NEET Physics

Short Notes

Oscillations

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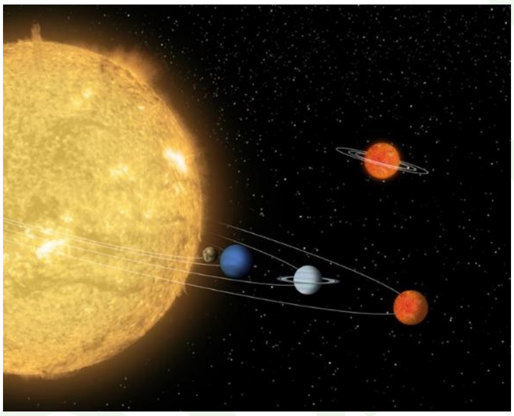
Oscillation is an important topic from NEET Exam Point of view. Every year there are 1-2 questions directly asked from this topic. This short notes on Oscillation will help you in revising the topic before the NEET Exam.

Oscillations

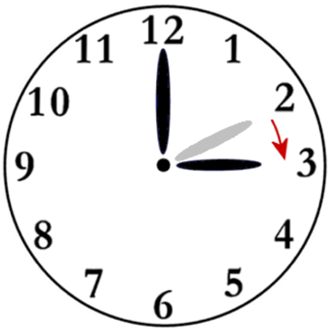
Periodic and Oscillatory Motion

Periodic Motion- Periodic motion of a body is that motion which is repeated identically after a fixed interval of time.

Example- The revolution of the planet around the sun in the solar system is periodic motion.

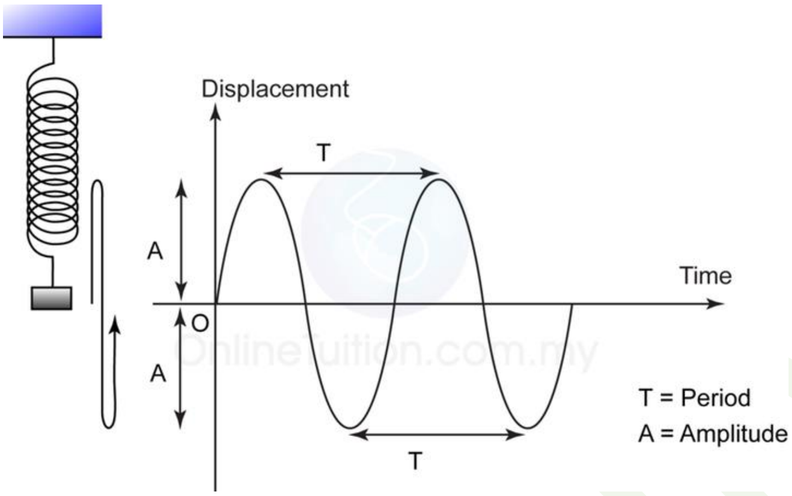


The motion of hands of a clock is a periodic motion.



Oscillatory or Vibratory motion- Oscillatory motion is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point (called mean position), in a definite interval of time.

In such a motion, the body is confined within well-defined limits (called extreme positions) on either side of the mean position. Thus a periodic and bounded motion of a body about a fixed point is called an oscillatory or vibratory motion.



Period and Frequency

Time period (T) - It is defined as the smallest interval of time after which the motion is repeated.

$$T = \frac{2\pi}{\omega}$$

, where ω is the angular velocity.

The earth revolves around the sun in 365 days. So the time period of the earth is 365 days. The minute hand takes 60 minutes to complete the cycle so the time period of the minute hand is 60 minute or 1 hour.

Similarly, the second hand of clock take 60 seconds to complete the cycle so the time period of the second hand is 60 second or 1 minute.

Frequency (f) - It is defined as the number of oscillations per unit time. The SI unit of the frequency is Hertz.

$$f = \frac{1}{T}$$

, where T is the time period.

Displacement as a Function of Time

Displacement- It refers to change in physical quantities with time such as position, angle, pressure, electric,.. etc. For an oscillating simple pendulum, the angle measured from the vertical as a function of time is a displacement variable.

