# JEE Main Maths

## Short Notes

**Solution of Triangle and Inverse Trigonometric Function**

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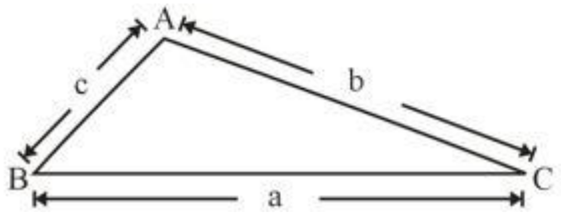
Consider a general triangle with sides a, b, c, and vertices A, B and C




semi perimeter $S = \frac{a+b+c}{2}$

**1. sine rule**

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

**2.Cosine rule**

$$\text{Cos A} = \frac{b^2+c^2-a^2}{2bc}$$

$$\text{Cos B} = \frac{a^2+c^2-b^2}{2ca}$$

$$\text{Cos C} = \frac{b^2+a^2-c^2}{2ab}$$

**3. Half angle formulas**

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$\sin\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}$$

$$\sin\frac{C}{2} = \sqrt{\frac{(s-b)(s-a)}{ab}}$$

$$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$$

$$\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}}$$

$$\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$$

$$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$$

$$\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

**4. sine of an angle in terms of sides of the triangle**

$$\sin A = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}$$

$$\sin B = \frac{2}{ac}\sqrt{s(s-a)(s-b)(s-c)}$$

$$\sin C = \frac{2}{ab}\sqrt{s(s-a)(s-b)(s-c)}$$

**5. Domain and Range of various inverse trigonometric functions**

| **Function** | **Domain** | **Range** |
|---|---|---|
| $\sin^{-1}x$ | [-1,1] | [-π/2 , π/2] |
| $\cos^{-1}x$ | [-1,1 | [0, π] |
| $\tan^{-1}x$ | R | (-π/2 , π/2) |
| $\cot^{-1}x$ | R | (0, π) |
| $\sec^{-1}x$ | R ~ (-1,1) | [0, π0~{ π/2 } |
| $\text{cosec}^{-1}x$ | R ~ (-1,1) | [-π/2 , π/2]~{0} |

**6. Relation between different inverse trigonometric functions**

For x > 0

a) $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}} = \cot^{-1}\frac{\sqrt{1-x^2}}{x}$

$= \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$

b) $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

$= \sec^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

c) $\tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \cot^{-1}\left(\frac{1}{x}\right)$

**7. Relating $F^{-1}$ (x) with $F^{-1}$ (-x)**

$\sin^{-1}(-x) = -\sin^{-1}x$

$\cos^{-1}(-x) = \pi - \operatorname{cosec}^{-1}x$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$

$\sec^{-1}(-x) = \pi - \sec^{-1}x$

$\tan^{-1}(-x) = \pi - \cot^{-1}x$

**8. Relation between $F^{-1}$ (x) and $F^{-1}$(1/x)**

a) $\sin^{-1}(1/x) = \operatorname{cosec}^{-1}x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

b) $\cos^{-1}(1/x) = \sec^{-1}x$. for all $x \in (-\infty, -1] \cup [1, \infty)$

c) $\tan^{-1}(1/x) = \{\cot^{-1}x$ , for $x>0$ ; $-\pi+\cot^{-1}x$ , for $x<0\}$

**9. Complementary Angles**

a) $\sin^{-1}x + \cos^{-1}x = \pi/2$ for all $x \in [-1,1]$

b) $\tan^{-1}x + \cot^{-1}x = \pi/2$ for all $x \in R$

c) $\sec^{-1}x + \operatorname{cosec}^{-1}x = \pi/2$ for all $x \in (-\infty, -1] \cup [1, \infty)$

**10. Sum and difference of angles in terms of $\sin^{-1}$ and $\cos^{-1}$**

a) $\sin^{-1}x + \sin^{-1}y = \begin{cases} \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right) x\geq 0, y\geq 0,\ x\geq 0,\ y\geq 0 \\ \pi - \sin^{-1}\left(x\sqrt{1-4^2} + y\sqrt{1-x^2}, x^2+y^2<1, x^2+y^2-1\right) \end{cases}$

b) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-x^2}\right)$, $x \geq 0$, $y \geq 0$

c) $2\sin^{-1}x = \begin{cases} \sin^{-1}\left(2x\sqrt{1-x^2}\right) -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}} \\ \pi - \sin^{-1}\left(2x\sqrt{1-x^2}\right) x > \frac{1}{\sqrt{2}} \\ -\pi - \sin^{-1}\left(2x\sqrt{1-x^2}\right) x < -\frac{1}{\sqrt{2}} \end{cases}$

d) $\sin^{-1}x = \begin{cases} \sin^{-1}(3x-4x^3) & \frac{-1}{2} \leq x \leq \frac{1}{2} \\ \pi - \sin^{-1}(3x-4x^3) & x > \frac{1}{2} \\ -\pi - \sin^{-1}(3x-4x^3) & x < \frac{-1}{2} \end{cases}$

**11. Multiple angle formulas**

$$2\tan^{-1}x = \begin{cases} \sin^{-1}\left(\frac{2x}{1+x^2}\right), & if -1 \le x \le 1 \\ \pi - \sin^{-1}\left(\frac{2x}{1+x^2}\right), & if\ x > 1 \\ -\pi - \sin^{-1}\left(\frac{2x}{1+x^2}\right), & if\ x < -1 \end{cases}$$

Or

$$2\tan^{-1}x = \begin{cases} \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), & if\ 0 < x < \infty \\ -\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), & if -\infty < x \le 0 \end{cases}$$



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