JEE Main Maths

Short Notes

ELLIPSE

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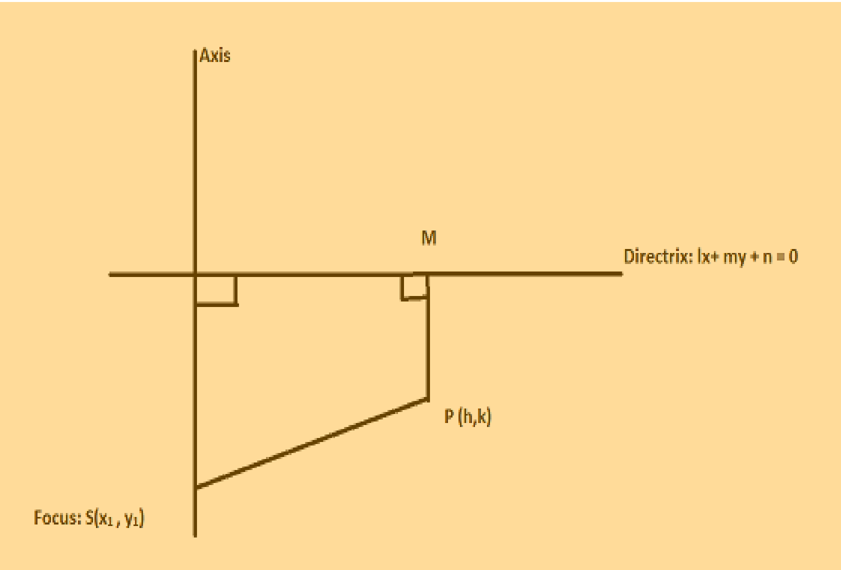
1.Basic of conic sections

Conic Section is the locus of a point which moves such that the ratio of its distance from a fixed point to its perpendicular distance from a fixed line is always constant.



The focus is the fixed point, directrix is the fixed straight line, eccentricity is the constant ratio and axis is the line passing through the focus and perpendicular to the directrix.

The point of intersection with axis is called vertex of the conic.

1.1 General Equation of the conic

$PS^2 = e^2 PM^2$

Simplifying above, we will get

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, this is the general equation of the conic.

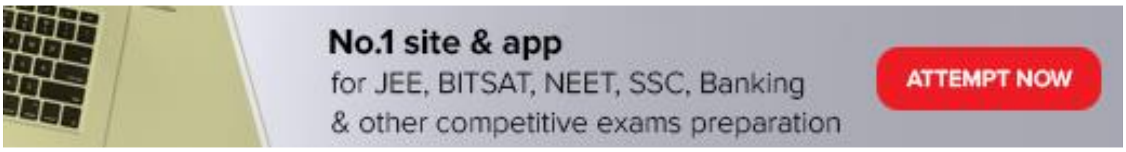
1.2 Nature of the conic

The nature of the conic depends on eccentricity and also on the relative position of the fixed point and the fixed line.

Discriminant 'Δ' of a second-degree equation is defined as:

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = abc + 2fgh - af^2 - bg^2 - ch^2$$

Nature of the conic depends on



Case 1: When the fixed point 'S' lies on the fixed line i.e., on directrix. Then the discriminant will be 0. Then the general equation of the conic will represent two lines

Case 2: When the fixed point 'S' does not lies on the fixed line i.e., not on directrix. Then the discriminant will not be 0. Then the general equation of the conic will represent parabola, ellipse, and hyperbola.