Displacement variable is measured as the function of time, and it can have both positive and negative values. In the case of periodic motion, the displacement is

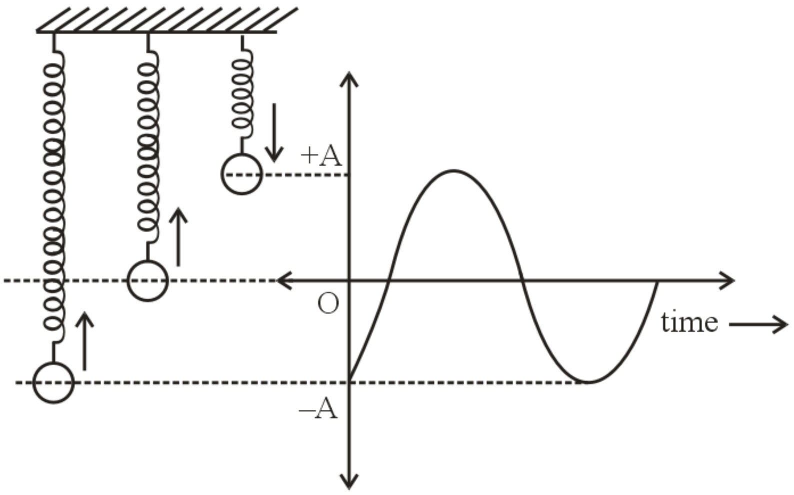
$$x = f(t) = A\cos(\omega t + \phi) \text{ or } y = f(t) = A\sin(\omega t + \phi)$$

where ω is the angular velocity, and ϕ is the phase change.

Simple Harmonic Motion

All oscillatory motions are simple harmonic motion. It is mainly two type-

Linear SHM- A particle executing linear simple harmonic motion oscillates in straight line periodically in such a way that the acceleration is proportional to its displacement from a fixed point, and is always directed towards that point.

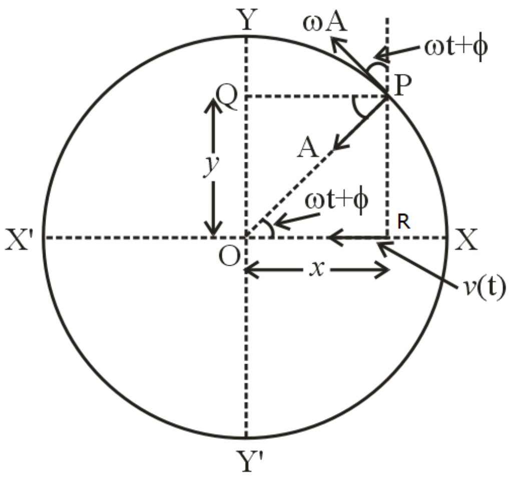


Angular SHM- If a body is describing rotational motion in such a way that direction of its angular velocity changes periodically and torque acting on it is always directed opposite to the angular displacement and magnitude of the torque is directly proportional to the angular displacement, then its motion is called angular SHM.

Velocity and Acceleration in SHM

Let us take a particle is moving in a Uniform circular motion of radius A. At time t = 0 particle is at position X and at time t is at point P. So the magnitude of the displacement of N from the means position at any instant is

$$x = A\cos(\omega t + \phi)$$



In right triangle OPR

$$\cos(\omega t + \phi) = \frac{x}{A}$$

where A is the radius of the reference circle and θ is the angle covered by the reference particle in time t.

If ω be the uniform angular velocity of the reference particle then, $x = A\cos(\omega t + \phi)$

If the projection N' of the reference particle is taken on the diameter YOY', then

$$y = A\cos(\omega t + \phi) \qquad \text{.......(1)}$$

On differentiating equation (1) with respect to time

$$\frac{dy}{dt} = \frac{d}{dt}\left(A\sin(\omega t + \phi)\right)$$

$$v = A\omega\cos(\omega t + \phi) \qquad \text{........(2)}$$

The velocity of the particle in SHM is

$$v = A\omega \frac{\sqrt{A^2 - y^2}}{A}$$
$$v = \omega\sqrt{A^2 - y^2}$$

On differentiating equation (2) with respect to time

$$\frac{dv}{dt} = \frac{d}{dt} A\omega\cos(\omega t + \phi)$$
$$a = -A\omega^2 \sin(\omega t + \phi)$$
$$a = -A\omega^2 y$$

Acceleration of the particle in SHM is $a = -A\omega^2 y$

Energy in SHM

In SHM kinetic and potential energy of the particle is vary between zero to maximum.

