ESE (Mains) 2019
Electromagnetic

Important Questions with Solutions

EMT

1. Two homogeneous isotropic dielectrics are separated by plane z = 0.

For $z \geq 0$; ϵ_r = 4 and $E = 3\hat{}\quad\hat{}\quad\hat{}$ kV/m

If ϵ_r = 3 for $z \leq 0$ then find,

A. E for $z \leq 0$

B. The angles that electric field makes with the normal to the plane z = 0 for $z \geq 0$ and for $z \leq 0$

C. The energy densities in both the dielectrics

D. The energy within a cube of side of 4 m and centred at (1, –3, 5)

Sol.

A) Let $\epsilon_{r_1} = 4$ and $E_1 = 3\hat{}\quad\hat{}\quad\hat{}$

$\epsilon_{r_2} = 3$

Since mediums are separated by z = 0 plane $\hat{}$ will be normal to the plane

$\therefore E_{n_1} = 6\hat{}$

$\& \ E_{t_1} = E_1 - E_{n_1}$

$\therefore E_{t_1} = 3\hat{}\quad\hat{}$

Now $E_{t_2} = E_{t_1}$

$\therefore E_{t_2} = 3\hat{}\quad\hat{}$ ...(i)

and $D_{n_2} = D_{n_1}$

$\therefore \epsilon_{r_2} E_{n_2} = \epsilon_{r_1} E_{n_1}$

$\therefore E_{n_2} = \frac{4}{3} \cdot 6\hat{}$

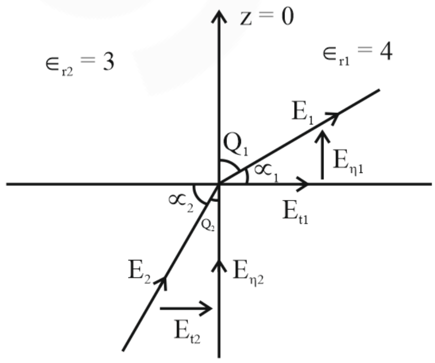
$\therefore E_{n_2} = 8\hat{}$ ...(ii)

Adding (i) and (ii)

$E_2 = E_{t_1} + E_{n_1}$

$\therefore E_2 = 3\hat{}\quad\hat{}\quad\hat{}$ kV/m For $z \leq 0$

B)



Now

$E_{n_1} = 6\hat{}$

$\therefore |E_{n_1}| = 6$

and $E_{t_1} = 3\hat{}\quad\hat{}$

$\therefore |E_{t_1}| = \sqrt{3^2 + 4^2}$

$\therefore |E_{t_1}| = 5$

$\therefore \tan Q_1 = \frac{E_{t_1}}{E_{n_1}}$

$= \frac{5}{6}$

$\therefore Q_1 = 39.80^0$

$\therefore \alpha_1 = 90 - Q_1$

= 90 – 39.80

$\therefore \alpha_1 = 50.19°$

So E_1 makes 50.19° angle with the normal to z = 0 plane

$E_{n_2} = 8\hat{}$

$\therefore |E_{n_2}| = 8$

and $E_{t_2} = 3\hat{}\quad\hat{}$

$\therefore |E_{t_2}| = \sqrt{3^2 + 4^2}$

$\therefore |E_{t_2}| = 5$

$\therefore \tan Q_2 = \frac{E_{t_2}}{E_{n_2}} = \frac{5}{8}$

$\therefore Q_2 = 32°$

$\therefore \alpha_2 = 90 - Q_2$

= 90 – 32

$\therefore \alpha_2 = 58°$

So E_2 makes 58° angle with the normal to z = 0 plane.