(a) Parabola: When eccentricity is 1; $h^2 = ab$

(b) Ellipse: When eccentricity is <1; $h^2 < ab$

(c) Hyperbola: When eccentricity is >1; $h^2 > ab$

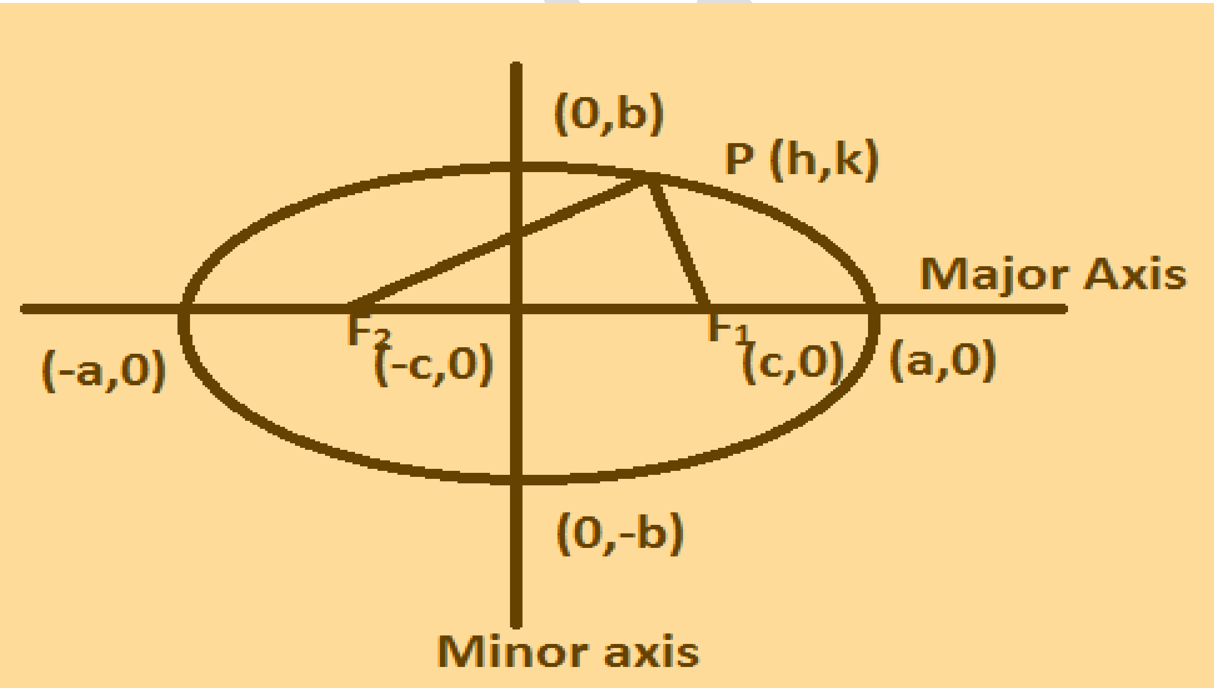
When a+ b = 0 then it becomes rectangular hyperbola.

2. Ellipse

Ellipse is the locus of a point in a plane which moves such that the sum of its distances from two fixed points in the same plane is always constant i.e., $|PF_1| + |PF_2| = 2a$

Simplifying above equation, the final equation of the ellipse will be,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$ where $b^2 = a^2 - c^2$



2.1 Nomenclature of the ellipse

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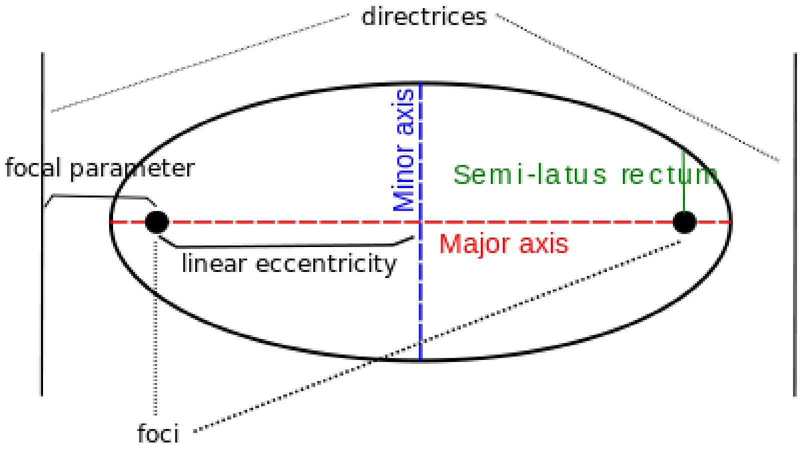
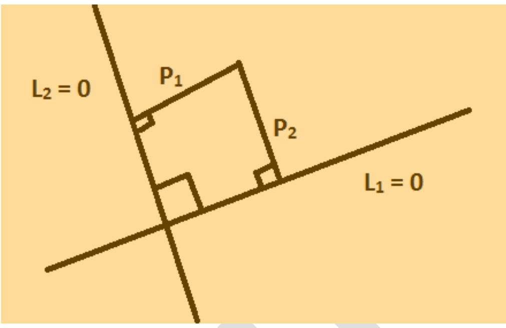


