gradeup

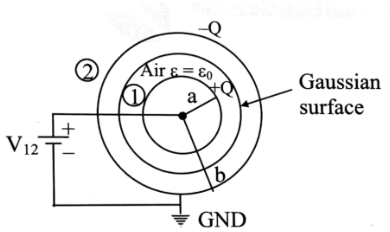
ESE (Mains) 2019 Electromagnetic

Important Questions with Solutions

Exmple:1

Show that the capacity of a spherical capacitor consisting of two concentric spheres of radii 'a' and 'b' and the dielectric medium between the two spheres being air can be given as $C = \frac{\varepsilon_0}{r}\sqrt{A_1 A_2}$ Farad. Where A_1 and A_2 are the surface areas of the two spheres and 'r' is the radial separation between spheres. If the radii of two spheres differ by 4cm and the capacity of the spherical capacitor is 88.88pF, then calculate the radii of spheres.

Sol.



Consider a spherical capacitor consists of two concentric spheres of radii a & b. (inner and outer respectively) as shown in fig.

From Gauss's law $Q_{enclosed}$ = D × Area

$D_r \times 4\pi^2 = Q$

$$\vec{D} = \frac{}{4\pi r^2}\hat{a}_r;\ a < r < b$$

$$\therefore \vec{E} = \frac{}{4\pi r^2}\hat{a}_r;\ a < r < b$$

Let 'V_{12}' be the potential applied between the sphere and is given by

$$V_{12} = -\int_2^1 \vec{E}.\vec{dl}$$

$$= \int_b^a \frac{Q}{4\pi\varepsilon r^2}\hat{a}_r.dr\ \hat{a}_r$$

$$V_{12} = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{a} - \frac{1}{b}\right]$$

Therefore the capacity or capacitance of spherical capacitor is given by

$$C = \frac{V_{12}}{Q} = \frac{4\pi\varepsilon_0 ab}{b-a}\text{Farad} \quad (\therefore \varepsilon_r = 1)$$

Surface area of inner sphere, $A_1 = 4\pi a^2$

Surface area of outer sphere $A_2 = 4\pi b^2$

Radial spacing between spheres, r = b – a

$$a = \sqrt{\frac{A_1}{4\pi}}, \quad b = \sqrt{\frac{A_2}{4\pi}}$$

$$C = \frac{4\pi\varepsilon_0 \sqrt{\frac{A_1}{4\pi}} \sqrt{\frac{A_2}{4\pi}}}{r}$$

$$\therefore C = \frac{\varepsilon_0}{r} \sqrt{A_1 A_2} \text{ Farad}$$

Given r = b – a = 4 × 10^{-12}m

C = 88.88pF

$$C = \frac{4\pi\varepsilon_0 ab}{r}$$

ab = 320 × 10^{-4} ....(1)

b – a = 4 × 10^{-2} ....(2)

From equation (1) & (2)

a^2 + 4 × 10^{-2}a – 320 × 10^{-4} =0

by solving

∴ a = 16cm & b = 20cm

Exmple:2

Point charges of 15nC each are symmetrically located at (4,4,0), (4,–4,0), (–4,4,0) and (–4, –4, 0) and a uniform line charge of 50nC/m lies at x = 0 , y = 8 all z.

A. Find $\overline{D}$ at the origin

B. How much electric flux crosses the y = 0 plane?

C. How much electric flux leaves the surfaces of a sphere of a sphere of radius 5m cantered at (0, 6, 0)

Sol. (a) Fig below shows the location of four charges. These charges are located on the corners of a square, origin being the centre of the square. Due to the charges located at A and C, the electric flux density vectors get cancelled out. Also due to the charges located at B and D, electric flux density is zero. Hence due to four point charges. $\overline{D}$ at origin is zero.

Now consider the line charge.