C) The energy density is given by,

$WE_1 = \frac{1}{2} \epsilon_1 |E_1|^2$

$= \frac{1}{2} \times 4 \times \frac{1}{36\pi \times 10^9} \times \left(3^2 + 4^2 + 6^2\right) \times 10^6$

$\therefore WE_1 = 1.07 \text{ mJ/m}^3$

∴ Energy density for $z \geq 0$ will be 1.07 mJ/m^3

now $WE_2 = \frac{1}{2} \epsilon_2 |E_2|^2$

$= \frac{1}{2} \times 3 \times \frac{1}{36\pi \times 10^9} \times (3^2 + 4^2 + 8^2) \times 10^6$

∴ WE^2 = 1.4 mJ/m^3

∴ Energy density for z ≤ 0 will be 1.4 mJ/m^3

D) Given cube has centre (1, −3, 5) and it has side of 4 m so it completely lies for z ≥ 0

∴ the cube will have energy density 1.07 mJ/m^3

As it has side of 4 m

It will cover the region

−1 ≤ x ≤ 3 ; −5 ≤ y ≤ −1 ; 3 ≤ z ≤ 7

$\therefore WE_2 = \oint$

$$= \int_{x=-1}^{3} \int_{y=-5}^{-1} \int_{z=3}^{7} WE_2 \, dx\, dy\, dz$$

= 1.07 × 4 × 4 × 4 × 10^{-3}

∴ W_E = 69.03 mJ

2. The magnetic field of a plane was travelling in free space is given by,

$H_s = 0.05 \cos(\omega t - j6\pi z)(\hat{a}_y - j\hat{a}_x)$ A/m

then find

A. Frequency and direction of propagation

B. Corresponding electric field vector

C. polarization of the wave

D. time average power density associated with wave

Sol.

A) Hs = 0.05 cos(ωt − j6πz) $(\hat{\ } \quad \hat{\ })$

A/m

∴ the wave is travelling along +ve z direction and β = 6π

now $V_p = \frac{\omega}{\beta}$

As the wave is travelling through free space

$V_p = \frac{\omega}{\beta} = 3 \times 10^8$

∴ ω = 6π × 3 × 10^8

∴ f = 9 × 10^8

∴ Frequency = 900 MHz

B) Now Hs=0.05cos(ωt − j6πz)$(\hat{\ } \quad \hat{\ })$

$\therefore H_s = 0.05 \cos(\omega t - j6\pi z)\hat{a}_y$

$- 0.05 j\cos(\omega t - j6\pi z)\hat{a}_x$

let $H_1 = 0.05 \cos(\omega t - j6\pi z)\hat{\ }$

and $H_2 = -0.05 j\cos(\omega t - j6\pi z)\hat{\ }$

now $\frac{|E|}{|H|} = 120\pi$ and $\bar{E} \times \bar{H} = \bar{P}$

$\therefore |E_1| = 0.05 \times 120\pi = 18.8$

∴ E_1 direction = H_1 direction × propagation direction

$= \hat{a}_y \quad \hat{a}_z$

$\therefore E_1$ direction $= \hat{\ }$

$\therefore E_1 = 18.8 \cos(\omega t - j6\pi z)\hat{\ }$

Similarly

$|E_2| = 0.05 \times 120\pi = 18.8$

And E_2 direction = H_2 direction × propagation direction $= -j(\hat{\ } \quad \hat{\ })$

$\therefore E_2$ direction $= j\hat{\ }$

Now E = E_1 + E_2

$\therefore E = 18.8 \cos(\omega t - j6\pi z)(\hat{\ } \quad \hat{\ })$

C) $E = 18.8 \cos(\omega t - j6\pi z)(\hat{\ } \quad \hat{\ })$

We calculate polarization at z = 0

$\therefore E(0,t) = 18.8\cos(\omega t)(\hat{\ } \quad \hat{\ })$

$\therefore E(0,t) = 18.8\cos\omega t \hat{\ } \quad \hat{\ }$

As E_1 = E_2 it will be circular polarization let's draw E vector at different time intervals.

