

JEE Main Maths Short Notes Binomial Theorem

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1. Binomial Theorem for positive integral index

If n is a positive integer and x, y is real or imaginary then

 $(x + y)^{n} = {^{n}C_{0}x^{n-0}y^{0}} + {^{n}C_{1}x^{n-1}y^{1}} + \dots + {^{n}C_{r}x^{n-r}y^{r}} + \dots + {^{n}C_{n}x^{0}y^{n}}$ i.e $(x + y)^{n} = \sum_{r=0}^{n} {^{n}C_{r}x^{n-r}y^{r}}$

General term is $t_{r+1} = {}^n C_r x^{n-r} y^r$ Here ${}^n C_0, {}^n C_1, {}^n C_2$... are called the binomial coefficients

Replacing y by -y the general term of $(x - y)^n$ is obtained as $t_{r+1} = (-1)^r \cdot {}^n C_r \cdot x^{n-r} y^r$

Similarly, the general term of $(1 + x)^n$ and $(1 - x)^n$ can be obtained by replacing x by 1 and x by 1 and y by -x respectively

• $.^n C_r = {}^n C_{(n-r)}$

•
$$.^{n}C_{r}+{}^{n}C_{r-1}={}^{n+1}C_{r}$$

•
$$.^{n}C_{r}/^{n}C_{r-1} = \frac{n-r+1}{2}$$

•
$$r_{r}^{n}C_{r} = n_{r}^{(n-1)}C_{r-1}$$

•
$$\frac{1}{r+1}^{n} C_{r} = \frac{1}{n+1}^{n+1} C_{r+1}$$

2. Middle Term

The middle term depends upon the even or odd nature of n

Case 1: When n is even Total number of terms in the expansion of $(x + y)^n$ is n+1 (odd) So there is only 1 middle term i.e $\left(\frac{n}{2} + 1\right)^{\text{th}}$ term is the middle term This is given by $t_{\frac{n}{2}+1} = {}^n C_{\frac{n}{2}} x^{\frac{n}{2}} y^{\frac{n}{2}}$

Case 2: When n is odd Total number of terms in the expansion of $(x + y)^n$ is n+1(even) So there are 2 middle terms i.e $\frac{n+1}{2}^{th}$ and $\frac{n+3}{2}^{th}$ terms are both middle terms They are given by

$$t_{\frac{n+1}{2}}, t_{\frac{n+3}{2}} = {}^{n}C_{\frac{n-1}{2}}x^{\frac{n+1}{2}}.y^{\frac{n-1}{2}}, {}^{n}C_{\frac{n+1}{2}}x^{\frac{n-1}{2}}.y^{\frac{n+1}{2}}$$

3. Greatest Term

If t_r and t_{r+1} br the rth and $(r + 1)^{th}$ terms in the expansion, $(1 + x)^n$ then $\frac{t_{r+1}}{t_r} = \frac{n - r + 1}{r} \cdot x$



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Let t_{r+1} be the numerically greatest term in the above expression

Then $t_{r+1} \ge t_r$ $Or \frac{n-r+1}{r} \ge 1$

Greatest Coefficient:

- If n is even, the greatest coefficient $=^{n} C_{\underline{n}}$
- If n is odd, then greatest coefficients are $\binom{n}{2} C_{\frac{n-1}{2}}$ and $\binom{n}{2} C_{\frac{(n+1)}{2}}$

4. Binomial Coefficients

In the binomial expansion of $(1 + x)^n$ let us denote the coefficients by C_0, C_1, \dots, C_n respectively

Since $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$ Put x=1 $C_0 + C_1 + C_2 + \cdots C_n = 2^n$

$$C_0 + C_2 + C_4 + \dots = C_1 + C_3 + C_5 + \dots$$

Sum of odd terms coefficients=Sum of even terms coefficients= 2^{n-1}

5. Series Summation

- $C_0 + C_1 + C_2 + C_3 + \dots + C_n = 2^n$
- $C_0 C_1 + C_2 C_3 + \dots + (-1)^n C_n = 0$
- $C_0 + C_2 + C_4 + ... = C_1 + C_3 + C_5 + ... = 2^{n-1}$ $C_0^2 + C_1^2 + C_2^2 + ... C_n^2 = \frac{(2n)!}{(n!)^2}$

6. Multinomial Expansion

If n is a positive integer and $a_1, a_2, a_3, \dots, a_m$ are real or imaginary then

$$(a_1 + a_2 + a_3 + \dots + a_m)^n = \sum \frac{n!}{n_1! n_2! n_3! \dots n_m!} a_1^{n_1} a_2^{n_2} \dots a_m^{n_m}$$

Where $n_1, n_2, n_3, \dots n_m$ are non-negative integers such that $n_1 + n_2 + n_3 + \dots + n_m = n$

- The coefficients of $a_1^{n_1}a_2^{n_2} \dots a_m^{n_m}$ in the expansion of $(a_1 + a_2 + \dots + a_m)^n$ is $\frac{n!}{n_1!n_2!\dots n_m!}$
- The number of dissimilar terms in the expansion of

$$(a_1 + a_2 + \cdots + a_m)^n =^{n+m-1} C_{m-1}$$

7. Binomial theorem for negative or fractional index



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When n is negative and/or fraction and |x|<1 then

$$(1+x)^{n} = 1 + nx + \frac{n(n-1)}{2!}x^{2} + \frac{n(n-1)(n-2)}{3!}x^{3} + \dots + n(n-1)(n-2)\dots\frac{n-r+1}{r!}x^{r} + \dots \text{ infinite terms}$$

The general term is $t_{r+1} = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}x^r$

For example, if |x| < 1 then,

- $(1+x)^{-1} = 1 x + x^2 x^3 + x^4 \cdots$
- $(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + ..$
- $(1+x)^{-2} = 1 2x + 3x^3 4x^3 + \cdots$
- $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$
- $(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}x^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}x^3 + \cdots$
- $(1+x)^{-\frac{1}{2}} = 1 \frac{1}{2}x + \frac{\frac{1}{2}(\frac{1}{2}+1)}{2!}x^2 \frac{\frac{1}{2}(\frac{1}{2}+1)(\frac{1}{2}+2)}{3!}x^3 + \cdots \dots$

8. Using Differentiation and Integration in Binomial Theorem

- (a) Whenever the numerical occur as a product of binomial coefficients, differentiation is useful.
- (b) Whenever the numerical occur as a fraction of binomial coefficients, integration is useful

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