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# GATE 2018
# Civil Engineering
## Questions & Solutions
## Session 1

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# GENERAL APTITUDE

1. The temperature $T$ in a room varies as a function of the outside temperature $T_0$ and the number of persons in the room $p$, according to the relation $T = K(\theta p + T_0)$, where $\theta$ is $K$ are constants. What would be the value of θ given the following data?

| $T_0$ | $p$ | $T$ |
|---|---|---|
| 25 | 2 | 32.4 |
| 30 | 5 | 42.0 |

A. 0.8 B. 1.0
C. 2.0 D. 10.0

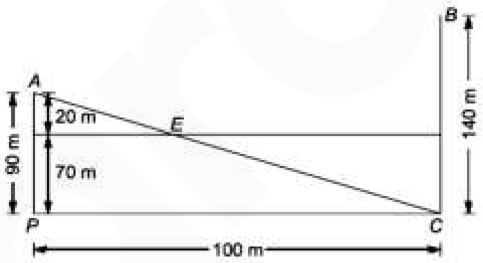
Ans. B

2. "The driver applied the ___________ as soon as she approached the hotel where she wanted to take a ___________."
The words that best fill the blanks in the above sentence are
A. brake, break B. break, break
C. brake, brake D. break, brake

Ans. A
- Brake is a device which is used for stopping or moving a vehicle.
- Break refers to a pause in work or during an activity.

3. Tower $A$ is 90 m tall and tower $B$ is 140 m tall. They are 100 m apart. A horizontal skywalk connects the floors at 70 m in both the towers. If a taut rope connects the top of tower $A$ to the bottom of tower $B$, at what distance (in meters) from tower $A$ will the rope intersect the skywalk?

Ans. (22.22)




$$\Delta AME = \Delta APC$$

$$\frac{AM}{AP} = \frac{ME}{PC}$$

$$\Rightarrow \frac{20}{90} = \frac{ME}{100}$$

$$\Rightarrow ME = 22.22$$

4. "It is no surprise that every society has had codes of behaviour; however; the nature of these codes is often _________."
The word that best fills the blank in the above sentence is
A. unpredictable B. simple
C. expected D. strict

Ans. A
Unpredictable – Contrary word required

5. Hema's age is 5 years more than twice Hari's age. Suresh's age is 13 years less than 10 times Hari's age. If Suresh is 3 times as old as Hema. How old is Hema?
A. 14 B. 17
C. 18 D. 19

Ans. D
Using options (a) and (c) cannot be answer.
As Hema's age is twice of a natural number +5.
(b) and (d) remain.
Let us take Hema's age as option (d) which is 19.
Hari's age = 7, and Suresh age = 57
Verifies all condition "answer (d)"

$$\text{Hema} = 2\ \text{Hari} + 5$$

$$\text{Suresh} = 10\ \text{Hari} - 13 = 3\ \text{Hema}$$

Solve :

$$\text{Hari} = 7$$

$$\text{Hema} = 19$$

$$\text{Suresh} = 57$$

6. Consider a sequence of number $a_1, a_2, a_3 \ldots, a_n$ where $a_n = \frac{1}{n} - \frac{1}{n+2}$, for each integer $n > 0$. What is the sum of the first 50 terms?

A. $\left(1+\frac{1}{2}\right) - \frac{1}{50}$

B. $\left(1+\frac{1}{2}\right) + \frac{1}{50}$

C. $\left(1+\frac{1}{2}\right) - \left(\frac{1}{51}+\frac{1}{52}\right)$

D. $1 - \left(\frac{1}{51}+\frac{1}{52}\right)$

Ans. C
Sum of series will

$$\left(1-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)\ldots\left(\frac{1}{48}-\frac{1}{50}\right)+\left(\frac{1}{49}-\frac{1}{51}\right)+\left(\frac{1}{50}-\frac{1}{52}\right)$$

All like terms will cancel out and we will be left with

$$\left(1+\frac{1}{2}\right)-\left(\frac{1}{51}+\frac{1}{52}\right)$$

7. A fruit seller sold a basket of fruits at 12.5% loss. Had he sold it for Rs. 108 more, he would have made a 10% gain. What is the loss in Rupees incurred by the fruit seller?

A. 48 B. 52
C. 60 D. 108

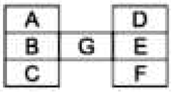
Ans. C

$$12.5\%x + 10\%x = 108$$

$$x = \frac{108}{22.5}$$

So loss $108 \times \frac{12.5}{22.5} = 60$

8. Each of the letters arranged as below represents a unique from 1 to 9. The letters are positioned in the figure such that $(A \times B \times C), (B \times G \times E)$ and $(D \times E \times F)$ are equal. Which integer among the following choices cannot be represented by the letters A, B, C, D, E, F or G?




A. 4 B. 5
C. 6 D. 9

Ans. B

$A \times B \times C = B \times G \times E = D \times E \times F = 72$

$8 \times 9 \times 1 = 9 \times 2 \times 4 = 3 \times 4 \times 6 = 72$

Any of $A, B, C, D, E, F, G$ cannot be 5.

9. The price of a wire made of a superalloy material is proportional to the square of its length. The price of 10 m length of the wire is Rs. 1600. What would be the total price (in Rs.) of two wires of lengths 4 m and 6 m?

A. 768 B. 832
C. 1440 D. 1600

Ans. B

$C \propto W^2$

$C = kW^2$

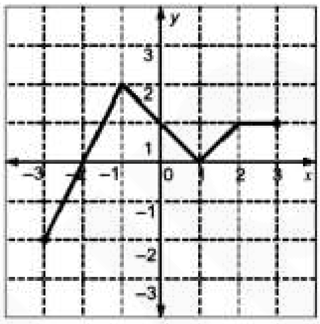
$\Rightarrow \quad C = k(10)^2 = 100k = 1600 \Rightarrow k = 16$

$C_1 = k(4)^2 = 16k$

$C_2 = k(6)^2 = 36k$

Now total cost $= 52k = 52 \times 16 = 832$

10. What of the following function(s) in an accurate description of the graph for the range(s) indicated?




A. $y = 2x + 4$ for $-3 \le x \le -1$

B. $y = |x - 1|$ for $-1 \le x \le 2$

C. $y = ||x| - 1|$ for $-1 \le x \le 2$

D. $y = 1$ for $2 \le x \le 3$

A. (i), (ii) and (iii) only
B. (i), (ii) and (iv) only
C. (i) and (iv) only
D. (ii) and (iv) only

Ans. B

Put value and verify

(i) $y = 2x + 4$ is true in $-3 \le x \le -1$

On putting $x = -2, y = -2$ and $x = -2, y = 0$ and $x = 1, y = 2$

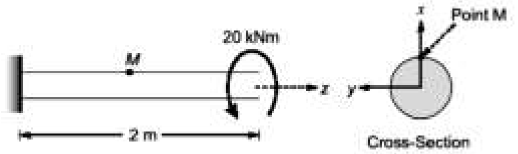
(ii) $y = |x - 1|$ is also true $(x = -1, y = 2)$, $(x = 0, y = 1)$ and $(x = 1, y = 0)$

(iv) $y = 1$ in $(2 \le x \le 3)$ always true

(i), (ii) and (iv) are true.