Above equations are valid when axes are x and y-axis. In the case of axes different from x and y-axis, the equation will be

$\frac{P_1^2}{a^2} + \frac{P_2^2}{b^2} = 1$, and $L_1 = 0$ and $L_2 = 0$ are the major and minor axis.



2.2 Terminology of the ellipse (when axes are x and y axis)

(a) The line joining two focuses (F_1 and F_2) are called as the focal axis or major axis.

(b) Distance between F_1 and F_2 is called as focal length.

(c) The points at (a,0) and (-a,0) are the coordinates of the vertex.

(d) Length of the major axis is 2a and that of the minor axis is 2b

(e) A chord of ellipse perpendicular to the major axis is called as double ordinate.

(f) A chord which passes through the focus of the ellipse is called as focal chord

(g) double ordinate passing through focus or a focal chord perpendicular to the major axis and passes through focus is called as latus rectum

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(h) Length of latus rectum = = $2a(1-e^2)$ = 2e (distance between focus and the directrix corresponding to the focus)

(i) Any chord of the ellipse passing through the centre (point of intersection of the major and minor axis) is bisected at this point and hence it is called as diameter.

2.3 Eccentricity

$$e = \frac{\text{Distance of focus from centre}}{\text{Distance of vertex from centre}} = \frac{c}{a}$$

⇒ c = ae

For the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

⇒ $b^2 = a^2 - c^2 = a^2(1 - e^2)$

⇒ $$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{(\text{Minor axis})^2}{(\text{Major Axis})^2}}$$

When a >b, then the equation of the ellipse will be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and rest will follow accordingly. However when a<b, then the equation of the ellipse will be $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ and thus the formula for different parameters will change accordingly.

For instance, Directrix now would be : $y = \pm b/e$

Vertices: $(0, \pm b)$

Length of latus rectum: $2a^2/b$

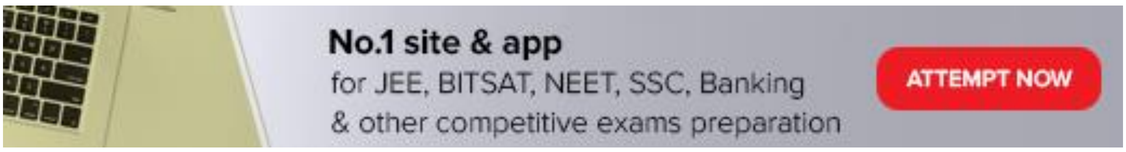
$$e = \sqrt{1 - \frac{a^2}{b^2}}$$

Focus: $(0, \pm be)$ since c = be now

2.4 Auxiliary Circle

The circle described on the major axis of the ellipse as the diameter is called an auxiliary circle.