At t = 0 $\quad E = 18.8 \hat{\ }$

At t = T/8

$E = 18.8\cos\left(\frac{2\pi}{T} \times \frac{T}{8}\right)\hat{a}_x - 18.8\text{in}\left(\frac{2}{T} \quad \frac{T}{8}\right)\hat{a}_y$

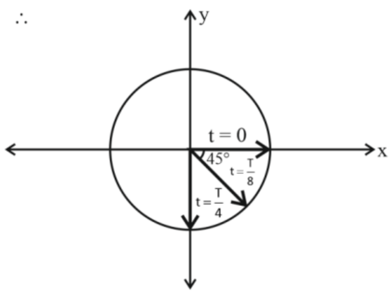
$E = 18.8\left(\frac{1}{\sqrt{2}}\hat{\ } \quad \frac{}{\sqrt{2}}\hat{\ }\right)$

At t = T/4

$E = 18.8\cos\left(\frac{2\pi}{T} \times \frac{T}{4}\right)\hat{a}_x - 18.8\sin\left(\frac{2}{T} \quad \frac{T}{4}\right)\hat{a}_y$

$E = -18.8\hat{\ }$

∴



∴It is left circular polarization.
D) Time average power density associated with wave is

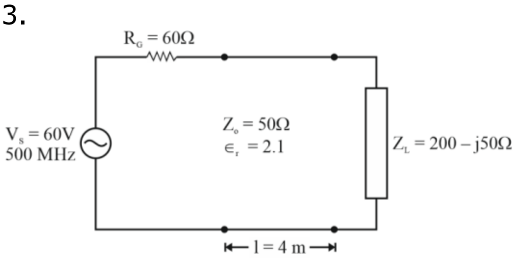
$$P = \frac{1}{2}\left(E_s \times H_s^*\right)$$

$$= \frac{1}{2} \times 18.8 \left(\hat{} \quad \hat{} \quad , \quad \hat{} \quad \hat{} \quad , \right)$$

$$= \frac{0.94}{2}\left(\hat{} \quad \hat{} \quad , \right)$$

$$P = 0.94 \hat{}$$

3.



For the circuit as shown above calculate
A. Reflection co-efficient and VSWR
B. Input impedance seen by transmitter
C. The time averaged power delivered to the transmission line by the transmitter
D. The time averaged power delivered to antenna
E. Total power supplied by the transmitter
Sol.
A)Reflection co-efficient (Γ)

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{(200 - j50) - 50}{(200 - j50) + 50}$$

= 0.6154 – j0.0769
∴ Γ = 0.6202 < –0.1244

$$\therefore |\Gamma| = 0.6202$$

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$= \frac{1 + 0.6202}{1 - 0.6202}$$

∴ VSWR = 4.27

B) Now $V_p = \frac{\omega}{\beta}$

$$\therefore \beta = \frac{\omega}{V_p} = \frac{2\pi \times f}{V_p}$$

$$= \frac{2\pi \times 5 \times 10^8 \times \sqrt{2.1}}{3 \times 10^8}$$

$$\therefore \beta = 15.18 \text{ rad/m}$$

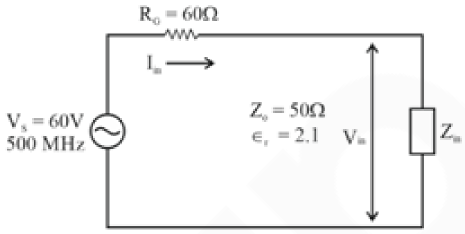
Now

$$Z_{in} = Z_o \cdot \frac{Z_L \cos\beta\ell \qquad \ell}{Z_o \cos\beta\ell \qquad \ell}$$

$$= 50 \cdot \frac{(200 - j50)\cos(15.18 \times 4) + j50\sin(15.18 \times 4)}{50\cos(15.18 \times 4) + (200 - j50)\sin(15.18 \times 4)}$$