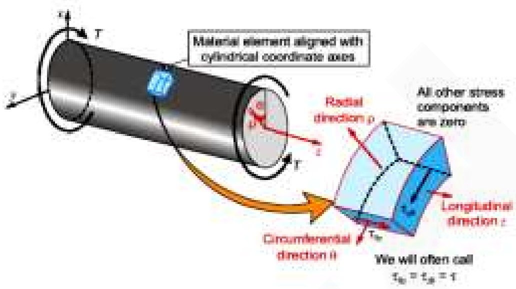
## CIVIL ENGINEERING

1. A solid circular beam with radius of 0.25 m and length of 2 m is subjected to a twisting moment of 20 kNm about the z-axis at the free end, which is the only load acting as shown in the figure. the stress component $\tau_{xy}$ at Point 'M' in the cross-section of the beam at a distance of 1 m from the fixed end is




A. 0.0 MPa B. 0.51 MPa
C. 0.815 MPa D. 2.0 MPa

Ans. A




The only non-zero stresses are $\tau_{\theta z} = \tau_{z\theta} = \tau$, if θ is 90° then $\theta = y$

Hence $\tau_{zy} = \tau_{yz} = \tau_{max} = 16T/\pi d^3 = 0.815 MPa$

But in rest of the planes shear stresses are zero, hence, $\tau_{xy} = \tau_{yz} = 0$

2. In a fillet weld, the direct shear stress and bending tensile stress are 50 MPa and 150 MPa, respectively. As per IS 800 : 2007, the equivalent stress (in MPa, up to two decimal places) will be _____________.

Ans. (173.21)

Direct bending tensile stress,

$$f_a = 150 \text{ MPa}$$

Direct shear stress, $q = 50$ MPa

According to IS 800 : 2007, clause 10.5.10.1.1

The equivalent stress $f_e = \sqrt{f_a^2 + 3q^2} \le \dfrac{f_u}{\sqrt{3}\gamma_{mw}}$

$$f_e = \sqrt{150^2 + 3 \times 50^2} = 173.21 \text{ MPa}$$

$$\Rightarrow \quad f_e \le \frac{f_u}{\sqrt{3}\gamma_{mw}} \left( = \frac{400}{\sqrt{3} \times 1.25} = 184.75 \text{MPa} \right)$$

(Hence OK)

Note : The above check for combination of stress need not be done for fillet welds where the sum of normal and shear stresses does not exceed $f_{wd}$ [Clause 10.5.10.1.2. (b)].

Thus, $f_a + q = 150 + 50 = 200$ MPa

$$f_a + q > f_{wd} \; (= 184.75 \text{ MPa})$$

$$\Rightarrow \quad f_e = 173.21 \text{ MPa}$$

Thus, the weld is designed for an equivalent stress of 173.21 MPa.

3. The percent reduction in the bearing capacity of a strip footing resting on sand under flooding condition (water level at the base of the footing) when compared to the situation where the water level is at a depth much greater than the width of footing, is approximately

A. 0 B. 25
C. 50 D. 100

Ans. B

For strip footing on sand $(c = 0)$

$$q_u = \gamma D_f N_q + 0.5 B \gamma N_\gamma$$

In flooding condition water level rises to base of footing hence IIIrd term unit weight of soil will change and IInd term unit weight will be unaffected.

$$\therefore \quad q_u = \gamma D_f N_q + 0.5 B \gamma N_\gamma$$

$$\therefore \quad \gamma \simeq$$

Hence third term reduced and second term will be same thereby percentage reduction will not be 50%.

According to option approach answer should be 25%.

Note : If water table rises to ground level then both γ will reduce to γ hence percentage reduction would be approximately 50%.

4. The deformation in concrete due to sustained loading is

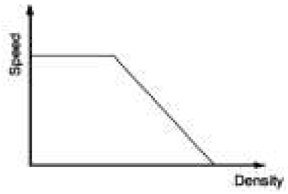
A. creep B. hydration

C. segregation D. shrinkage

Ans. A

Creep is inelastic deformation with time due to sustained loading.

5. The speed-density relationship for a road section is shown in the figure.




The shape of the flow-density relationship is

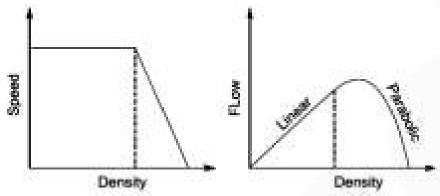
A. piecewise linear

B. parabolic

C. initially linear then parabolic

D. initially parabolic then linear

Ans. C




6. A steel column of ISHB 350 @ 72.4 kg/m is subjected to a factored axial compressive load of 2000 kN. The load is transferred to a concrete pedestal of grade M20 through a square base plate. Consider bearing of concrete as 0.45 $f_{ck}$, where $f_{ck}$ is the characteristic strength of concrete. Using limit state method and neglecting the self weight of base plate and steel column, the length of a side of the base plate to be provided is

A. 39 cm B. 42 cm

C. 45 cm D. 48 cm

Ans. D

Area required for base plate

$$= \frac{\text{Factored load}}{\text{Bearing capacity of concrete}}$$

$$= \frac{2000 \times 10^3}{0.45 \times 20} 222222.222 \text{ mm}^2$$

So, side of base plate $= \sqrt{\text{Area}}$

$= 471.4$ mm

$= 47.14$ cm

Since, provided area must be more than required so answer should be 48 cm.

7. For routing of flood in a given channel using the Muskingum method, two of the routing coefficients are estimated as $C_0 = -0.25$ and $C_1 = 0.55$. The value of the third coefficient $C_2$ would be ___________.

Ans. (0.7)

In Muskingum flood routing method $C_0 + C_1 + C_2 = 1$

$$\Rightarrow \quad C_2 = 1 - (-0.25) - 0.55 = 0.7$$

8. A column of height h with a rectangular cross-section of size $a \times 2a$ has a buckling load of P. If the cross-section is changed to $0.5a \times 3a$ and its height changed to 1.5 h, the buckling load of the redesigned column will be

A. $\frac{P}{12}$ B. $\frac{P}{4}$

C. $\frac{P}{2}$ D. $\frac{3P}{4}$

Ans. A

For column, $P = \frac{\pi^2 EI_{min}}{L^2}$

$$= \frac{\pi^2 E\left(\frac{2a \times a^3}{12}\right)}{h^2} = \frac{\pi^2 Ea^4}{6h^2}$$

For new column, $P' = \frac{\pi^2 E\left[\frac{3a \times (0.5a)^3}{12}\right]}{(1.5h)^2}$

$$= \frac{1}{2} \times \frac{\pi^2 Ea^4}{6h^2} = \frac{P}{12}$$

9. Bernoulli's equation is applicable for

A. viscous and compressible fluid flow

B. inviscid and compressible fluid flow

C. inviscid and incompressible fluid flow

D. viscous and income pressible fluid flow

Ans. C

10. The frequency distribution of the compressive strength of 20 concrete cube specimen is given in the table.

| f (MPa) | Number of specimens with compressive strength equal to f |
|---|---|
| 23 | 4 |
| 28 | 2 |
| 22.5 | 5 |
| 31 | 5 |
| 29 | 4 |

If μ is the mean strength of the specimens and σ is the standard deviation, the number of specimens (out of 20) with compressive strength less than $\mu - 3\sigma$ is __________.

Ans. (0)

Average strength,

$$\mu = \frac{(4\times23)+(2\times28)+(5\times22.5)+(5\times31)+(4\times29)}{20} = 26.575 \text{ MPa}$$

$$\sigma = \sqrt{\frac{\Sigma(\mu - f)^2}{n-1}}$$

$$= \sqrt{\frac{(26.575-23)^2\times4+(26.575-28)^2\times2+(26.575-22.5)^2\times5+(26.575-31)^2\times5+(26.575-29)^2\times4}{(20-1)}} = 3.4$$

Now, $\mu - 3\sigma = 26.575 - 3\times3.7 = 15.48$

Thus, no specimen is having compressive strength less than $\mu - 3\sigma$.