P: (a cosθ, b sin θ)



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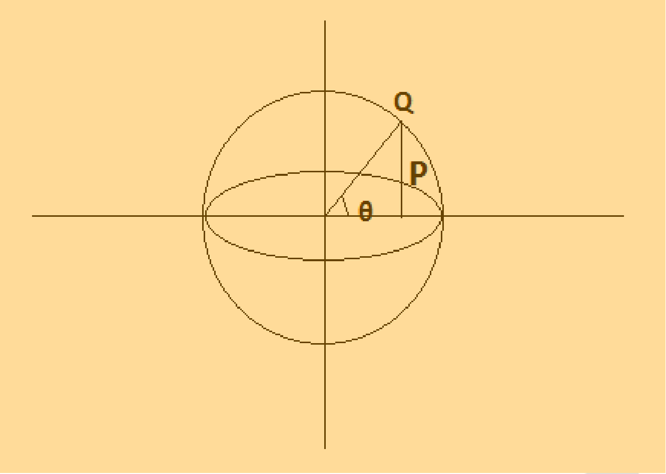
Q: (a cosθ, a sin θ)

Equation of the circle is:

$x^2 + y^2 = a^2$

When a perpendicular is drawn from a point Q on the auxiliary circle to cut the ellipse at P then P, Q are called as corresponding points and θ is called as eccentric angle of the point P.



2.5 Parametric Representation of an ellipse

In the auxiliary topic, we saw that P is on the ellipse and thus it satisfies the equation of the ellipse.

Parameter here is θ.

This gives us the parametric representation of the ellipse with point P: (a cosθ, b sin θ)

2.6 Position of a point w.r.t to an ellipse

Let S: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Point P lies outside the circle,

$$\frac{y_1^2}{b^2} > \frac{y_2^2}{b^2}$$
$$\frac{x_1^2}{b^2} + \frac{y_1^2}{b^2} - 1 > \frac{x_1^2}{b^2} + \frac{y_2^2}{b^2} - 1$$
$$\frac{x_1^2}{b^2} + \frac{y_1^2}{b^2} - 1 > 0$$

$S_1 > 0$: Point is outside the curve

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$S_1 < 0$: Point is inside the curve

$S_1 = 0$: Point is on the curve.

2.7 Line and Ellipse

L: $y = mx + c$

S: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

A line can be a tangent to the ellipse, it may cut the ellipse or it may not cut/touch the ellipse at all.

To find that, we will make a quadratic in 'x' using equations of L and S.

If discriminant D> 0 ⇒ Two roots ⇒ Intersect ⇒ Secant

If D= 0 ⇒ One root ⇒ Touching the ellipse ⇒Tangent

If D< 0 ⇒ No roots ⇒ Neither secant nor tangent

2.8 Condition of Tangency

D = 0

$\Rightarrow c = \pm\sqrt{a^2m^2 + b^2}$

Thus the equation of the tangent would be

L: $y = mx + c$

$$y = mx \pm\sqrt{a^2m^2 + b^2}$$

2.9 Number of tangents from a given point (h,k) to the ellipse

We have,

$$y = mx \pm\sqrt{a^2m^2 + b^2}$$

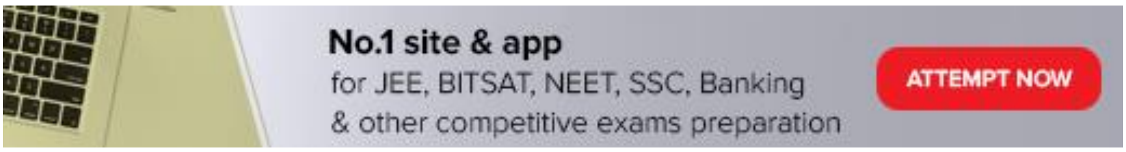
p (h,k)

Thus,

$(k - mh)^2 = a^2m^2 + b^2$ which is a quadratic in m.

This suggests that from a given point P(h,k) we can draw at max two tangents.

2.10 Angle between the two tangents



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Let the angle between the tangents be θ

From above quadratic equation, we know slope: m_1 and m_2

Thus, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\sqrt{(m_1+m_2)^2 - 4m_1m_2}}{1+m_1m_2}\right|$

From the quadratic equation in m,

$$m_1 + m_2 = \frac{2hk}{h^2 - a^2};\ m_1m_2 = \frac{k^2 - b^2}{h^2 - a^2}$$

2.11 Director circle of the ellipse

When θ = 90° then

$m_1m_2 = -1$

$x^2 + y^2 = a^2 + b^2$: Equation of director circle.