$$Z_{in} = 14.664 - j24.17\Omega$$

C) Equivalent circuit can be drawn as,



$$\therefore V_{in} = V_s \cdot \frac{Z_{in}}{Z_{in} + R_G}$$

$$= 60 \times \frac{14.664 - j24.17}{14.664 - j24.17 + 60}$$

$$\therefore V_{in} = 16.3574 - j14.127V$$

∴ V_{in} = 21.613 < –40.81 V

$$\therefore |V_{in}| = 21.613 \text{ V}$$

now $I_{in} = \frac{V_{in}}{Z_{in}}$

$$= \frac{21.613 < -40.81}{14.664 - j27.14}$$

$$I_{in} = 0.655 + j0.298$$

∴ I_{in} = 0.7 < 20.8

$$\therefore |I_{in}| = 0.7 \text{ A}$$

Now power will be dissipated only across resistive part of the load

$$= \frac{1}{2} \times (I_{in})^2 \times \text{Real}(Z_{in})$$

$$\therefore P_{in} = \frac{1}{2} \times (0.7)^2 \times 14.664$$

∴ P_{in} = 3.592 watt
D) As the transmission line is lossless, total power delivered to the transmission line will be completely delivered to the antenna. The time average power delivered to the antenna is 3.592 watt.
E) Power dissipated in internal impedance R_G is

$$P_G = \frac{1}{2} \times (I_{in})^2 \times R_G$$

$$= \frac{1}{2} \times (0.7)^2 \times 60$$

∴ P_G = 14.7 watt

F) Total power supplied by the transmitter will be,

$P_T = P_{in} + P_G$

= 3.592 + 14.7

$P_T = 18.292$ watt

4. A. Define dominant mode in a rectangular waveguide and find the power carried by the dominant mode for a > b by using transverse components.
B. Plot the TE_{10} mode and how to extract the TE_{10} mode effectively.
Sol.
A) Dominant mode: The mode which is having highest cut off wave length (or) lowest cut off frequency.

$$\lambda_C = \frac{2ab}{\sqrt{(mb)^2 + (na)^2}}$$

Lowest TM mode is TE_{11}

$$\therefore \lambda_C TM_{11} = \lambda_C TE_{11} = \frac{2ab}{\sqrt{b^2 + a^2}}$$

$$\lambda_C TE_{01} = 2b$$

$$\lambda_C TE_{10} = 2a$$

As a > b i.e., TE_{10} is the dominant mode
Field components for TE_{10}

$$H_Z = C\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

$$E_X = -\frac{j\omega\mu}{h^2}\frac{\partial H_Z}{\partial y}$$

$$= \frac{j\omega\mu}{h^2}\frac{n\pi}{b}C\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_Z}{\partial x}$$

$$= \frac{-j\omega\mu}{h^2}\frac{m\pi}{a}C\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

$$H_X = -\frac{\bar{\gamma}}{h^2}\frac{\partial H_Z}{\partial x}$$

$$= \frac{\bar{\gamma}}{h^2}\left(\frac{m\pi}{a}\right)C\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

$$H_y = -\frac{\bar{\gamma}}{h^2}\frac{\partial H_Z}{\partial y}$$

$$= \frac{\bar{\gamma}}{h^2}\left(\frac{n\pi}{b}\right)C\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

For, $TE_{10} \Rightarrow H_z = C\cos\left(\frac{\pi}{a}x\right)$

$$and\ E_x = H_y = 0$$

$$E_y = \frac{-j\omega\mu}{h^2}C\left(\frac{\pi}{a}\right)\sin\left(\frac{\pi}{a}x\right)$$

$$H_x = \frac{j\bar{\beta}}{h^2}C\left(\frac{\pi}{a}\right)\sin\left(\frac{\pi}{a}x\right)$$