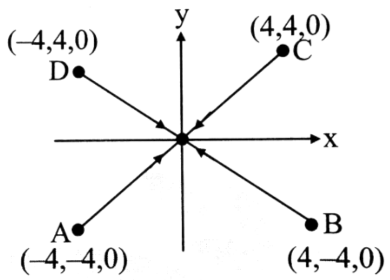
$$\overline{D} = \varepsilon_0 \overline{E} = \varepsilon_0 \left[\frac{\rho_L}{2\pi\varepsilon_0 d} \hat{a}_{12} \right] = \frac{\rho_L}{2\pi d} \hat{a}_{12}$$

Point 2 is that point at which $\overline{D}$ is desired = (0,0,0)

Point 1 is the foot of perpendicular dropped from point 2 on the line charge = (0, 8,0)

$\therefore\ \overline{R}_{12} = (0-8)\hat{a}_y = -8\hat{a}_y$

$d = \left|\overline{R}_{12}\right| = \sqrt{(-8)^2} = 8$



$$\hat{a}_{12} = \frac{R_{12}}{|R_{12}|} = -\frac{8\hat{a}_y}{8} = -\hat{a}_y$$

$$\therefore \overline{D} = \frac{50\times10^{-9}}{2\pi(8)}\left(-\hat{a}_y\right) = -994.718\hat{a}_y pC/m^2$$

(b) Any point in y = 0 plane is given by (x, 0 , z). At this point, $\overline{D}$ due to four point charges is given by

$$\overline{D} = \frac{15\times10^{-9}}{4\pi}\left\{\begin{array}{l}\dfrac{(x-4)\hat{a}_x+(0-4)\hat{a}_y+(z-0)\hat{a}_z}{\left[(x-4)^2+16+z^2\right]^{3/2}}+\dfrac{(x-4)\hat{a}_x+(0+4)\hat{a}_y+(z-0)\hat{a}_z}{\left[(x-4)^2+16+z^2\right]^{3/2}}\\ +\dfrac{(x+4)\hat{a}_x+(0-4)\hat{a}_y+(z-0)\hat{a}_z}{\left[(x-4)^2+16+z^2\right]^{3/2}}+\dfrac{(x+4)\hat{a}_x+(0+4)\hat{a}_y+(z-0)\hat{a}_z}{\left[(x-4)^2+16+z^2\right]^{3/2}}\end{array}\right\}$$

$$= \frac{15\times10^{-9}}{4\pi}\left\{\frac{2[x-4]\hat{a}_x+z\hat{a}_z}{\left[(x-4)^2+16+z^2\right]}+\frac{2\left[(x+4)\hat{a}_x+z\hat{a}_z\right]}{\left[(x+4)^2+16+z^2\right]^{3/2}}\right\}$$

$$\overline{dS} = \pm\left(dx\,dy\right)\hat{a}_y$$

$\therefore \overline{D}.\overline{dS} = 0$ as $\hat{a}_x.\hat{a}_y$ and $\hat{a}_z.\hat{a}_y$ is equal to zero. Hence the flux crossing the y = 0 plane due to four charges is zero. $\overline{D}$ at (x, 0, z) due to line charge 50nC/m at x = 0 y = 8 is given by

$$\overline{D} = \frac{\rho_L}{2\pi d}\hat{a}_{12}$$

Point 2 is that point at which $\overline{D}$ is desired = (x, 0, z)

Point 1 is the foot of perpendicular dropped from point 2 on the line charge = (0, 8, z)

$$\overline{R}_{12} = (x-0)\hat{a}_x + (0-8)\hat{a}_y = x\hat{a}_x - 8\hat{a}_y$$

$$d = \left|\overline{R}_{12}\right| = \sqrt{x^2+64}$$

$$\hat{a}_{12} = \frac{x\hat{a}_x - 8\hat{a}_y}{\sqrt{x^2+64}}$$

$$\therefore \overline{D} = \frac{50\times10^{-9}}{2\pi\sqrt{x^2+64}}\left[\frac{x\hat{a}_x - 8\hat{a}_y}{\sqrt{x^2+64}}\right]$$

$$\overline{dS} = \pm\left(dx\,dz\,\hat{a}_y\right); \overline{D}.\,\overline{dS} = \frac{\pm50\times10^{-9}(-8\,dx\,dz)}{2\pi\left(x^2+64\right)}$$

$$\therefore \psi = \int \overline{D}.\,\overline{dS} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \pm\left[\frac{400\times10^{-9}\,dx\,dz}{2\pi\left(x^2+64\right)}\right]$$

$$= \pm\left(\frac{-50\times10^{-9}}{2\pi}\right)\left[\tan^{-1}\left(\frac{x}{8}\right)\right]_{-\infty}^{\infty}[z]_{-\infty}^{\infty} = \infty$$