11. The width of a square footing and the diameter of a circular footing are equal. If both the footings are placed on the surface of sandy soil, the ratio of the ultimate bearing capacity of circular footing to that of square footing will be

A. $\frac{4}{3}$ B. 1

C. $\frac{3}{4}$ D. $\frac{2}{3}$

Ans. C

Footing placed on surface

∴ $D_f = 0$

For square footing,

$$q_u = CN_c + \gamma D_f N_q + 0.4B\gamma N\gamma$$

For circular footing,

$$q_u = CN_c + \gamma D_f N_q + 0.3B\gamma N\gamma$$

For sandy soil, $C = 0$

$$Ratio\ \frac{(q_u)_{circular}}{(q_u)_{square}} = \frac{3}{4}$$

12. In a shrinkage limit test, the volume and mass of a dry soil pat are found to be 50 $cm^3$ and 88 g. respectively. The specific gravity of the soil solids is 2.71 and the density of water is 1 g/cc. The shrinkage limit (in % up to two decimal places) is__________.

Ans. (19.90)

Dry soil mass = 88 gm

Volume of dry soil = 50 cc

Dry density of soil mass,

$$\rho_d = \frac{M_d}{V_d} = \frac{88}{50} \text{gm/cc}$$

∵ Shrinkage limit,

$$w_s = \frac{1}{R} - \frac{1}{G} = \frac{1}{(\rho_d / \rho_w)} - \frac{1}{G}$$

$$w_s = \frac{\rho_w}{\rho_d} - \frac{1}{G} = \frac{1}{1.76} - \frac{1}{2.71} = 0.1990 = 19.90\%$$

13. A core cutter of 130 mm height has inner and outer diameters of 100 mm and 106 mm respectively. The area ratio of the core cutter (in %, up to two decimal places) is ______________.

Ans. (12.36)

$$\text{Area ratio} = \frac{A_{outer} - A_{inner}}{A_{inner}} \times 100$$

$$= \frac{\frac{\pi}{4}D_0^2 - \frac{\pi}{4}D_i^2}{\frac{\pi}{4}D_i^2} \times 100$$

$$= \frac{\frac{\pi}{4}(106)^2 - \frac{\pi}{4}(100)^2}{\frac{\pi}{4}(100)^2} \times 100 = 12.36\%$$

14. Two rectangular under-reinforced concrete beam sections X and Y are similar in all aspects except that the longitudinal compression reinforcement in section Y is 10% more. Which one of the following is the correct statement?

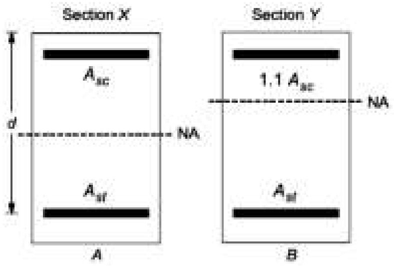
A. Section X has less flexural strength and is less ductile than section Y

B. Section X has less flexural strength but is less ductile than section Y

C. Section X and Y have equal flexural strength but different ductility

D. Section X and Y have equal flexural strength and ductility.

Ans. A



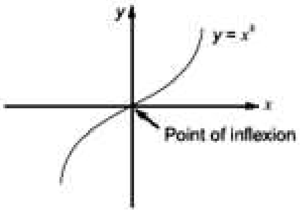

Due to presence of more compression steel in section Y, NA of section of Y is above than as of X. It means Y is more under-reinforced than X so ductility of Y is more. Since compression steel of Y is more so flexure resistance of X is less than as of Y.

15. At the point $x = 0$, the function $f(x) = x^3$ has

A. local maximum
B. local minimum
C. both local maximum and minimum
D. neither local maximum nor local minimum

Ans. D

$f(x) = x^3$ at $x = 0$




At $x = 0$, the function $y = x^3$ has neither minima nor maxima.

16. A well-designed signalized intersection is one in which the

A. crossing conflicts are increased
B. total delay is minimized
C. cycle time is equal to the sum of red and green times in all phases
D. cycle time is equal to the sum of red and yellow times in all phases

Ans. B

17. A flow field is given by Value of the z-component of the angular velocity (in radians per unit time, up to two decimal places) at the point (0, -1, 1) is _________.

Ans. (1.5)

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$
$$= \frac{1}{2}\left[\frac{\partial}{\partial x}(-xy) - \frac{\partial}{\partial y}(y)^2\right]$$
$$= \frac{1}{2}\left[-y - 2y\right]$$
$$= -\frac{3y}{2}$$

At point (0,-1,1) $\omega_z = -\frac{3}{2} \times -1 = 1.50$ rad/s

18. A bitumen sample has been graded as VG30 as per IS : 73-2013. The '30' in the grade means that

A. penetration of bitumen at 25°C is between 20 and 40
B. viscosity of bitumen at 60°C is between 2400 and 3600 Poise
C. ductility of bitumen at 27°C is more than 30 cm
D. elastic recovery of bitumen at 15°C is more than 30%

Ans. B

19. The Le Chatelier apparatus is used to determine

A. Compressive strength of cement
B. fineness of cement
C. setting time of cement
D. soundness of cement

Ans. D

Le Chatelier Apparatus is used to determine the soundness of cement as per IS code 4031 (part 3); this cement testing procedure is called Le Chatelier test for determining the unsoundness properties of cement due t presence of "free lime".

20. A city generates $40 \times 10^6$ kg of municipal solid waste (MSW) per year, out of which only 10% is recovered/recycled and the rest goes to landfill. The landfill has a single lift of 3 m height and is compacted to a density of 550 kg/m$^3$. If 80% of the landfill is assumed to be MSW, the landfill area (in m$^2$, up to one decimal place) required would be ___________.

Ans. (27272.7)

Total weight generated by city $= 40 \times 10^6$ kg/year

Weight of MSW going into landfill

$$= 0.9 \times 40 \times 10^6 \text{ kg/year}$$
$$= 36 \times 10^6 \text{ kg/year}$$

Compacted density $= 550$ kg/m$^3$

Compacted volume of MSW

$$= \frac{36 \times 10^6 \text{ kg/year}}{550 \text{ kg/m}^3} = 65454.545 \text{ m}^3/\text{year}$$

Total landfill volume = Volume of MSW + Volume of cover

Given,

Volume of MSW $= 0.8 \times$ Total landfill volume

∴ Volume of cover $= 0.2 \times$ Total landfill volume

$\therefore$ Total landfill volume $= \frac{65454.5454}{0.8}$ $m^3$/year $= 81818.18175$ $m^3$/year

Height of landfill = 3 m

$\therefore$ Area of landfill $= \frac{81818.18175}{3}$

$= 27272.7$ $m^2$/year

21. There are 20,000 vehicles operating in a city with an average annual travel of 12,000 km per vehicle. The $NO_x$ emission rate is 2.0 g/km per vehicle. The total annual release of $NO_x$ will be

A. 4,80,000 kg
B. 4,800 kg
C. 480 kg
D. 48 kg

Ans. A

Total no. of kms. travelled by all the vehicles

$$= 20000 \times 12000 \text{ km}$$
$$= 24 \times 10^7 \text{ km}$$

Total $NO_x$ emission $= 2\,\text{g/km} \times 24 \times 10^7$ km

$$= 48 \times 10^7 \text{ g}$$
$$= 48 \times 10^4 \text{ kg}$$

22. For the given orthogonal matrix Q,

$$Q = \begin{bmatrix} \frac{3}{7} & \frac{2}{7} & \frac{6}{7} \\ -\frac{6}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{6}{7} & -\frac{3}{7} \end{bmatrix}$$

The inverse is

A. $Q = \begin{bmatrix} \frac{3}{7} & \frac{2}{7} & \frac{6}{7} \\ -\frac{6}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{6}{7} & -\frac{3}{7} \end{bmatrix}$

