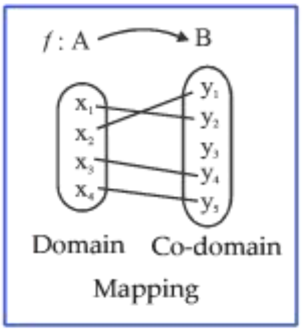
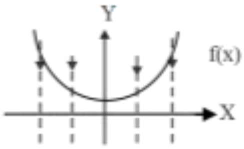
FUNCTIONS

A relation f from a set A to a set B is said to be function if every element of set A has one and only one image in set B.

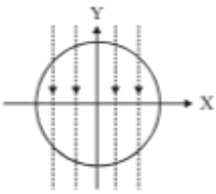


Graphically, if any relation is a function then any line parallel to y-axis intersect the curve at unique point throughout the graph (Vertical Line Test).

Graphical Method



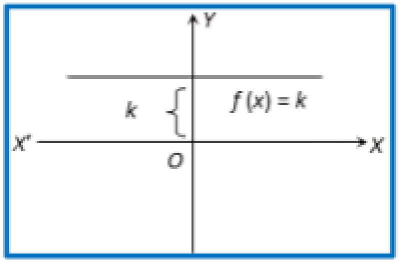
Graphical Method



Domain and range of a function

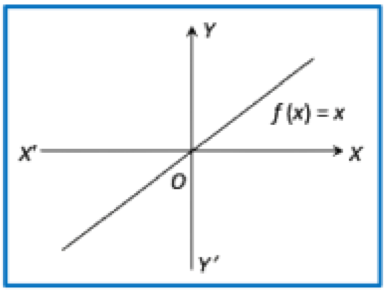
CLASSIFICATION OF FUNCTIONS

Constant function: Let k be a fixed real number. Then a function $f(x)$ given by $f(x) = k$ for all $x \in R$ is called a constant function.



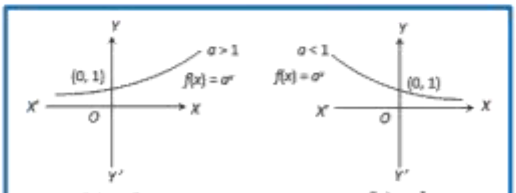
Identity function:

$f(x) = x$ for all $x \in R$,



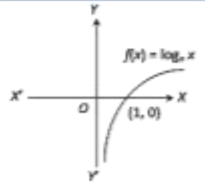
CLASSIFICATION OF FUNCTIONS

Exponential function : Let $a \neq 1$ be a positive real number. Then $f : R \to (0, \infty)$ defined by $f(x) = a^x$ called exponential function. Its domain is R and range is $(0, \infty)$.



Graph of $f(x) = a^x$, when $a > 1$ Graph of $f(x) = a^x$, when $a<1$

Logarithmic function: Let $a \neq 1$ be a positive real number. Then $f:(0,\infty) \to R$ defined by $f(x) = \log_a x$ is called logarithmic function. Its domain is $(0,\infty)$ and range is *R*.



Graph of $f(x) = \log_a x$, when $a > 1$

Graph of $f(x) = \log_a x$, when $a < 1$

Properties of Logarithmic Functions

(i) $\log_a 1 = 0$

(ii) $\log_a a = 1$

(iii) $a^{\log_a x} = x$

(iv) $\log_a (m\,n) = \log_a m + \log_a n$

(v) $\log_a (m/n) = \log_a m - \log_a n$

(vi) $\log_a m^k = k \log_a m$

Base change formula

$\log_a m = \dfrac{\log_b m}{\log_b a}$

1 $\log_a b = \dfrac{1}{\log_b a}$

2 $\log_{a^k} m = \dfrac{1}{k} \log_a m$

$a^{\log_b c} = c^{\log_b a}$

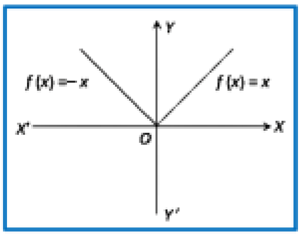
Logarithmic Inequalities

(i) $\log_a x > \log_a y \Leftrightarrow \begin{cases} x > y, & \text{if} \quad a > 1 \\ x < y, & \text{if} \quad 0 < a < 1 \end{cases}$