Since the length of line charge is infinite, infinite amount of flux is crossing through the y = 0 plane.

(C) Equation of the sphere of radius 5m cantered at (0,6,0) is given by

$(x-0)^2 + (y-6)^2 + (z-0)^2 = (5)^2$ or

$X^2 + (y-6)^2 + z^2 = 25$ ....(1)

Substitute (4,−4,0) in equation (1)

LHS = $(4)^2 + (4-6)^2 + (0)^2 = 16 + 4 = 20 <$ RHS of equation (1)

Hence the point charge located at (4, 4, 0) lies inside the sphere

$\therefore \Psi_1 = 15nC$

Substitute (4, −4,0) in equation (1)

LHS = $(4)^2 + (-4-6)^2 + (0)^2 = 16 + 100 = 116 >$ RHS of equation (1)

Hence the point charge located at (4, −4,0) lies outside the sphere

$\therefore \Psi_2 = 0$

Substitute (−4,4,0) in equation (1)

LHS = $(-4)^2 + (4-6)^2 + (0)^2 = 20 <$ RHS of equation (1)

Hence the charge located at (−4,4,0) lies inside the sphere

$\therefore \Psi_3 = 0$

Substitute (−4,4,0) in equation (1)

LHS = $(-4)^2 + (4-6)^2 + (0)^2 = 116 >$ RHS of equation (1)

Hence the charge located at (−4,4,0) lies outside the sphere

$\therefore \Psi_4 = 0$

Substitute x =0, y = 8 (equations of line charge) in equation (1)

$(0)^2 + (8-6)^2 + z^2 = 25$

Z2 = 21 or z = ± 4.583. Hence the portion of the line charge from z = −4.583 to z =4.583− (4.583) = 91.166

Charge on above length = 50 × 9.166 = 458.3 nC

$$\left[\text{or use } Q = \int dQ \int \rho_L dL = \int_{-4.583}^{4.583} 50 dz \, nC\right]$$

Hence due to line charge flux leaving the sphere is $\therefore \Psi_5 = 458.3nC$

∴ Total flux leaving the sphere $\therefore \Psi = \therefore \Psi_1 + \Psi_2 + \Psi_3 + \Psi_4 + \Psi_5$

= 15+0+15+0+458.3 = 488.3 nC

Example: 3

Consider a 100kHz plane wave travelling downward along the +z direction into the sea water. The x-y plane at z = 0 represents the sea surface with any point z > 0 representing the surface into the sea water. The constitutive parameters of the sea water are given as $\varepsilon_r = 81, \mu_r = 1, \text{and } \sigma = 4S/m.$ The electric field at z = 0^+ (just below the surface inside water) is given by

$$\overline{E}\left(0^+, t\right) = 0.2\cos(2\pi\times10^5 t + 75^0)\hat{a}_x \ \ V/m$$

A. For this wave. Determine whether the sea water can be considered a good dielectric or a good conductor, and why?

B. Calculate the propagation constant, the intrinsic impedance and the phase velocity for this medium

C. Calculate the depth at which the amplitude of $\overline{E}$ gets to half its value at the surface.

D. Obtain the time-domain expression for the electric and magnetic fields associated with this in sea water (z >0).

Sol. The loss tangent is given by,

$$\frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{\omega\varepsilon_0\varepsilon_r}$$
$$= \frac{4}{2\times\pi\times100\times10^3\times8.854\times10^{-12}\times81}$$
$$= 8.8\times10^3 >> 1$$

From the above equation it is obvious that sea water can be considered a good conductor at 100kHz.