B. $Q = \begin{bmatrix} -\frac{3}{7} & -\frac{2}{7} & -\frac{6}{7} \\ \frac{6}{7} & -\frac{3}{7} & -\frac{2}{7} \\ -\frac{2}{7} & -\frac{6}{7} & \frac{3}{7} \end{bmatrix}$

C. $Q = \begin{bmatrix} \frac{3}{7} & -\frac{6}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{3}{7} & \frac{6}{7} \\ \frac{6}{7} & \frac{2}{7} & -\frac{3}{7} \end{bmatrix}$

D. $Q = \begin{bmatrix} -\frac{3}{7} & \frac{6}{7} & -\frac{2}{7} \\ -\frac{2}{7} & -\frac{3}{7} & -\frac{6}{7} \\ -\frac{6}{7} & -\frac{2}{7} & \frac{3}{7} \end{bmatrix}$

Ans. C

$$|Q| = \frac{3}{7}\left(-\frac{9}{49} - \frac{12}{49}\right) - \frac{2}{7}\left(\frac{18}{49} - \frac{4}{49}\right) + \frac{6}{7}$$
$$\left(\frac{-36}{49} - \frac{6}{49}\right) = -1$$

$$\text{Adj. Q} = \begin{bmatrix} -\frac{21}{49} & \frac{42}{49} & -\frac{14}{49} \\ -\frac{14}{49} & -\frac{21}{49} & -\frac{42}{49} \\ -\frac{42}{49} & -\frac{14}{49} & \frac{21}{49} \end{bmatrix}$$

$$\therefore \quad Q^{-1} = \frac{AdjQ}{|Q|} = Q = \begin{bmatrix} \frac{3}{7} & -\frac{6}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{3}{7} & \frac{6}{7} \\ \frac{6}{7} & \frac{2}{7} & -\frac{3}{7} \end{bmatrix}$$

23. A 10 m wide rectangular channel carries a discharge of 20 $m^3$/s under critical condition. Using $g = 9.81$ m/$s^2$, the specific energy (in m, up to two decimal places) is ____________

Ans. (1.11)

$$E_c = \frac{3}{2} y_c$$
$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

Here, $q = \frac{20}{10} = 2$

$$\Rightarrow \quad E_c = \frac{3}{2}\left(\frac{2^2}{9.81}\right)^{1/3} = 1.11$$

24. A 1:50 model of a spillway is to be tested in the laboratory. The discharge in the prototype spillway is 1000 $m^3$/s. The corresponding discharge (in $m^3$/s up to two decimal places) to be maintained in the

model, neglecting variation in acceleration due to gravity, is ___________.

Ans. (0.06)

Froude law is valid

$$Q_r = L_r^{2.5}$$

$$\frac{Q_m}{Q_p} = \left(\frac{1}{50}\right)^{2.5}$$

$$\frac{Q_m}{1000} = \left(\frac{1}{50}\right)^{2.5}$$

$$Q_m = 0.0566 \text{ m}^3/\text{s}$$

So, $Q_m \simeq$ n³/s

25. Which one of the following matrices is singular?

A. $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ B. $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$ D. $\begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix}$

Ans. C

Option (a): $|A| = 6 - 5 = 1$

Option (b): $|A| = 9 - 4 = 5$

Option (c): $|A| = 12 - 12 = 0$

Option (d): $|A| = 8 - 18 = -10$

Hence matrix (c) is singular.

26. A waste activated sludge (WAS) is to be blended with green waste (GW). The carbon (C) and nitrogen (N) contents, per kg of WAS and GW on dry basis are given in the table.

| Parameter | WAS | GW |
|---|---|---|
| Carbon (g) | 54 | 360 |
| Nitrogen (g) | 10 | 6 |

The ratio of WAS to GW required (up to two decimal places) to achieved a blended C:N ratio of 20:1 on dry basis is _________.

Ans. (1.64)

Let 20 kg xof C and 1 kg of N is required

Let x kg of WAS is taken

∴ Carbon in x kg = 0.054 x kg

Nitrogen in x kg = 0.010 x kg

Let y kg of GW is taken

∴ Carbon in x kg = 0.360 y kg

Nitrogen in x kg = 0.006 x kg

Total Carbon,

$$0.054x + 0.36y = 20 \text{ kg}$$

Total Nitrogen,

$$0.01x + 0.0\ 06\ y = 1 \text{ kg}$$

$$x = 73.26$$

$$y = 44.566$$

$$\therefore \quad \frac{x}{y} = \frac{\text{WAS}}{\text{GW}} = 1.6438 \simeq$$

27. Variation of water depth (y) in a gradually varied open channel flow is given by the first order differential equation

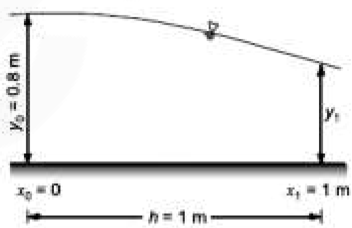
$$\frac{dy}{dx} = \frac{1 - e^{-\frac{10}{3}\ln(y)}}{250 - 45e^{-3\ln(y)}}$$

Given initial conditions: $y(x = 0) = 0.8$ m.

The depth (in m, up to three decimal places) of flow at a downstream section at $x = 1$ from one calculation step of Single Step Euler Method is ____________.

Ans. (0.793)

$$\frac{dy}{dx} = \frac{1 - e^{-\frac{10}{3}\ln y}}{250 - 45e^{-3\ln y}}$$




$$y_1 = y_0 + \text{hf}(x_0, y_0)$$

$$\Rightarrow \quad y_1 = 0.8 + 1\left[\frac{1 - e^{-\frac{10}{3}\ln 0.8}}{250 - 45e^{-3\ln 0.8}}\right]$$

$$= 0.8 + 1\left(\frac{-1.1039}{162.109}\right)$$

$$= 0.793 \text{ m}$$

28. A rapid sand filter comprising a number of filter beds is required to produce 99 MLD of potable water. Consider water loss during backwashing as 5%, rate of filtration as 6.0 m/h and length to width ratio of filter bed as 1.35. The width of each filter bed is to be kept equal to 5.2 m. One additional filter bed is to be provided to take care of break-down repair and maintenance. The total number of filter required will be

A. 19 B. 20

C. 21 D. 22

Ans. C

Total water to be filtered $= 99 \times 1.05$ MLD $= 103.95$ MLD

(Addition of 5% to be used for backwashing)

$$\frac{L}{B} = 1.35 \qquad \text{where } B = 5.2 \text{ m}$$

$\therefore \qquad L = 7.02$ m

$\therefore$ Surface area of each filter $= 36.504 \text{ m}^2$

Total surface area required

$$= \frac{\text{Discharge through filter}}{\text{Rate of filtration}} = \frac{10.3.95 \times 10^3}{6 \times 24}$$

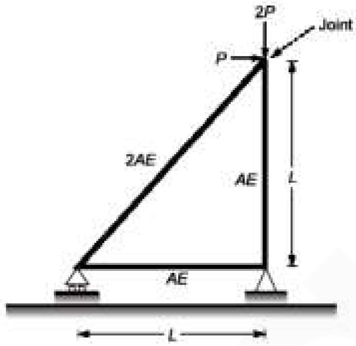
$= 721.875 \text{ m}^2$

Total no. of working units required

$$= \frac{721.875}{36.504} = 19.77 \text{ filters} = 20 \text{ filters}$$

1 unit is to added as standby, thus total no. of units required = 21

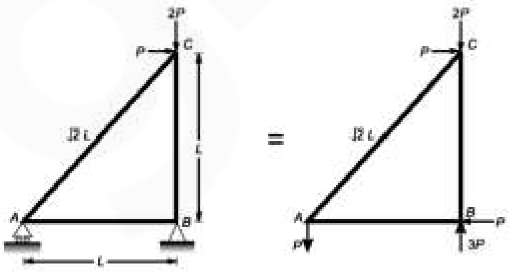
29. Consider the deformable pin-jointed truss with loading, geometry and section properties as shown in figure.