(ii) $\log_a x > b \quad \Leftrightarrow \quad \begin{cases} x > a^b, & \text{if} \quad a > 1 \\ x < a^b, & \text{if} \quad 0 < a < 1 \end{cases}$

Modulus function:

$f(x) = |x| = \begin{cases} x, & \text{when} \quad x \geq 0 \\ -x, & \text{when} \quad x < 0 \end{cases}$ is



Basic Properties

(i) $||x|| = |x|$

(ii) $|x| > a \Rightarrow x > a$ or $x < -a$ if $a \in R^+$ and $x \in R$ if $a \in R^-$

(iii) $|x| < a \Rightarrow -a < x < a$ if $a \in R^+$ and no solution if $a \in R^- \cup \{0\}$

(iv) $|x \ y| = |x||y|$

(v) $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$, $y \neq 0$

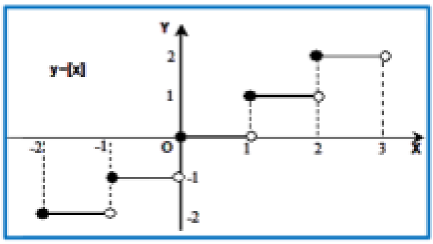
(vi) $|x + y| \leq |x| + |y|$. Here the equality sign holds if x and y either both are non-negative or non-positive in other words $x.y \geq 0$.

(vii) $|x - y| \geq |x| \sim |y|$ Here the equality sign holds if x and y either both are non-negative or non-positive in other words $x.y \geq 0$.

The last two properties can be put in one compact form namely, $|x| \sim |y| \leq |x \pm y| \leq |x| + |y|$

(12) Greatest integer function: Let

$f(x) = [x]$



Greatest Integer Function

(i) $[x] \leq x, \quad x \in R$

(ii) $[x] = x \quad$ if $\quad x \in I$

(iii) $[-x] = -[x] \quad$ if $\quad x \in I$

(iv) $[-x] = -[x] - 1 \quad$ if $\quad x \notin$ Integer

(v) $[x \pm I] = [x] \pm I$

(vi) $[x] \geq I \Rightarrow x \geq I$

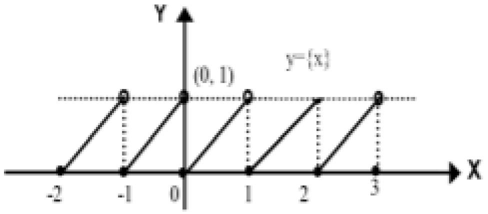
(vii) $[x] > I \Rightarrow x \geq I + 1$

(viii) $[x] \leq I \Rightarrow x < I + 1$

(ix) $[x] < I \Rightarrow x < I$

Fractional Part function

$\{x\} = x - [x]$



Fractional Function:

(i) $0 \leq \{x\} < 1, \ x \in R$

(ii) $\{x\}$ is periodic function with period 1

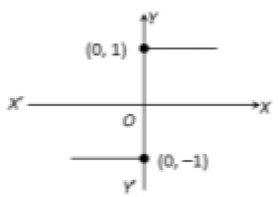
(iii) $\{x\} = 0 \quad$ if $\quad x \in I$

(iv) $\{-x\} = 1 - \{x\}$ if $\quad x \notin I$

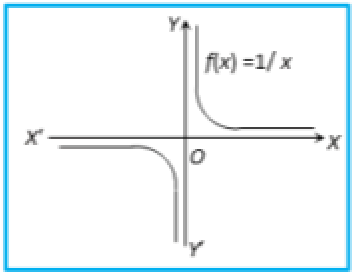
(v) $\{x\} = x - I$ if $\quad I \leq x < I + 1$

Signum function : $f(x) = \begin{cases} \frac{|x|}{x}, x \neq 0 \\ 0, x = 0 \end{cases}$ or

$$f(x) = \begin{cases} 1, \ x > 0 \\ 0, \ x = 0 \\ -1, \ x < 0 \end{cases}$$



Reciprocal function



Power function $f(x) = x^{\alpha}$, $\alpha \in R$