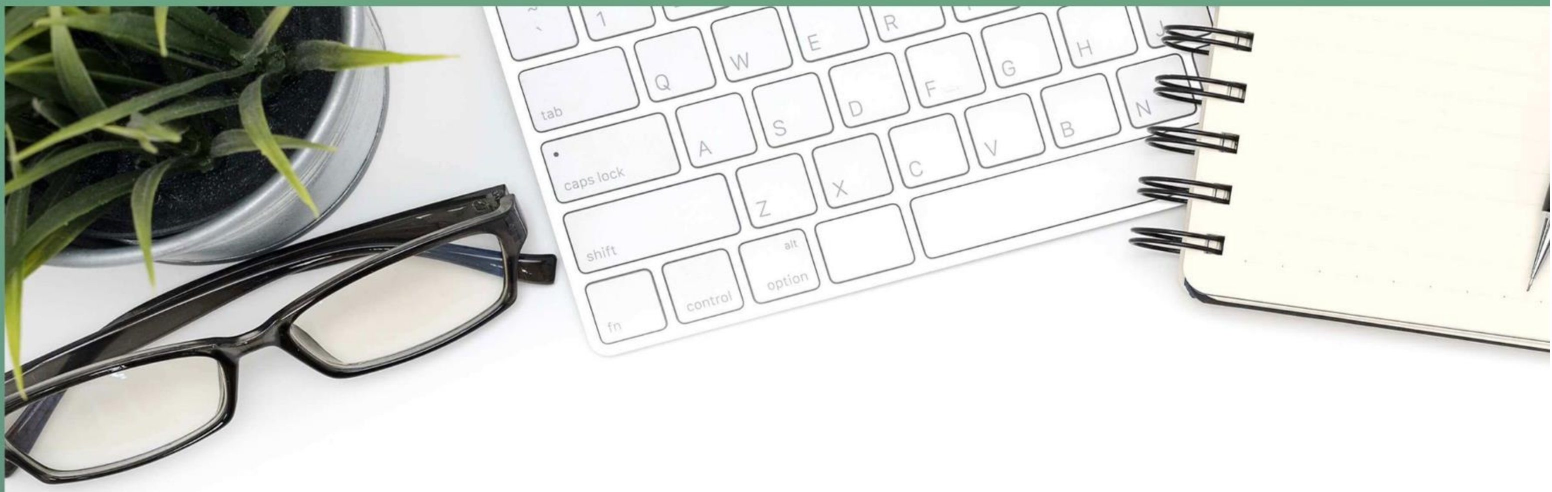


# JEE Main Maths

## Short Notes

### Binomial Theorem

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## 1. Binomial Theorem for positive integral index

If n is a positive integer and x, y is real or imaginary then

$$(x+y)^n = {}^nC_0x^{n-0}y^0 + {}^nC_1x^{n-1}y^1 + \cdots + {}^nC_rx^{n-r}y^r + \cdots + {}^nC_nx^0y^n$$

i.e $(x+y)^n = \sum_{r=0}^{n} {}^nC_rx^{n-r}y^r$

General term is $t_{r+1} = {}^nC_rx^{n-r}y^r$

Here ${}^nC_0, {}^nC_1, {}^nC_2$ ... are called the binomial coefficients

Replacing y by -y the general term of $(x-y)^n$ is obtained as

$$t_{r+1} = (-1)^r.{}^nC_r.x^{n-r}y^r$$

Similarly, the general term of $(1+x)^n$ and $(1-x)^n$ can be obtained by replacing x by 1 and x by 1 and y by -x respectively

- ${}^nC_r = {}^nC_{(n-r)}$
- ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- ${}^nC_r / {}^nC_{r-1} = \frac{n-r+1}{r}$
- $r.{}^nC_r = n.{}^{(n-1)}C_{r-1}$
- $\frac{1}{r+1}{}^nC_r = \frac{1}{n+1}{}^{n+1}C_{r+1}$

## 2. Middle Term

The middle term depends upon the even or odd nature of n

Case 1: When n is even

Total number of terms in the expansion of $(x+y)^n$ is n+1 (odd)

So there is only 1 middle term i.e $\left(\frac{n}{2}+1\right)^{th}$ term is the middle term

This is given by $t_{\frac{n}{2}+1} = {}^nC_{\frac{n}{2}}x^{\frac{n}{2}}y^{\frac{n}{2}}$

