



# JEE Main Physics Short Notes

## Rotational Motion

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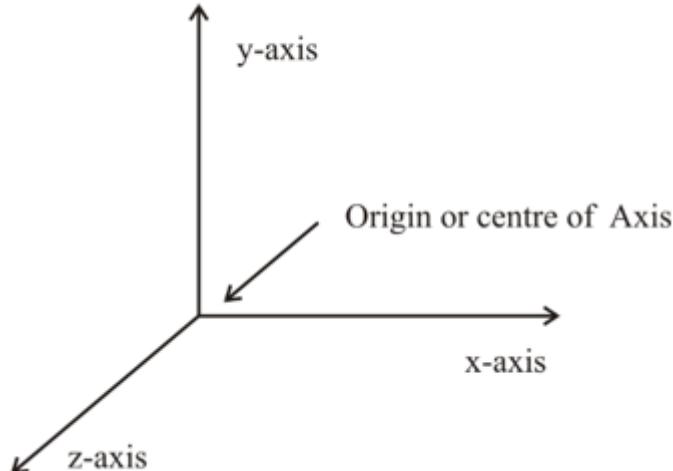
# System of Particles and Rotational Motion

## 1. Motion of a Rigid Body

### Motion and Centre of Axis Visualization

**Motion-** Motion is defined as the change in position of an object with respect to time and its surrounding.

**Axis-** Axis is a fixed imaginary lines to describe a position of an object in space. In Cartesian coordinate system centre of axis is taken as the point of intersection where all three axes mutually perpendicular to each other. It is also known as the origin.



## 2. Centre of Mass and its Motion

Centre of mass of a body is a point where the whole mass of the body is supposed to be concentrated. If all the forces acting on the body were applied on the Centre of mass, the nature of the motion of the body shall remains unaffected.

### A. Centre of Mass of a two Particle System

Centre of mass of a two-particle system is a point where the whole mass of the system is supposed to be concentrated.

Let us assume a system of two particles A and B which has masses  $m_1$  and  $m_2$  respectively. Let their position vectors  $\vec{r}_1$  and  $\vec{r}_2$  with respect to the origin O.



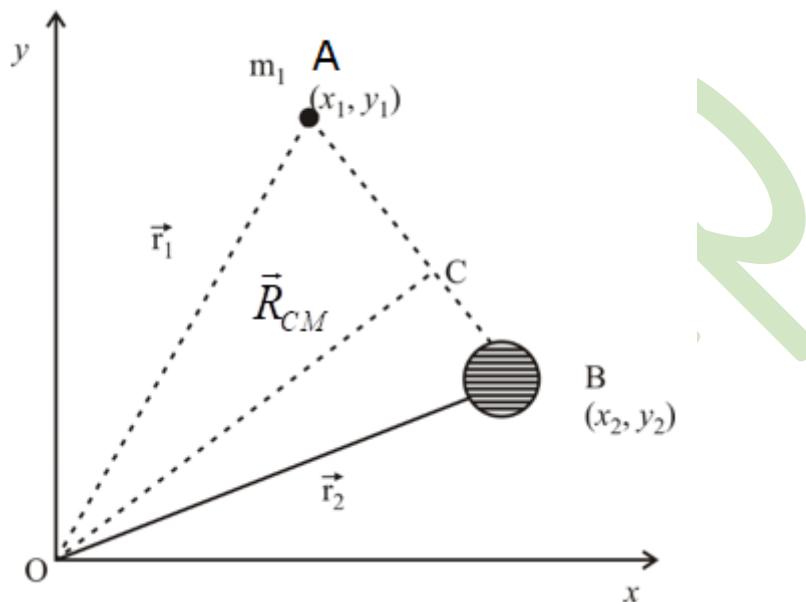
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The position vector  $\vec{R}_{CM}$  of the centre of mass C of the two-particle system is given by

$$\vec{R}_{CM} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$



Let  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  are the coordinates of their locations of the two particles, then the coordinates of their centre of mass is given by