Given that $E = 2 \times 10^{11} \text{ N/m}^2$, $A = 10 \text{ mm}^2$, $L = 1$ m and $P = 1$ kN. the horizontal displacement of Joint C (in mm, up to one decimal place) is ____________.

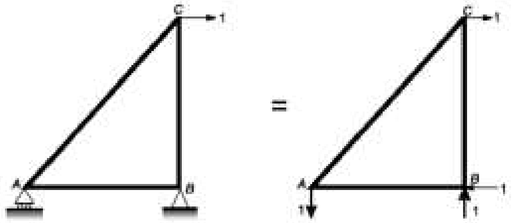
Ans. (3.4)

Force is each member due to applied loading.



$$F_{AB} = P(\text{Comp.})$$
$$F_{BC} = 3P(\text{Comp.})$$
$$F_{AC} = \sqrt{2}\,P(\text{Tension})$$

Force in each member due to unit load.



$$F_{AB} = 1(\text{Comp.})$$
$$F_{BC} = 1(\text{Comp.})$$
$$F_{AC} = \sqrt{2}\ (\text{Tension})$$

| Member | P | k | L | AE | $\frac{PkL}{AE}$ |
|---|---|---|---|---|---|
| AB | $-P$ | $-1$ | L | AE | $\frac{PL}{AE}$ |
| BC | $-3P$ | $-1$ | L | AE | $\frac{3PL}{AE}$ |
| CA | $\sqrt{2}P$ | $\sqrt{2}$ | $\sqrt{2}L$ | 2AE | $\frac{2\sqrt{2}PL}{2AE}$ |
| | | | | | $\Sigma = \frac{5.414PL}{AE}$ |

$\therefore$ Total deflection $= \delta_{H_c} = \Sigma \frac{PkL}{AE}$

$$= \frac{5.414 \times (1000\text{N}) \times (1000\,\text{mm})}{(10 \text{ mm}^2) \times (2 \times 10^5 \text{ N/m}^2)} = 2.7 \text{ mm}$$

30. Rainfall depth over a watershed is monitored through six number of well distributed rain gauges. Gauged data are given below:

| Rain Gauge Number | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Rainfall Depth (mm) | 470 | 465 | 435 | 525 | 480 | 510 |
| Area of Thiessen Polygon $(\times 10^4 \text{m}^2)$ | 95 | 100 | 98 | 80 | 85 | 92 |

The Thiessen mean value (in mm, up to one decimal place) of the rainfall is ____________

Ans. (479.1)

Thiessen mean value;

$$P_{avg} = \frac{\sum_{i=1}^{6} P_i A_i}{\sum_{i=1}^{6} A_i}$$

$$\Rightarrow \quad P_{avg} = \frac{470 \times 95 + 465 \times 100 + 435 \times 98 + 525 \times 80 + 480 \times 85 + 510 \times 92}{95 + 100 + 98 + 80 + 85 + 92}$$

$= 479.09$ mm

31. The infiltration rate $f$ in a basin under ponding condition is given by $f = 30 + 10e^{-2t}$, where, $f$ is in mm/h and $t$ is time in hour. Total depth of infiltration (in mm, up to one decimal place) during the last 20 minutes of a storm of 30 minutes duration is __________

Ans. (11.7)

Infiltration rate $f(t) = 30 + 10e^{-2t}$

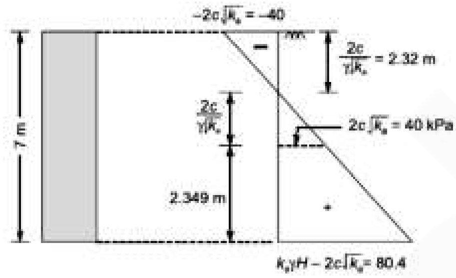
Total infiltration depth in time 10 min. to 30 min. i.e., 0.166 hour to 0.5 hour

$$= \int_{0.166}^{0.5} \left(30 + 10e^{-2t}\right) dt$$

$$= 11.74 \text{ mm}$$

32. A rigid smooth retaining wall of height 7 m with vertical backface retains saturated clay as backfill. The saturated unit weight and undrained cohesion of the backfill are $17.2 \text{ kN/m}^3$ and 20 kPa, respectively. The difference in the active lateral forces on the wall (in kN per meter length of wall, up to decimal places), before and after the occurrence of tension cracks is __________

Ans. (46.72)

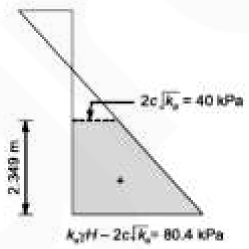



For clay $\phi = 0$

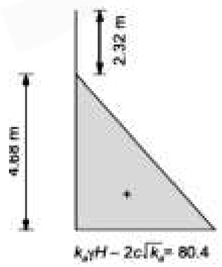
$$\therefore \quad k_a = \frac{1-\sin 0}{1+\sin 0} = 1$$

Earth pressure when tension cracks are not developed.

$$P_a = \frac{1}{2}(40 + 80.4) \times 2.349 = 141.4098$$




Earth pressure when tension cracks are developed




$$P_a = \frac{1}{2} \times 80.4 \times 4.68 = 188.136$$

$$\text{Difference} = 188.136 - 141.4098$$

$$= 46.7262 \text{ kN/m}^2$$

33. An aircraft approaches the threshold of a runway strip at a speed of 200 km/h. The pilot decelerates the aircraft at a rate of 1.697 m/s$^2$ and takes 18 s to exit the runway strip. If the deceleration after exiting the runway is $1 \text{ m/s}^2$, then the distance (in m, up to one decimal place) of the gate position from the location of exit on the runway is ______________

Ans. (312.8)

Speed of aircraft, $u_i = 200$ km/hr

$$= \frac{200 \times 1000}{3600} = 55.56 \text{ m/s}$$

Deceleration of aircraft on the runway

Aircraft takes 18 sec to exit the runway strip.

Speed of the aircraft at the exit of the runway

$$u_f = u_i + at$$

$$= 55.56 - 1.697 \times 18 = 25.014 \text{ m/s}$$

After runway aircraft decelerate with

Total distance travelled by the aircraft from the location of exit on the runway

$$U^2 = u^2 + 2aS$$

$$0 = 25.014^2 - 2 \times 1 \times S$$

$$S = 312.8 \text{ m}$$

34. A $0.5 \text{ m} \times 0.5 \text{ m}$ square concrete pile is to be driven in a homogeneous clayed soil having undrained shear strength $c_u = 50 \text{kPa}$ and unit weight, $\gamma = 18.0 \text{ kN/m}^3$. The design capacity of the pile is 500 kN. The adhesion factor α is given 0.75. The length of the pile required for the above design load with a factor of safety of 2.0 is

A. 5.2 m
B. 5.8 m
C. 11.8 m
D. 12.5 m

Ans. C

$$C_u = 50 \text{ kPa}, \gamma = 18 \text{ kN/m}^3, \text{FOS} = 2$$

$$Q_{up} = 9C \times B^2 + \alpha\overline{C}(4BL) = 1000 \text{ kN}$$

$$1000 = 9 \times 50 \times 0.5 \times 0.5 + 0.75 \times 50 (4 \times 0.5L)$$

$$L = 11.83 \text{ m}$$

35. An RCC beam of rectangular cross-section has factored shear of 200 kN at its critical section. Its width b is 250 mm and effective depth d is 350 mm. Assume design shear strength $\tau_c$ of concrete as $0.62\ N/mm^2$ and maximum allowable shear stress $\tau_{c\ max}$ in concrete as $2.8\ N/mm^2$. If two legged 10 mm diameter vertical stirrups of Fe250 grade steel are used, then the required spacing (in cm, up to one decimal place) as per unit state method will be ______________

