JEE Main Maths

Short Notes

Point and Straight Line

Powered by :

gradeup

1. Basics of 2D geometry

Coordinate geometry is the branch of Mathematics which describes geometrical properties by means of algebra.

Cartesian Coordinate system: There is a reference frame which consists of two mutually perpendicular lines in a plane. The horizontal line is called the x-axis and the vertical line is called the y-axis and their point of intersection is called the origin. The location of a point P with respect to the system is described by an ordered pair of real numbers i.e. (x, y) where x is the algebraic distance from the y-axis and y is the distance from the x-axis. x is called the abscissa and y the ordinate.

Coordinates of origin are (0,0)

The x-axis and the y-axis of the Cartesian system divide the xy plane into 4 parts which are known as quadrants. For x coordinate to the right of the y-axis, it is positive and those to the left are negative. Similarly, y coordinate above x-axis are positive and below are negative.

(a) Distance formula

Let the points be $P(x_1,y_1)$ and $Q(x_2, y_2)$, then the distance formula is

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(b) Section formula

If a point divides the line joining two points $P(x_1,y_1)$ and $Q(x_2, y_2)$ in the ratio m: n, the point of division is

$$X = \frac{mx_2 + nx_1}{m+n}$$

$$Y = \frac{my_2 + ny_1}{m+n}$$

2. Shifting of origin

The coordinate system is versatile and can be shifted to anywhere
Let (0,0) be the origin and another point P be at (x,y)
If the origin is shifted to (x_0, y_0) the coordinates of P are affected independently
The new coordinates of P will be $(x-x_0 , y-y_0)$

3. The slope of a line and angle between 2 lines

Slope of the gradient of a line is defined as tanθ where θ is the angle made by the line with the positive direction of the x-axis in the anticlockwise direction. It is defined by m. The slope is undefined for a line parallel to the y-axis.

$m=\tan\theta$ where $0 \le \theta \le \pi$

When $\theta=\pi/2$ the line will be parallel to the y-axis

If m_1 and m_2 be the slopes of the lines AB and CD which intersect at a point P and makes angles θ_1 and θ_2 respectively with the positive direction of x-axis then the angle between them is given by

$$\tan\theta = \pm\frac{m_1-m_2}{1+m_1m_2}$$

The acute angle θ between two straight lines is given by

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

Two lines are said to be parallel if their slopes are equal

Any line parallel to ax+by+c=0 will be of the form ax+by+k=0

The shortest distance between two parallel lines $ax+by+c_1 = 0$ and $ax+by+c_2 = 0$ is given by

$$\left|\frac{c_1-c_2}{(a^2+b^2)^{0.5}}\right|$$

Two lines are said to be perpendicular if the product of their slopes is equal to -1

Any line perpendicular to the line ax+by+c=0 will be of the form bx-ay+k=0

4. Equations of a straight line in various forms

(a) Parallel to axes

A line parallel to x-axis will be horizontal and have its y coordinate constant
So the equation of the horizontal line is y=constant
Similarly, the equation of the line parallel to the y-axis is x=constant

(b) Point-slope form

The equation of the line with slope m and passing through the point $A(x_1, y_1)$ is given as

$(y-y_1) = m(x-x_1)$

Slope intercept form

The equation of a straight line with slope m and y-intercept c is given as y=mx+c

If the line passes through the origin then the equation becomes y=mx

Two point form

The equation of the line which passes through the two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$y - y_1 = \frac{y_2-y_1}{x_2-x_1}.(x - x_1)$$

Slope of the line is

$$m = \frac{y_2-y_1}{x_2-x_1}$$

(e) Intercepts form

The equation of the line whose x-intercept and y-intercept are a and b respectively is given by $\frac{x}{a}+\frac{y}{b}=1$

(f). Normal form

The equation of the line where the length of the perpendicular to the line from the origin is p and the angle made by this perpendicular with the positive direction of x-axis taken in the counter-clockwise direction is q is given by

xcosq+ysinq=p, (p>0)

(g) The general equation of a line

The general equation of a line is ax+by+c=0 (a, b, c are real numbers) where a and b both are not zero at the same time.

6. The equation of the family of lines passing through the point of intersection of two lines

If L_1: $a_1x+by+c_1=0$ and L_2: $a_2x+b_2y+c_2=0$ be intersecting straight lines and a third line L_3 passes through the intersection of these two lines then the equation of L_3 will be of the form L_3:L_1 + pL_2 where p is an arbitrary constant. Depending on the value of p there may be infinite such lines passing through the point of intersection of L_1 and L_2.

7. The distance of a point

The distance of a point P(x_1, y_1) from the line L:ax+by+c=0 is given by

$$\frac{ax_1+by_1+c}{(a^2+b^2)^{0.5}}$$

The distance between two points A(x_1, y_1) and B(x_2, y_2) is given by

$$[(x_2-x_1)^2+(y_2-y_1)^2]^{0.5}$$

8. The equations of the lines passing through a point (x_1, y_1)and making an angle α with a line y=mx+c

$y-y_1=\tan(\theta\pm\alpha)(x-x_1)$

9. The image of a point w.r.t. a plane mirror represented as a line

Let A(x_1, y_1) and B(x_2, y_2) be the image of each other w.r.t. the line mirror ax+by+c=0

$$\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{(a^2 + b^2)}$$

Subscribe to YouTube Channel for JEE Main

All the best!
Team Gradeup

All About JEE Main Examination: https://gradeup.co/engineering-entrance-exams/jee-main

Download Gradeup, the best IIT JEE Preparation App