Power carried by waveguide

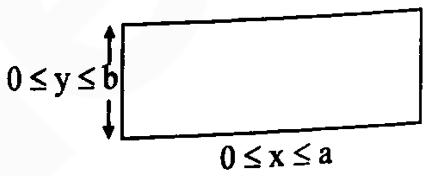
$$p = \bigcirc$$

$$\bar{p} = \frac{1}{2}\mathrm{Re}[\bar{E} \times \bar{H}^*]$$

$$= \frac{1}{2}\mathrm{Re}\left[\frac{-j\omega\mu}{h^2}C\left(\frac{\pi}{a}\right)\sin\left(\frac{\pi}{a}x\right)\hat{y} \times \hat{x}\frac{-j\bar{\beta}}{h^2}C\left(\frac{\pi}{a}\right)\sin\left(\frac{\pi}{a}x\right)\right]$$

$$\bar{p} = \frac{\bar{\beta}}{2}\frac{1}{\omega\mu}\frac{(\omega\mu)^2 C^2\left(\frac{\pi}{a}\right)^2}{(h^2)^2}\sin^2\left(\frac{\pi}{a}x\right)\hat{z}$$

$$p = \frac{E_{\max}^2}{2Z_{TE}}\sin^2\left(\frac{\pi}{a}x\right)\hat{z}$$



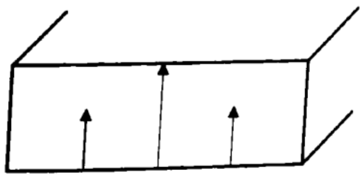
$$P = \bigcirc \qquad \frac{ax}{TE} b \int_{x=0}^{a} \sin^2\left(\frac{\pi}{a}x\right)dx$$

$$= \frac{E_{\max}^2}{2Z_{TE}} \times b \times \frac{a}{2}$$

$$\therefore P_{TE10} = \frac{E_{\max}^2}{4Z_{TE}}ab$$

B) We have

$$E_y \propto \sin\left(\frac{\pi}{a}x\right)$$



As the maximum E is existing at midpoint of the broader dimension, it is effective if making use of probe or E-coupling.

5. A. Find the radiation efficiency of an Hertzian dipole metal wire having radius 1.8 mm, length 2 meter and σ = 6 × 10^9 s/m and it is operating at frequency of 2 MHz.

B. An end-fire array consists of λ/2 radiations with axes at right angles to the line of the array required to have a power gain of 30. Determine the array length and the width of a major lobe between the nulls.

Sol.

A) For Hertzian dipole, Radiation resistance (R_r)

$$R_r = 80\pi^2\left(\frac{dl}{\lambda}\right)^2$$

$$= 80\pi^2\left(\frac{dl\ f}{c}\right)^2$$

$$= 80\pi^2\left(\frac{2\times2\times10^6}{3\times10^8}\right)^2$$

R_r = 0.14 Ω

And loss resistance (R_L)

$$R_L = \frac{dl}{2\pi a}\sqrt{\frac{\pi f\mu}{\sigma}}$$

Now dl = 2m, a = 1.8 × 10^{-3},

μ = 4π × 10^{-7}

f = 2 × 10^6, σ = 6 × 10^9

$$\therefore R_L = \frac{2}{2\pi\times1.8\times10^{-3}}\times\sqrt{\frac{\pi\times2\times10^6\times4\pi\times10^{-7}}{6\times10^9}}$$

∴ R_L = 6.41 × 10^{-3} Ω

∴ R_L = 0.00641 Ω

∴ Radiation efficiency (η)

$$\eta = \frac{R_r}{R_r + R_L}$$

$$= \frac{0.14}{0.14 + 0.00641}$$

∴ η = 0.9561

∴ Radiation efficiency = 95.61%

B) For an end fire array

$$D = 4\left(\frac{L}{\lambda}\right)$$

$$30 = 4\left(\frac{L}{\lambda}\right)$$

∴ L = 7.5λ

∴ Array length = 7.5λ

Now width of a major lobe between the nulls is given as,

$$BWFN = 114.6\sqrt{\frac{2}{L/\lambda}}$$

$$= 114.6\sqrt{\frac{2}{7.5}}$$

∴ BWFN = 59.18°

∴ Width of major lobe between the nulls is 59.18°.

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