Ans. (8.2)

Nominal shear stress $= \tau_v = \frac{V_u}{bd} = \frac{200 \times 10^3}{250 \times 350}$

$2.286\ N/mm^2 < \tau_{c,\ max}$ (OK)

SF taken by stirrups $= (\tau_v - \tau_c)bd$

$= (2.286 - 0.62) \times 250 \times 350 = 145.75$ kN

Now, $V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$

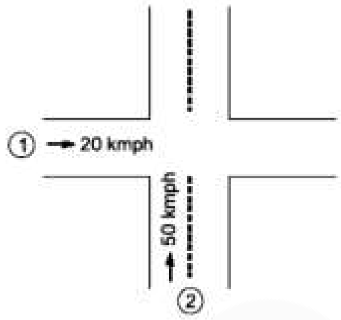
$\Rightarrow \quad S_v = \frac{0.87 \times 250 \times 350 \times 2 \times \frac{\pi}{4} \times 10^2}{145.75 \times 10^3}$

$= 82$ mm $= 8.2$ cm

36. A priority intersection has a single-lane one-way traffic road crossing an undivided two-lane two-way traffic road. The traffic stream speed on the single-lane road is 20 kmph and the speed on the two-lane road is 50 kmph. The perception-reaction time is 2.5 s, coefficient of longitudinal friction is 0.38 and acceleration due to gravity is $9.81\ m/s^2$. A clear sight triangle has to be ensured at this intersection. The minimum lengths of the sides of the sight triangle along the two-lane road and the single-lane road, respectively will be

A. 50 m and 20 m
B. 61 m and 18 m
C. 111 m and 15 m
D. 122 m and 36 m

Ans. B




$SSD_2 = 0.278 \times Vt_R + \frac{V^2}{254f}$

$= 0.278 \times 50 \times 2.5 + \frac{50^2}{254 \times 0.38} \simeq$

$SSD_1 = 0.278 \times 20 \times 2.5 + \frac{20^2}{254 \times 0.38} \simeq$

37. An RCC short column (with lateral ties) of rectangular cross-section of in $250\ mm \times 300\ mm$ reinforced with four members of 16 mm diameter longitudinal bars. The grades of steel and concrete are Fe415 and M20, respectively. Neglect eccentricity effect. Considering limit state of collapse in compression (IS 456 : 2000), the axial load carrying capacity of the column (in kN, up to one decimal place) is ______________

Ans. (918.1)

Since eccentricity effect is being neglected so column can be considered as concentrically loaded.

Ultimate axial load carrying capacity of column.

$P_u = 0.45 f_{ck} A_g + (0.75 f_y - 0.45 f_{ck}) A_{sc}$

$= 0.45 \times 20 \times 250 \times 300 +$

$(0.75 \times 415 - 0.45 \times 20) 4 \times \frac{\pi}{4} \times 16^2$

$= 91.1$ kN

38. The ultimate BOD $(L_0)$ a wastewater sample is estimated as 87% of COD. The COD of this wastewater is 300 mg/L. Considering first order BOD reaction rate constant k (use natural log) = 2.3 per day and temperature coefficient $\theta = 1.047$, the BOD value (in mg/L, up to one decimal place) after three days of incubation at 27°C for the wastewater will be __________

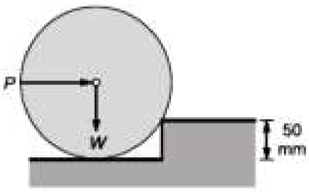
Ans. (160.2)

Ultimate BOD

$= 0.87\,COD = 0.87 \times 300 = 261\,mg/l$

$$BOD_3 = L_0\left(1 - e^{-k_{27}\times 3}\right)$$
$$k_{27} = k_{20}(1.047)^{T-20}$$
$$= 0.23(1.047)^{27-20} = 0.317 \text{ days}^{-1}$$
$$BOD_3 = 261\left(1 - e^{0.317\times 3}\right) = 160.226 \text{ mg}/l$$

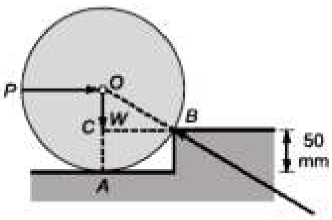
39. A cylinder of radius 250 mm and weight, W = 10 kN is rolled up an obstacle of height 50 mm by applying a horizontal force P at its centre as shown in the figure.




All interfaces are assumed frictionless. The minimum value of P is

A. 4.5 kN B. 5.0 kN
C. 6.0 kN D. 7.5 kN

Ans. D




Given: $r = 250$ mm, $W = 10$ kN

Note:

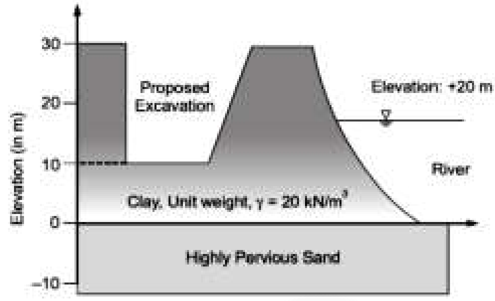
1. When the cylinder will be about to move out of the cylinder, it will loose its contact at point A, only contact will be at point B.
2. Considering equation of cylinder of that instant under P, W and $R_B$ (contact force at B).

$$\Sigma \vec{M}_B = 0$$
$$P \times OC - W \times BC = 0$$

In $\Delta OCB$

$$CB = \sqrt{250^2 - 200^2} = 150$$
$$P = \frac{10\times 10^3 \times 150}{200} = 2.5 \times 3 \text{ kN} = 7.5 \text{ kN}$$

40. At a construction site, a contractor plans to make an excavation as shown in the figure.




The water level in the adjacent river is at an elevation of +20.0 m. Unit weight of water is 10 kN/m$^3$. The factor of safety (up to two decimal places) against sand boiling for the proposed excavation is ___________

Ans. (1)

$$\text{FOS} = \frac{10\times\gamma_{sat}}{20\times\gamma_w} = \frac{10\times 20}{20\times 10} = 1$$

41. The solution (up to three decimal places) at $x = 1$ of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ subject to boundary conditions $y(0) = 1$ and $\frac{dy}{dx} = (0) = -1$ is ____________

Ans. (0.36)

$(D^2 + 2D + 1)y = 0$ ($\therefore$ Roots are $-1, -1$)

$$CF = (C_1 + C_2x)e^{-x}$$
$$y = C_1e^{-x} + C_2x\,e^{-x} \quad \text{...(i)}$$
$$y(0) = 1 \quad 1 = C_1 \quad \text{...(ii)}$$
$$y' = C_1e^{-x} + C_2\left(e^{-x} - xe^{-x}\right)$$
$$y'(0) = -1, -1 = C_1 + C_2 \quad \text{...(iii)}$$

From eq. (ii) and (iii),

$$C_1 = 1, C_2 = 0$$
$$\therefore \quad y = e^{-x}$$

At $x = 1, y = e^{-1} = \frac{1}{e} = 0.368$

42. The void ratio of a soil is 0.55 at an effective normal stress of 140 kPa. The compression index of the soil is 0.25. In order to reduce the void ratio to 0.4, an increase in the magnitude of effective normal stress (in kPa, up to one decimal place) should be ______________