(b) Skin depth,

$$\delta = \sqrt{\frac{1}{\pi f\mu\sigma}} = \frac{1}{\sqrt{\pi\times100\times10^3 4\pi\times10^{-7}\times4}} = 0.7919\text{m}$$

Now for good conductor ,

$$\alpha = \beta = \frac{1}{\delta} = 1.2626\text{Np}/\text{m}$$

The propagating constant can be expressed as

$$\gamma = \alpha + j\beta = (1.2626 + j1.2626)\text{m}^{-1}$$

The intrinsic impedance inside the sea water can be determined as

$$\eta = \frac{(1+j)}{\sigma\delta} = \frac{(1+j)}{4\times0.7919} = (0.3157 + j3157) = 0.446e^{j\pi/4}\Omega$$

The phase velocity inside the medium is

$$V_p = \omega\delta = 2\times\pi\times100\times10^3\times0.7919 = 4.975\times10^5\,\text{m}/\text{s}$$

(C) The amplitude of electric field at the surface is 0.2V/m the depth z_1 at which this amplitude of E gets reduced to half its value at the surface can be obtained using the following expression:

$$0.1 = 0.2e^{-1.2626z_1}$$

$$\Rightarrow z_1 = \frac{1}{1.2626}\ln 2 = 54.89\text{cm}$$

(d) From the given E- field equation

$$E_s(0^+) = 0.2\angle75^o\,\hat{a}_x$$
$$= E_{x0}\hat{a}_x$$
$$= |E_{x0}|\angle75^o\hat{a}_x$$

From the above equation it obvious that the value of the electric field at z = 0 is complex quantity. Now the general phaser expression of the electric field for plane wave polarized along x-direction and propagating along +z direction can be given as

$$\overline{E}_s(z) = E_{x0}e^{-j\beta z}\hat{a}_x \ V/m$$

$$\overline{E}_s(z) = 0.2e^{-1.2626z}e^{-j1.2626z}e^{j75^o}\hat{a}_x \ \ V/m$$

The phasor form of the magnetic filled in the present case can be written using the above equation as

$$\overline{H}_s(z) = \frac{0.2}{|\eta|}e^{-1.2626z}\ e^{-j\theta_n}\ e^{j75^o}\ \ e^{-j1.2626z}\ \hat{a}_y \ \ A/m$$

$$= \frac{0.2}{0.446}e^{-1.2626z}e^{-j\pi/4}e^{j75^o}\ \ e^{-j1.2626z}\ \hat{a}_y \ \ A/m$$

$$= 0.448e^{-1.2626z}e^{-j1.2626z+30^o}\ \hat{a}_y \ \ A/m$$

Finally, the time-domain expression for the electric and magnetic fields associated with this wave in sea water (z>0) can be given as

$$\overline{E}(z,t) = 0.2e^{-1.2626z}\cos\left(2\pi\times10^5 t - 1.2626z + 75^0\right)\hat{a}_x \ V/m$$

$$\overline{H}(z,t) = 0.448e^{-1.2626z}\ \cos\left(2\pi\times10^5 t - 1.2626z + 30^o\right)\hat{a}_y \ A/m$$

Example: 4

A coaxial lossless transmission line consists of inner and outer conductors made of copper and having of a = 1.3 mm and b = 5.5 mm. the space between the cylindrical conductors being filled with air. The line is to be used at 1GHz. Find the values of the inductance (in H/m), capacitance (in F/m) characteristic impedance, and phase velocity on the line.

Sol. Given: Inner conductor radius, a = 1.3 mm

Outer conductor radius, 5.5 mm

f = 1GHz

As the line is operating at very high frequency (f = 1GHz), hence internal inductance cab be inflected.