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}, Y_{CM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} \text{ and } Z_{CM} = \frac{m_1 z_1 + m_2 z_2}{m_1 + m_2}$$

**For n particle system-** Let a system of n particles of masses  $m_1, m_2, m_3, \dots$  having their position vectors  $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots$  respectively with respect to the origin of the coordinate system. Then the Position vector of the centre of mass

$$\vec{R}_{CM} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$



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Let  $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), \dots$  are the coordinates of their locations of the n particles, then the coordinates of their centre of mass is given by

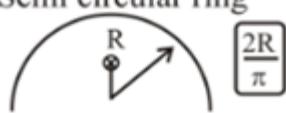
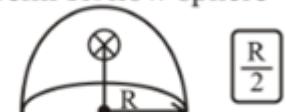
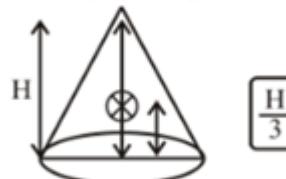
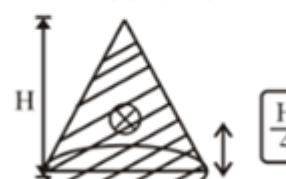
$$X_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\sum m_i x_i}{\sum m_i} , Y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\sum m_i y_i}{\sum m_i} \text{ and}$$

$$Z_{CM} = \frac{m_1 z_1 + m_2 z_2 + m_3 z_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\sum m_i z_i}{\sum m_i}$$

## B. Centre of Mass of a Rigid Body

The centre of mass of a rigid body is a point whose position is fixed with respect to the body. It doesn't change with time because the positions of the particles in a rigid body remains fixed.

### Centre of mass of geometrical rigid body

Semi circular ring		$\frac{2R}{\pi}$
Semi circular disc		$\frac{4R}{3\pi}$
Semi Hollow sphere		$\frac{R}{2}$
Semi Solid sphere		$\frac{3R}{8}$
Hollow cone		$\frac{H}{3}$
Solid cone		$\frac{H}{4}$
Triangular Plate		$\frac{H}{3}$



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## C. Motion of Centre of Mass

$$\vec{v}_{cm} = \frac{d\vec{R}_{cm}}{dt}$$

The velocity of the centre of mass for a system of n particles is

$$\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt}$$

Similarly, acceleration of the centre of mass for a system of n particles is

## 3. Basic Concept of Rotational Motion

### A. Linear Momentum of System of Particles

Let a system of n particles of masses  $m_1, m_2, m_3, \dots$  having their position vectors  $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots$  respectively with respect to the origin of the coordinate system. Then the Position vector of the centre of mass

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\sum m_i\vec{r}_i}{\sum m_i}$$

$$\vec{p}_{net} = M \frac{d\vec{R}_{CM}}{dt} = M\vec{v}_{cm}$$

Then the linear momentum of system of particle

### B. Linear and Angular Velocity

**Linear velocity** – The rate of change of linear displacement of a body in motion is known as linear velocity.

$$v = \frac{ds}{dt}, \text{ where } ds \text{ is the linear displacement}$$

**Angular velocity ( $\omega$ )** – Angular velocity of a particle is the rate of change of angular displacement in a rotational motion.



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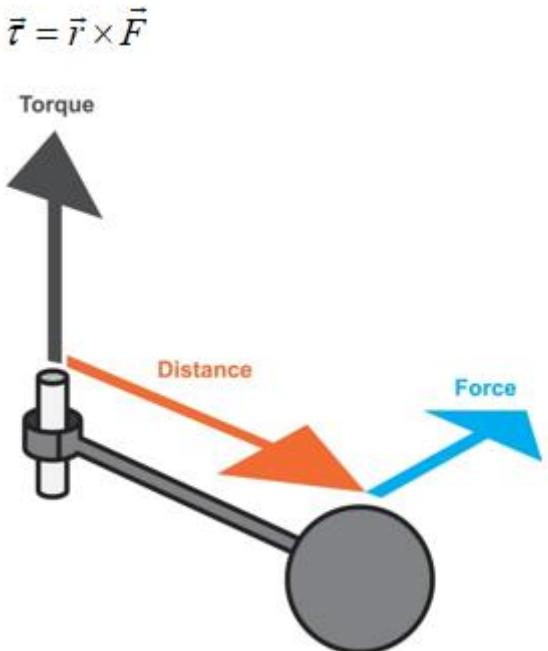
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$$\omega = \frac{d\theta}{dt}$$