Director circle is the locus of all those point from where the ellipse can be seen at angle 90°.

2.12 Equation of the chord of an ellipse joining α and β on it

$$\frac{x}{a}\cos\frac{\alpha+\beta}{2} + \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha-\beta}{2}$$

2.13 The equation of Tangent, Normal and Chord of Contact

Tangents:

(a) Cartesian Tangent

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

(b)Slope form

$$y = mx \pm \sqrt{a^2m^2 + b^2}$$

(c) Parametric form

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$$

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Normal:

(a) Cartesian Normal

$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 e^2 = a^2 - b^2$$

(b) Slope form

$$y = mx - \frac{(a^2 - b^2)m}{\sqrt{a^2 + b^2m^2}}$$

(c) Parametric form

$$ax \sec\theta - by \operatorname{cosec}\theta = a^2 e^2 = a^2 - b^2$$

2.14 Chord of contact:

The equation of chord of contact will be like that of the tangent. Thus, a line when touches the ellipse will be tangent and the same line when cuts the ellipse will be the chord of contact.

Equation: T = 0 (Similar to that of tangent equation)

2.15 Pair of tangents:

Equation of pair of tangents would be

$SS_1 = T^2$, where S is the equation of the ellipse, S_1 is the equation when a point P (h, k) satisfies S, T is the equation of the tangent.

2.16 Equation of the chord whose middle point is (x_1, y_1):

$T = S_1$

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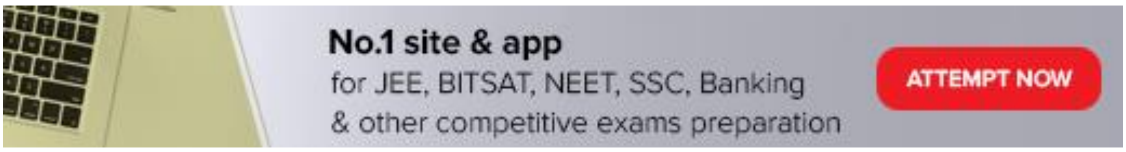

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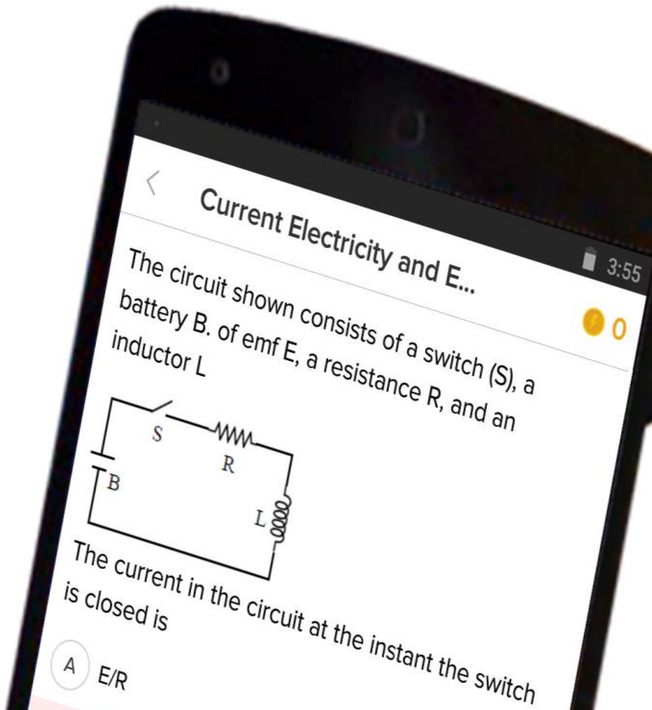

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