Kinetic energy in Simple harmonic motion is

$$KE = \frac{1}{2}mv^2$$
$$KE = \frac{1}{2}m\left(\omega\sqrt{A^2 - y^2}\right)^2$$
$$KE = \frac{1}{2}m\omega^2\left(A^2 - y^2\right)$$

$$KE = \frac{1}{2}m\omega^2\left(A^2 - \left(A\cos(\omega t + \phi)\right)^2\right)$$
$$KE = \frac{1}{2}m\omega^2\left(A^2 - A^2\cos^2(\omega t + \phi)\right)$$
$$KE = \frac{1}{2}m\omega^2 A^2\left(1 - \cos^2(\omega t + \phi)\right)$$
$$KE = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$

Potential energy in SHM is

$$PE = \frac{1}{2} m\omega^2 y^2$$

$$PE = \frac{1}{2} m\omega^2 \left(A\cos\left(\omega t + \phi\right)\right)^2$$

$$PE = \frac{1}{2} m\omega^2 A^2 \cos^2\left(\omega t + \phi\right)$$

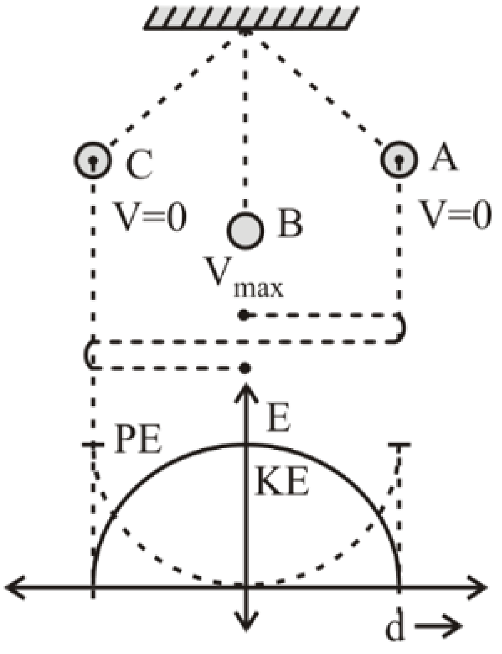
The total energy in SHM is

$$TE = KE + PE$$

$$TE = \frac{1}{2} m\omega^2 A^2 \sin^2\left(\omega t + \phi\right) + \frac{1}{2} m\omega^2 A^2 \cos^2\left(\omega t + \phi\right)$$

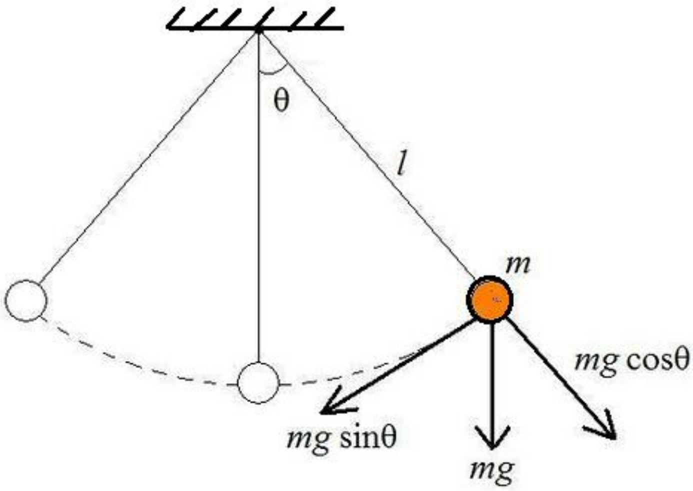
$$TE = \frac{1}{2} m\omega^2 A^2 \left(\sin^2\left(\omega t + \phi\right) + \cos^2\left(\omega t + \phi\right)\right)$$

$$TE = \frac{1}{2} m\omega^2 A^2$$



The Simple Pendulum: Time period Expression

Let us assume that a mass suspended by a light, long and inextensible string forms a simple pendulum. Length of the simple pendulum is the distance between the point of suspension and the centre of mass of the suspended mass. Consider the bob when string deflects through a small angle θ. Force acting on the bob are tension T in the string and weight mg of the bob.

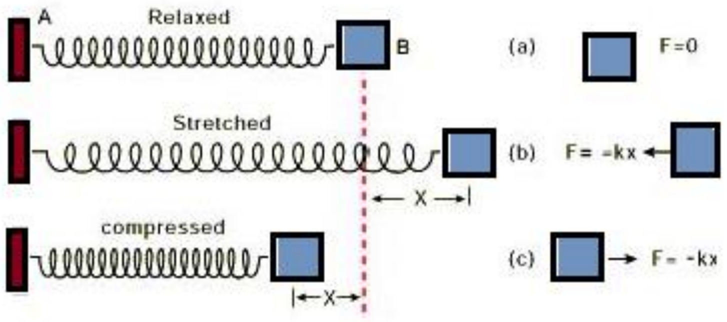


Acceleration of the pendulum is, $a = -\omega^2 x$

The time period of the simple pendulum is, $T = 2\pi\sqrt{\frac{l}{g}}$, where l is the length of the simple pendulum.