, where  $d\theta$  is the angular displacement

### C. Torque or Moment of a Force

The torque or moment of force is the turning effect of the force about the axis of rotation. It is measured as the product of the magnitude of force and the perpendicular distance between the line of action of the force and the axis of rotation.



### D. Angular Momentum a particle

Angular momentum of a particle rotating about an axis is defined as the moment of linear momentum of the particle about that axis. It is measured as product of linear momentum and the perpendicular distance of its line of action from the axis of rotation.

$$\vec{L} = \vec{r} \times \vec{p}$$

### E. Torque and Angular Momentum of a System of Particles

Angular momentum of system of particles is the vector sum of angular momentum of individual particles.



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$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \dots + \vec{L}_n = \sum_i \vec{L}_i$$

The angular momentum of  $i^{th}$  particle is,  $\vec{L}_i = \vec{r}_i \times \vec{p}_i$ , where  $\vec{r}_i$  is the position vector and  $\vec{p}_i$  is the linear momentum of the  $i^{th}$  particle.

$$\vec{L}_i = \sum_i \vec{r}_i \times \vec{p}_i$$

$$\frac{d\vec{L}}{dt} = \sum_i \vec{r}_i \times \frac{d\vec{p}_i}{dt}$$

$$\frac{d\vec{L}}{dt} = \sum_i \vec{r}_i \times \vec{F}_i$$

$$\frac{d\vec{L}}{dt} = \sum_i \vec{\tau}_i$$

where  $\vec{\tau}_i$  is the torque of  $i^{th}$  particle.

## F. Conservation of Angular Momentum and its Applications

If the net external torque on a system is zero, then the total angular momentum of the system remains constant.

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

$$\frac{d\vec{L}}{dt} = 0$$

$$\vec{L} = \text{constant}$$



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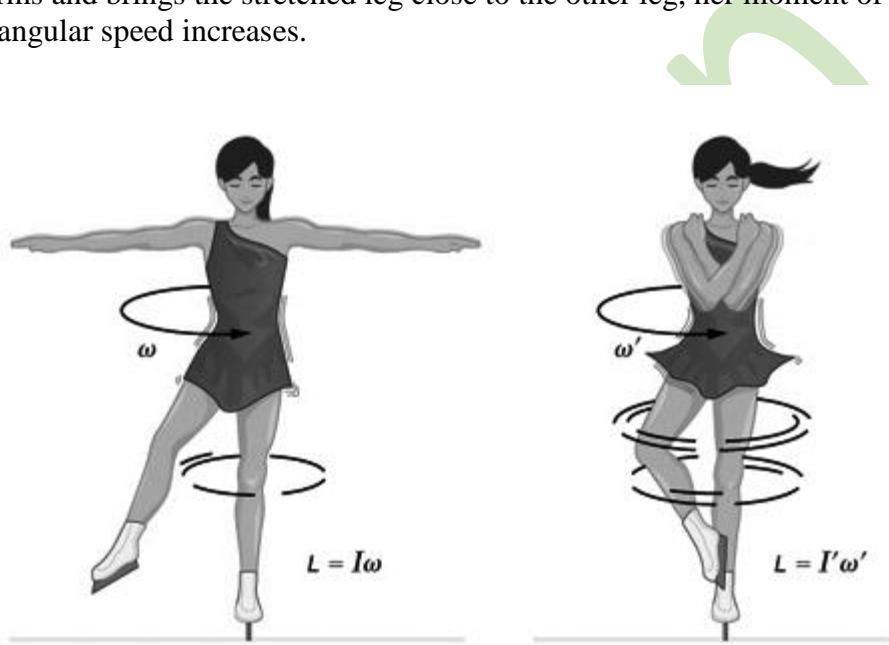
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In absence of external torque angular momentum of the system is,  $\vec{L} = I\omega$ , where  $I$  is the Moment of inertia and  $\omega$  is angular velocity.