Case 2: When n is odd

Total number of terms in the expansion of $(x+y)^n$ is n+1(even)

So there are 2 middle terms i.e $\frac{n+1}{2}^{th}$ and $\frac{n+3}{2}^{th}$ terms are both middle terms

They are given by

$$t_{\frac{n+1}{2}}, t_{\frac{n+3}{2}} = {}^nC_{\frac{n-1}{2}}x^{\frac{n+1}{2}}.y^{\frac{n-1}{2}}, {}^nC_{\frac{n+1}{2}}x^{\frac{n-1}{2}}.y^{\frac{n+1}{2}}$$

## 3. Greatest Term

If $t_r$ and $t_{r+1}$ br the $r^{th}$ and $(r+1)^{th}$ terms in the expansion, $(1+x)^n$ then

$$\frac{t_{r+1}}{t_r} = \frac{n-r+1}{r}.x$$

Let $t_{r+1}$ be the numerically greatest term in the above expression
Then $t_{r+1} \geq t_r$
Or $\frac{n-r+1}{r} x \geq 1$

Greatest Coefficient:

- If n is even, the greatest coefficient $= {}^n C_{\frac{n}{2}}$
- If n is odd, then greatest coefficients are $.^n C_{\frac{n-1}{2}}$ and $.^n C_{\frac{(n+1)}{2}}$

**4. Binomial Coefficients**

In the binomial expansion of $(1+x)^n$ let us denote the coefficients by $C_0, C_1, \ldots . C_n$ respectively

Since $(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots + C_nx^n$
Put x=1

$$C_0 + C_1 + C_2 + \cdots C_n = 2^n$$

Put x=-1

$$C_0 + C_2 + C_4 + \cdots = C_1 + C_3 + C_5 + \cdots$$

Sum of odd terms coefficients=Sum of even terms coefficients= $2^{n-1}$

**5. Series Summation**

- $C_0 + C_1 + C_2 + C_3 + \cdots + C_n = 2^n$
- $C_0 - C_1 + C_2 - C_3 + \cdots + (-1)^n C_n = 0$
- $C_0 + C_2 + C_4 + .. = C_1 + C_3 + C_5 + .. = 2^{n-1}$
- $C_0^2 + C_1^2 + C_2^2 + \cdots C_n^2 = \frac{(2n)!}{(n!)^2}$

**6. Multinomial Expansion**

If n is a positive integer and $a_1, a_2, a_3 \ldots .. a_m$ are real or imaginary then

$$(a_1 + a_2 + a_3 + \cdots a_m)^n = \sum \frac{n!}{n_1!\, n_2!\, n_3! \ldots n_m!} a_1^{n_1} a_2^{n_2} \ldots a_m^{n_m}$$

Where $n_1, n_2, n_3, \ldots n_m$ are non-negative integers such that $n_1 + n_2 + n_3 + \cdots n_m = n$

- The coefficients of $a_1^{n_1} a_2^{n_2} \ldots a_m^{n_m}$ in the expansion of $(a_1 + a_2 + \cdots a_m)^n$ is $\frac{n!}{n_1! n_2! \ldots n_m!}$
- The number of dissimilar terms in the expansion of

$$(a_1 + a_2 + \cdots a_m)^n = {}^{n+m-1} C_{m-1}$$

**7. Binomial theorem for negative or fractional index**

When n is negative and/or fraction and |x|<1 then

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + n(n-1)(n-2)..\frac{n-r+1}{r!}x^r + \cdots \text{infinite terms}$$

The general term is $t_{r+1} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r$

For example, if |x|<1 then,

- $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \cdots$
- $(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 +..$
- $(1+x)^{-2} = 1 - 2x + 3x^3 - 4x^3 + \cdots.$
- $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$
- $(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}x^2 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}x^3 + \cdots.$
- $(1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{\frac{1}{2}\left(\frac{1}{2}+1\right)}{2!}x^2 - \frac{\frac{1}{2}\left(\frac{1}{2}+1\right)\left(\frac{1}{2}+2\right)}{3!}x^3 + \cdots ....$

## 8. Using Differentiation and Integration in Binomial Theorem

(a) Whenever the numerical occur as a product of binomial coefficients, differentiation is useful.

(b) Whenever the numerical occur as a fraction of binomial coefficients, integration is useful



All the best!

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