Oscillations of a Spring

Let us assume that spring of spring constant k is attached with a block of mass m on a smooth horizontal surface as shown below. At equilibrium, position spring is released. When the block is displaced through a distance x towards the right, it experiences a net restoring force, F = -kx toward left. The negative sign shows that the restoring force is always opposite to the displacement. That is when x is positive, F is negative the force is directed to the left. When x is negative, F is positive, the force always tends to restore the block to its equilibrium position x = 0.



$$F = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

Comparing equation (i) with, F = ma

$$a = \frac{d^2x}{dt^2}$$

$$a = -\omega^2 x$$

$$-\frac{kx}{m} = -\omega^2 x$$

$$\omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

The time period of the simple harmonic motion is,

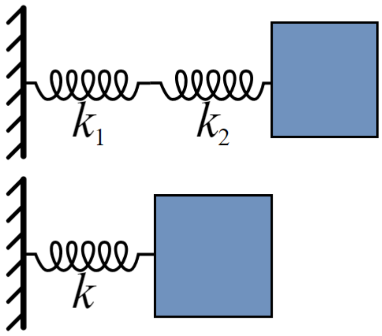
$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

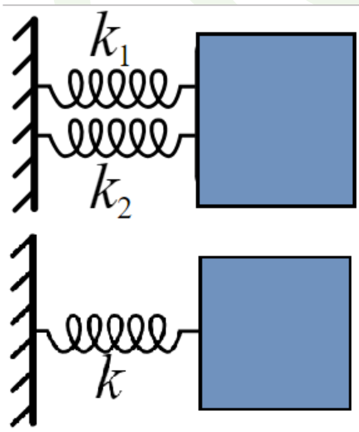
Restoring Force and Force Constant

Series and Parallel combination of spring

When two spring are joined in series, the equivalent stiffness of the combination is $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$



When two spring are joined in parallel, the equivalent stiffness of the combination is $k = k_1 + k_2$



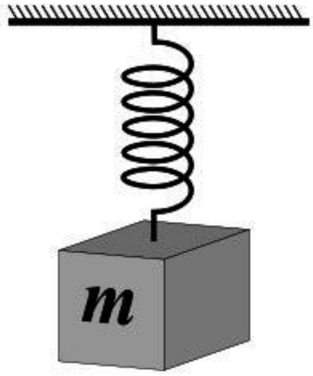
Free, Forced and Damped SHM, Resonance

Damped SHM

In damped SHM the amplitude of oscillating body is reduced and eventually comes to its mean position. In damped oscillations, the energy of the system is dissipated continuously but for small damping, the oscillations remain approximately periodic.

Let us consider a spring-mass system block of mass m connected to an elastic spring of spring constant k oscillates vertically. If the block is pushed down a little and released, its angular frequency of oscillation is,

$$\omega = \frac{k}{m}$$



The total force acting on the mass at any time t, $F = -kx - bv$

the motion of the block under the influence of a damping force, $x(t) = Ae^{-bt/2m}\cos(\omega' t + \phi)$

where A is the amplitude and ω′ is the angular frequency of the damped oscillator. $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

Forced Oscillations and Resonance

Free Oscillations- When a system oscillates with its own natural frequency, without the help of any external periodic force, its oscillations are called free oscillations.

Forced Oscillation- When a system oscillates with the help of an external periodic force, other than its own natural angular frequency, its oscillations are called forced or driven oscillations. The differential equation of forced damped harmonic oscillator is given by

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_o \cos \omega_o t$$

where ω_o is the angular frequency of the external force. The displacement of the forced damped harmonic oscillator at any instant t is given by

$$x = A\cos(\omega_o t + \phi)$$

where, $A = \frac{F_o}{\left[m^2\left(\omega^2 - \omega_d^2\right)^2 + \omega_d^2 b^2\right]^{1/2}}$ and $\tan\phi = -\frac{v_o}{\omega_d x_o}$

where ω is the natural angular frequency of the oscillator, x_o and v_o are the displacement and velocity of the oscillator at time t = 0, when the periodic force is applied.

Resonance- It is the phenomenon in which a system is made to oscillate by external force whose frequency is equal to the natural frequency of the system. At resonance, the amplitude of the system is maximum. It is a special case of forced oscillation. Condition for resonance is $\omega = \omega_d$

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