Complex Number

Any number of the form a+ib is known as complex number where $a,b \in R$. It is denoted by z.

a = Real part of z = Re(z)

b = Imaginary part of z = Im(z)

Where

$i=\sqrt{-1}$. $i^2=-1$; $i^3=-i$, $i^4=1$

$i^{4n+1}=i$; $i^{4n+2}=-1$; $i^{4n+3}=-i$; $i^{4n}=1$

If Re(z) = 0 ; z is said to be purely imaginary

If Im(z) = 0 ; z is said to be purely real.

If $z_1 = z_2 \Rightarrow$ Re(z_1) = Re(z_2) and Im (z_1) = Im (z_2).

CONJUGATE OF A COMPLEX NUMBER $(\bar{z})$

If z = a + ib , then $\bar{z}$ = a – ib

Properties

(i) $\bar{\bar{z}}=z$

(ii) $Re(z)=\frac{z+\bar{z}}{2}$

(iii) $Im(z)=\frac{z-\bar{z}}{2i}$

(iv) If $z=\bar{z}$ then z is purely real.

(v) If $z=-\bar{z}$ then z is purely imaginary.

(vi) $\overline{z_1 \pm z_2}=\bar{z}_1 \pm \bar{z}_2$; $\overline{z_1 z_2 z_3}=\bar{z}_1 \bar{z}_2 \bar{z}_3$; $\overline{\left(\frac{z_1}{z_2}\right)}=\frac{\bar{z}_1}{\bar{z}_2} (z_2 \neq 0)$

ARGAND PLANE

MODULUS OF A COMPLEX NUMBER

Modulus is the distance of the complex number from the origin in the argand plane. It is denoted by $|z|$.

Properties

(i) If $|z| = 0 \Rightarrow z = 0$

(ii) $|\bar{z}| = |z|$

(iii) $z\bar{z} = |z|^2$

(iv) $|z^n| = |z|^n$

(v) $|z_1 z_2 z_3| = |z_1||z_2||z_3|$

(vi) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

(vii) $||z_1| - |z_2|| \leq |z_1 + z_2| \leq |z_1| + |z_2|$

$||z_1| - |z_2|| \leq |z_1 - z_2| \leq |z_1| + |z_2|$

(viii) $|z_1 + z_2 + ... + z_n| \leq |z_1| + |z_2| + ...|z_n|$

(ix) $|z_1 \pm z_2|^2 = (z_1 \pm z_2)(\overline{z_1} \pm \overline{z_2})$

$= |z_1|^2 + |z_2|^2 \pm z_1\overline{z_2} \pm \overline{z_1}z_2$

Q.1. Find the value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary.

Q.2. If $|z_1|=1$; $|z_2|=2$; $|z_3|=3$ and $|9z_1z_2+4z_1z_3+z_2z_3|=12$ find the value of $|z_1+z_2+z_3|$.

Q.3. If $|z|=1$; $w=\frac{z-1}{z+1}(z\neq-1)$. Find Re(w)

Q.4. If $\frac{w-\overline{w}z}{1-z}$ is purely real where $w=\alpha+i\beta$; $\beta\neq0$; $z\neq1$. Find the value of $|z|$.

Q.5. Let z is a complex number such that $\text{Im}(z)\neq0$ and $a=z^2+z+1$ is real. Find value of a.

Q.6. If z_1 and z_2 are two complex numbers such that $|z_1|<1<|z_2|$. Prove that $\left|\frac{1-z_1\overline{z_2}}{z_1-z_2}\right|<1$.

Q.7. Prove that these exists no complex number z such that $|z|<\frac{1}{3}$ and $\sum_{r=1}^{n}a_rz^r=1$ where $|a_r|<2$.

Q.8. If z is a complex number such that $\left|z+\frac{1}{z}\right|=2$. Find the range of $|z|$.