Ans. (417.3)

$$\Delta H = \frac{H_0 \Delta e}{1+e_0} = \frac{H_0 C_c}{1+e_0}\log\left(\frac{\sigma_0+\Delta\sigma}{\sigma_0}\right)$$

$$\Delta H = H_0\left(\frac{0.55-0.40}{1+0.50}\right) = \frac{H_0\times 0.25}{1+0.50}\log$$

$$\left(\frac{140+\Delta\sigma}{140}\right)$$

$$\frac{0.15}{0.25} = \frac{3}{5} = \log\left(\frac{140+\Delta\sigma}{140}\right)$$

$$\Delta\sigma = 417.35 \text{ kPa}$$

43. A water sample analysis data is given below:

| Ion | Concentration, Mg/L | Atomic Weight |
|---|---|---|
| $Ca^{2+}$ | 60 | 40 |
| $Mg^{2+}$ | 30 | 24.31 |
| $HCO_3^-$ | 400 | 61 |

The carbonate hardness (expressed as mg/L of $CaCO_3$, up to one decimal place) for the water sample is ____________

Ans. (273.4)

Carbonate hardness = min. (Total hardness, alkalinity)

Total hardness

$$= \left(\frac{60}{20}\times 50 + \frac{30}{12.155}\times 50\right) \text{mg/}l \text{ as } CaCO_3$$

$$= 150 + 123.406$$

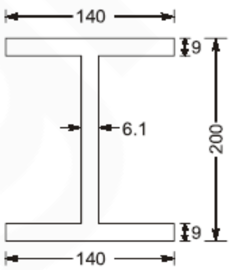
$$= 273.406 \text{ mg/}l \text{ as } CaCO_3$$

$$\text{Alkalinity} = \left(\frac{400}{61}\times 50\right)\text{mg/}l \text{ as } CaCO_3$$

$$= 327.868 \text{ mg/}l \text{ as } CaCO_3$$

$\therefore$ $CH = 273.4$ mg/$l$ as $CaCO_3$

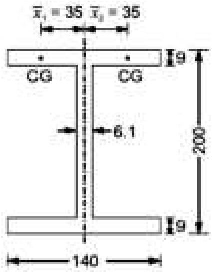
44. The dimensionless of a symmetrical welded I-section are shown in the figure.




(All dimension are in mm)

The plastic section modulus about the weaker axis (in $cm^3$, up to one decimal place) is ____________

Ans. (89.9)




(All dimension are in mm)

$$Z_p = \frac{A}{2}\left(\bar{x}_1+\bar{x}_2\right)$$

$$= \left[\frac{140\times 9}{2}(35+35)\right]\times 2 + \left[\frac{(200-18)\times 6.1}{2}\right]$$

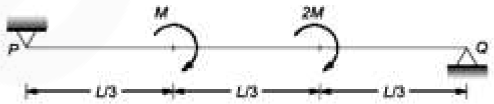
$$\times\left(\frac{3.05}{2}+\frac{3.05}{2}\right)$$

$$= 88200 + 1693.055$$

$$= 89893.055 \text{ mm}^3$$

$\simeq$ m$^3$

45. The figure shows a simply supported beam PQ of uniform flexural rigidity EI carrying two moments M and 2M




The slop at P will be

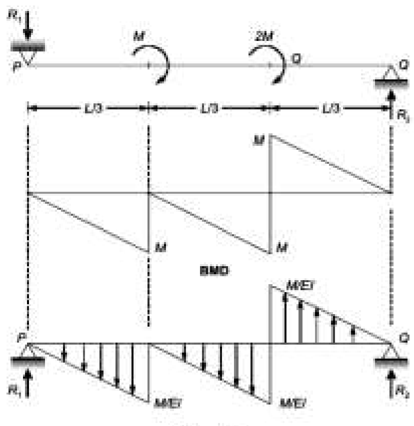
A. 0

B. ML/(9EI)

C. ML(6EI)

D. ML/(3EI)

Ans. C




Now $\quad R_1 + R_2 = \frac{1}{2}\times\frac{L}{3}\times\frac{M}{EI} = \frac{ML}{6EI}$

$$\Sigma M_Q = 0$$

$$R_1 L - \frac{1}{2}\times\frac{L}{3}\times\frac{M}{EI}\left(\frac{2L}{3}+\frac{L}{9}\right) - \frac{1}{2}\times\frac{L}{3}\times\frac{M}{EI}\left(\frac{L}{3}+\frac{L}{9}\right)$$

$$+\frac{1}{2}\times\frac{L}{3}\times\frac{M}{EI}\times\frac{2L}{9} = 0$$

$$R_1 = \frac{7\,ML}{54\,EI} + \frac{4ML}{54EI} - \frac{ML}{27EI} = \frac{ML}{6EI}$$

46. A conventional drained triaxial compression test was conducted on a normally consolidated clay sample under an effective confining pressure of 200 KPa. The deviator stress at failure was found to be 400 kPa. An identical specimen of the same clay sample is isotropically consolidated to a confining pressure of 200 kPa and subjected to standard undrained triaxial compression test. If the deviator stress at failure is 150 kPa, the pore pressure developed (in kPa, up to one decimal place) is ________________

Ans. (125)

I[st] Specimen : Drained condition

$\overline{\sigma_3} = 200$ kPa : $\overline{\sigma_d} = 400$ kPa : $\overline{\sigma_1} = 600$ kPa

II[nd] Specimen : Undrained condition

$\overline{\sigma_3} = 200$ kPa : $\overline{\sigma_d} = 150$ kPa : $\overline{\sigma_1} = \overline{\sigma_3} + \sigma_d$

$= 350$ kPa

Let pore pressure developed is u

$$\therefore \quad \overline{\sigma_3} = (200 - u)$$

$$\overline{\sigma_1} = (350 - u)$$

From stress relationship

$$\overline{\sigma_1} = \overline{\sigma_3}\tan^2\left(45^o + \frac{\phi}{2}\right) + 2c\tan\left(45^o + \frac{\phi}{2}\right)$$

For clay under drained condition

$$\therefore \quad \overline{\sigma_1} = \overline{\sigma_3}\tan^2\left(45^o + \frac{\phi}{2}\right)$$

$$600 = 200\tan^2\left(45^o + \frac{\phi}{2}\right)$$

$$\phi = 30^o$$

For second specimen,

$$\overline{\sigma_1} = \overline{\sigma_3}\tan^2\left(45^o + \frac{\phi}{2}\right)$$

$$(350 - u) = (200 - u)\tan^2\left(45^o + \frac{30^o}{2}\right)$$

$$u = 125 \text{ kPa}$$

47. Given the following data: design life $n = 15$ years, lane distribution factor $D = 0.75$, annual rate of growth of commercial vehicles $r = 6\%$, vehicle damage factor $F = 4$ and initial traffic in the year of completion of construction = 3000 Commercial Vehicles Per Day (CVPD). As power IRC : 37-2012, the design traffic in terms of cumulative number of standard axles (in million standard axles, up to two decimal places) is __________

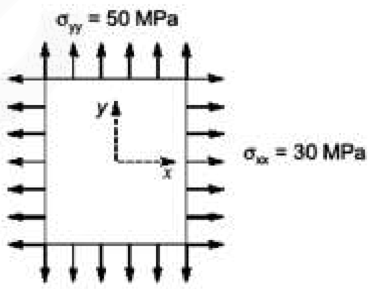
Ans. (76.45)

$$N_s = \frac{365A\left[\left(1+\frac{r}{100}\right)^n - 1\right]\times VDF\times LDF}{\frac{r}{100}}$$

$$= \frac{365\times 3000\left[1.06^{15} - 1\right]\times 4\times 0.75}{0.06}$$

$$= 76.45 \text{ msa}$$

48. A plate in equilibrium is subjected to uniform stresses along its edges with magnitude $\sigma_{xx} = 30$ MPa and $\sigma_{yy} = 50$ MPa as shown in the figure




