df$$

$$= 0.50 \int_{-\infty}^{\infty} |H(f)|^2 df = 0.50 \int_{-\infty}^{\infty} |h(f)|^2 dt$$

Given the impulse response of the filter being used,

$$h(t) = \frac{1}{2} e^{-t^2/2}$$

So,

$$P_v = \frac{1}{2} \int_{-\infty}^{\infty} \left(\frac{1}{2} e^{-t^2/2} \right)^2 dt = \frac{1}{8} \int_{-\infty}^{\infty} e^{-t^2} dt$$

$$= \frac{\sqrt{\pi}}{8} = 0.22 \text{ W}$$

61. Ans. B.

According to data given in question, we can get the $I_{s(\min)}$ as follows,

$$V_1 = 6 \text{ V} \pm 5\% = 6 \text{ V} \pm 0.3 \text{ V} = 5.7 \text{ V to } 6.3 \text{ V}$$

$$I_L = \frac{5 \text{ V}}{1 \text{ k}\Omega} = 5 \text{ mA}$$

$$I_{s(\min)} = I_L + I_{z(\min)} = 5 \text{ mA} + 2 \text{ mA} = 7 \text{ mA}$$

$$I_s = \frac{V_1 - V_2}{R}$$

$$I_{s(\min)} = \frac{V_{1(\min)} - V_2}{R_{\max}} = 7 \text{ mA}$$

Now when $I_{s(\min)}$ flows that means Resistance is

$$R_{\max} = \frac{5.7 - 5}{7} \text{ k}\Omega = \frac{700}{7} \Omega = 100 \Omega$$

maximum,

Now obtaining Maximum current that could flow through Zener Diode due to fluctuation in Source and

removal of Load while $R = 100 \Omega$,

$$I_{s(\max)} = \frac{6.3 - 5}{100} \text{ A} = 13 \text{ mA}$$

$$I_{z(\max)} = I_{s(\max)} - I_L = 13 \text{ mA} - 5 \text{ mA} = 8 \text{ mA}$$

$$P_{z(\min)} = V_{z(\max)} = (5 \times 8) \text{ mW} = 40 \text{ mW}$$

62. Ans. B.

Attenuation constant is related with skin depth as follows, And according to given condition of 20 dB attenuation we can get required depth by following calculation,

$$\alpha = \frac{1}{\text{skin depth}} = 10 \text{ Np/m}$$

$$20 \log_{10} \left(\frac{E_o}{E_x} \right) = 20 \text{ dB}$$

$$\frac{E_o}{E_x} = 10 \Rightarrow (E_x) = \frac{E_o}{10}$$

$$E_x = E_o e^{-\alpha x} = E_o e^{-10x} = \frac{E_o}{10}$$

$$e^{-10x} = \frac{1}{10}$$

$$x = \frac{1}{10} \ln(10) = 0.23 \text{ m}$$

63. Ans. D.

According to given input signal, we can obtain an output signal as follows,

$$v_i(t) = A_c \cos(2\pi f_c t) + \cos(2\pi f_m t)$$

$$v_o(t) = av_i(t) + bv_i^2(t)$$

$$= [aA_c \cos(2\pi f_c t) + a \cos(2\pi f_m t)] + b$$

$$[A_c^2 \cos^2(2\pi f_c t) + \cos^2(2\pi f_m t) + 2A_c \cos(2\pi f_c t) \cos(2\pi f_m t)]$$

When the signal is passed through given Band Pass Filter,

$$y(t) = aA_c \cos 2\pi f_c t + 2bA_c \cos(2\pi f_c t) \cos(2\pi f_m t)$$

$$= aA_c \left[1 + \frac{2b}{a} \cos(2\pi f_m t) \right] \cos(2\pi f_c t)$$

The Modulation index can be obtained through output of the BPF,

$$\mu = \frac{2b}{a}$$

We have been given in the problem statement that Side Band contains half the carrier power,

$$P_{sb} = \frac{\mu^2}{2} P_c = \frac{1}{2} P_c$$

So, $\mu^2 = 1 \Rightarrow \mu = 1$

Comparing with the value obtained in form of a and b,

$$\frac{2b}{a} = 1$$

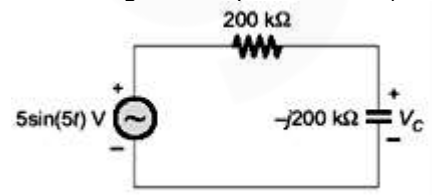
$$\frac{a}{b} = 2$$

64. Ans. C.

Reactance of capacitor with respect to given value of C and ω ,

$$\frac{1}{\omega C} = \frac{1}{5 \times 10^{-6}} = 200 \text{ k}\Omega$$

Redrawing the simplified circuit,



Applying Voltage division rule to get voltage across Capacitor,

$$V_c = \frac{5\angle 0^\circ}{200 - j200} \times (-j200)$$

$$V = \frac{5\angle 0^\circ \times 1\angle -90^\circ}{\sqrt{2}\angle -45^\circ} V$$

$$= \frac{5}{\sqrt{2}} \angle -45^\circ V = 2.5\sqrt{2} \sin\left(5t - \frac{\pi}{4}\right) V$$

$$= 2.5\sqrt{2} \sin(5t - 0.25\pi)$$

65. Ans. A.

Given Differential equation,

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2y} + \frac{y}{x}$$

We need to use suitable substitution here,

$$\text{Put, } \frac{y}{x} = t$$

$$\frac{dy}{dx} = t + x \frac{dt}{dx}$$

$$1 + x \frac{dt}{dx} = \frac{x}{2t} + \frac{tx}{2} + t$$

$$x \frac{dt}{dx} = x \left(\frac{1}{2t} + \frac{t}{2} \right)$$

$$x \frac{dt}{dx} = x \left(\frac{1+t^2}{2t} \right)$$

$$\int \frac{2t}{1+t^2} dt = \int \frac{dx}{x} + C$$

$$\ln(1+t^2) = x + C$$

$$t = \frac{y}{x}$$

After simplification we obtain the following relation,

$$\ln\left(1 + \frac{y^2}{x^2}\right) = x + C$$

Given that the curve passes through points,

$x = 1, y = 0$, we can obtain the value of constant C.

$$\ln\left(1 + \frac{0}{1}\right) = \ln(1) = 0 = 1 + C$$

$$C = -1$$

So, $\ln\left(1 + \frac{y^2}{x^2}\right) = x - 1$