Inductance per unit length of the line is

$$L = \frac{\mu_0}{2\pi}\ell\left(\ \right)\quad /\,m)$$

$$= \frac{4\pi\times10^{-7}}{2\pi}\ell\left(\ \right)$$

$$\therefore L \simeq \quad H/m$$

Capacitance per unit length of the cable is given by

$$C = \frac{2\pi\varepsilon_0}{\ell\left(\ \right)}\quad (F/m)$$

$$= \frac{2\pi\times\frac{10^{-9}}{36\pi}}{\ell\left(\ \right)}$$

$$\therefore C = 38.5\rho F/m$$

Characteristic impedance of the line is given by

(∴ lossless transmission line)

$$z_0 = \sqrt{\frac{L}{C}}$$

$$= \sqrt{\frac{\frac{\mu_0}{2\pi}\ell\left(\ \right)}{\frac{2\pi\varepsilon_0}{\ell\left(\ \right)}}}$$

$$= 60\ell\left(\ \right)$$

$$= 60\ell\left(\ \right)$$

$$\therefore Z_0 = 86.64\Omega$$

(or)

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$= \sqrt{\frac{0.289\times10^{-6}}{38.5\times10^{-12}}}$$

$$= 86.64\Omega$$

Phase velocity of the wave on the line is given by

$v_p \cong 3\times10^8 \, m/sec$

Note: Phase velocity, we need not to calculate because the cable is having conductor filled with air (free space), hence the wave can travel with velocity of light (V_p = 3 × 10^8 m/sec)

Example: 5

In a rectangular waveguide for which a = 1 cm, b = 0.8cm, σ = 0, μ = μ_0, and ε = $3\varepsilon_0$,

$$H_x = 2\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{2\pi}{b}\right)\sin\left(\pi\times10^{11}t-\beta z\right)A/m$$

Determine

(i) The mode of operation

(ii) The cutoff frequency

(iii) The phase contant $\overline{\beta}$

(iv) The propagation content $\overline{\gamma}$

(v) The intrinsic wave impedance η

Sol:

We know that

$$H_x = H_{x0}.\sin\left(\frac{m\pi}{a}x\right).\cos\left(\frac{n\pi}{b}y\right)\sin\left(\omega t-\beta z\right)$$

From the given expression, it is clear that m = 1 and n = 2

H_x field component exists for both TE and TM waves, the mode can be either TE_{12} (or) TM_{12}.

(ii)

$$f_{c_{mn}} = \frac{c}{2\sqrt{\varepsilon_r}}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$$

$$f_{c_{13}} = \frac{c}{2\sqrt{3}}\sqrt{\frac{1}{\left[1\times10^{-2}\right]^2}+\frac{4}{\left[0.8\times10^{-2}\right]^2}} = \frac{3\times10^8}{2\sqrt{3}}\left(\sqrt{1+6.25}\right)\times10^2$$

f_c = 23.31GHz

(iii)

$$\overline{\beta} = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left[\frac{f_c}{f}\right]^2} = \frac{\omega\sqrt{\varepsilon_r}}{c}\sqrt{1-\left[\frac{f_c}{f}\right]^2}$$

ω = 2πf = π × 10^{11}

$$\overline{\beta} = \frac{\pi \times 10^{11}\sqrt{3}}{3 \times 10^{8}}\sqrt{1-\left[\frac{23.31}{50}\right]^{2}}$$

$$\overline{\beta} = 1604.63\, rad/m$$

(iv) $\overline{\gamma} = j\overline{\beta} = j1604.63m$

(v) $$\eta_{TM_{13}} = \eta'\sqrt{1-\left[\frac{f_c}{f}\right]^{2}}$$

$$= \frac{377}{\sqrt{\varepsilon_r}}\sqrt{1-\left[\frac{23.31}{50}\right]^{2}}$$

= 192.56 Ω

$$\eta_{TE_{13}} = \frac{\eta'}{\sqrt{1-\left(\frac{f_c}{f}\right)^{2}}} = \frac{377}{\sqrt{\epsilon_r}} \times \frac{1}{\sqrt{1-\left(\frac{23.31}{50}\right)^{2}}}$$

= 246.03 Ω s

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