The Young's modulus of the material is $2\times 10^{11}$ N/m$^2$ and the Poisson's ratio is 0.3. If $\sigma_{zz}$ is negligibly small and assumed to be zero, then the strain $\varepsilon_{zz}$ is

A. $-120\times 10^{-6}$
B. $-60\times 10^{-6}$
C. 0.0
D. $120\times 10^{-6}$

Ans. A

$\sigma_{xx} = 30$ MPa, $\sigma_{yy} = 50$ MPa, $\sigma_{zz} = 0$

$$\varepsilon_{zz} = \frac{\sigma_{zz}}{E} - \mu\frac{\sigma_{xx}}{E} - \mu\frac{\sigma_{yy}}{E} = -\frac{\mu}{E}\left(\sigma_{xx} + \sigma_{yy}\right)$$

$$= -\frac{0.3}{2\times 10^5}(30 + 50) = -120\times 10^{-6}$$

49. In a laboratory, a flow experiment is performed over a hydraulic structure. The measured values of discharge and velocity are 0.05 m$^3$/s and 0.25 m/s, respectively. If the full scale structure (30 times bigger) is subjected to a discharge of 270 m$^3$/s, then the time scale (model to full scale)

value (up to two decimal places) is ______________

Ans. (0.18)

Froude Law $(\text{Fr})_m = (\text{Fr})_p$

$$\left(\frac{V}{\sqrt{Lg}}\right)_m = \left(\frac{V}{\sqrt{Lg}}\right)_p \qquad (g_m = g_p)$$

$$V_r = \sqrt{L_r}$$

or $$\frac{L_r}{T_r} = \sqrt{L_r}$$

$$T_r = \sqrt{L_r}$$

$$T_r = \sqrt{\frac{1}{30}} = 0.1826$$

50. The following details refer to a closed traverse :

| Line | Consecutive coordinate | | | |
|---|---|---|---|---|
| | Northing (m) | Southing (m) | Easting (m) | Westing (m) |
| PQ | - | 437 | 173 | - |
| QR | 101 | - | 558 | - |
| RS | 419 | - | - | 96 |
| SP | - | 83 | - | 634 |

The length and direction (whole circle bearing) of closure, respectively are

A. 1 m and 90° B. 2 m ans 90°

C. 1 m and 270° D. 2 m and 270°

Ans. A

$$\Sigma L = 101 + 419 - 437 - 83 = 0$$

$$\Sigma D = 173 + 558 - 96 - 634 = 1$$

Therefore departure of closure is 1 m and WCB is 90°.

51. A square area (on the surface on the earth) with side 100 m and uniform height, appears as 1 $cm^2$ on a vertical aerial photograph. The topographic map shows that a contour of 650 m passes through the area. If focal length of the camera lens is 150 mm, the height from which the aerial photograph was taken, is

A. 800 m B. 1500 m

C. 2150 m D. 3150 m

Ans. C

$$A = 100 \times 100 \text{ m}^2$$

Area on photo, $a = 1 \text{ cm}^2$

Scale $1 \text{ cm} = 100 \text{ m}$

$$f = 150 \text{ mm}$$

$$h = 650 \text{ m}$$

Scale $= \frac{1}{100} = \frac{1}{100 \times 10^2} = \frac{1}{10000} = \frac{f}{H-h}$

$$\Rightarrow \quad \frac{1}{10000} = \frac{150 \times 10^{-3}}{H - 650}$$

$$\Rightarrow \quad H = 2150 \text{ m}$$

52. The solution at $x = 1, t = 1$ of the partial differential equation $\frac{\partial^2 u}{\partial x^2} = 25\frac{\partial^2 u}{\partial t^2}$ subject to initial conditions of $u(0) = 3x$ and $\frac{\partial u}{\partial t}(0) = 3$ is ______________

A. 1 B. 2

C. 4 D. 6

Ans. (*)

Data Insufficient.

53. The value of the integral $\int_0^{\pi} x\cos^2 x\,dx$ is

A. $\frac{\pi^2}{8}$ B. $\frac{\pi^2}{4}$

C. $\frac{\pi^2}{2}$ D. $\pi^2$

Ans. B

The value of $\int_0^{\pi} x\cos^2 x\,dx$

$$= \int_0^{\pi}\left(\frac{x}{2} + \frac{x\cos 2x}{2}\right)dx$$

$$= \frac{x^2}{4}\Big|_0^{\pi} + \frac{1}{2}\left(\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right)$$

$$= \frac{\pi^2}{4} + \frac{1}{2}\left\{\left(0 + \frac{1}{4}\right) - \left(0 + \frac{1}{4}\right)\right\}$$

$$= \frac{\pi^2}{4} + \frac{1}{2}\left(\frac{1}{4} - \frac{1}{4}\right)$$

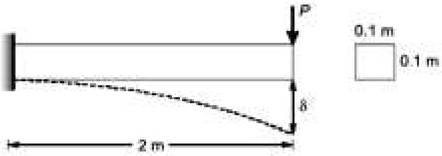
$$= \frac{\pi^2}{4}$$

54. A cantilever beam of length 2 m with a square section of side length 0.1 m is loaded vertically at the free end. The vertical displacement at the free end is 5 mm. The beam is made of steel with Young's modulus of $2.0 \times 10^{11}$ N/$m^2$. The maximum bending stress at the fixed end of the cantilver is

A. 20.0 MPa B. 37.5 MPa

C. 60.0 MPa D. 75.0 MPa

Ans. B




$$I = \frac{(0.1)^4}{12}$$

Deflection $\delta = \frac{Pl^3}{3EI}$

$$\Rightarrow \quad 5 \times 10^{-3} \text{ m} = \frac{P(2)^3}{3 \times 2 \times 10^{11} \times \frac{(0.1)^4}{12}}$$

$$\Rightarrow \quad P = 3125 \text{ N}$$

Now, $M = Pl = 3125 \times 2 = 6250$ Nm

As, $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

$$\Rightarrow \sigma_{max} = \frac{M}{Z} = \frac{6250}{\frac{(0.1)^3}{6}} = 37.5 \times 10^6 \text{ N/m}^2$$

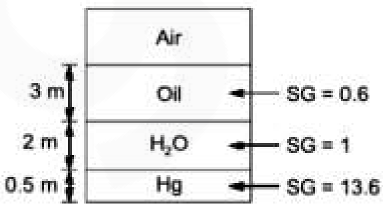
$$= 37.5\text{MPa}$$

55. A closed tank contains 0.5 m thick layer of mercury (specific gravity = 13.6) at the bottom. A 2.0 m thick layer of water lies above the mercury layer. A 3.0 m thick layer of oil (specific gravity = 0.6) lies above the water layer. The space above the oil layer contains air under pressure. The gauge pressure at the bottom of the tank is 196.2 kN/m$^2$. The density of water is 1000 kg/m$^3$ and the acceleration due to gravity is 9.81 m/s$^2$. The value of pressure in the air space is

A. 92.214 kN/m$^2$ B. 95.644 kN/m$^2$

C. 98.922 kN/m$^2$ D. 99.321 kN/m$^2$

Ans. A




$P_{air}$ is in gauge pressure.

$$P_{air} + (0.6 \times 10^3)(9.81)(3) + (10^3)(9.81)(2) + (13.6 \times 10^3)(9.81)(0.5) = 196.2 \times 10^3$$

$$P_{air} = 92.214 \text{ kN/m}^2$$