### Applications of Angular Momentum

- An Ice skater can increase her angular velocity by folding her arm and bringing the stretched leg close to each other leg. When she stretches her hands and a leg outward, her moment of inertia increases and hence angular speed decreases to conserve angular momentum. When she folds her arms and brings the stretched leg close to the other leg, her moment of inertia decreases and hence angular speed increases.



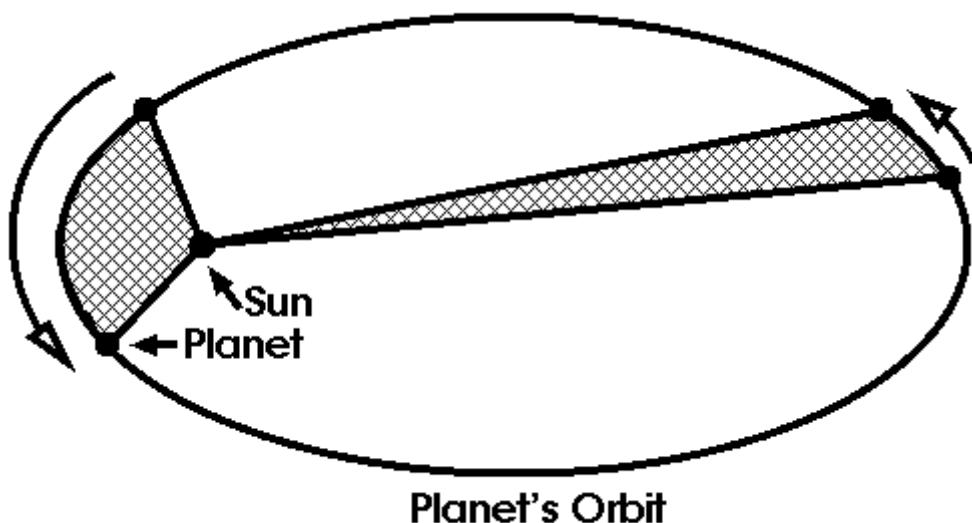
- A planet revolves around the sun in an elliptical path. When it comes near the sun, the moment of inertia of the planet about the sun decreases. In order to conserve the angular momentum, the angular velocity shall increase. Similarly, when the planet is away from the sun, there will be decrease in the angular velocity.



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#### 4. Equilibrium of a Rigid Body

A rigid body is in mechanical equilibrium if,

- It is in translational equilibrium, i.e. the total external force on it is zero.  $\sum \vec{F} = 0$
- It is in rotational equilibrium, i.e. the total external torque on it is zero.  $\sum \vec{\tau} = 0$

#### 5. Moment of Inertia and Radius of Gyration

Moment of inertia of a rigid body about a fixed axis is defined as the sum of the products of the masses of the particles constituting the body and the squares of their respective distances from the axis of rotation.

$$I = M \sum_i R_i^2$$

The moment of inertia of the body depends on the size and shape of the body, mass of the body, distribution of mass about axis of rotation, position and orientation of the axis of rotation with respect to the body.

##### Radius of Gyration

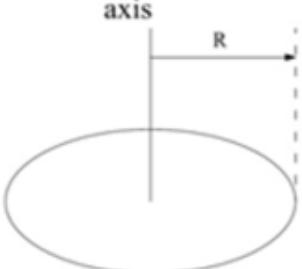
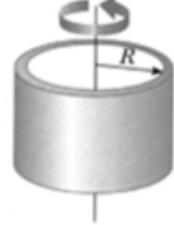
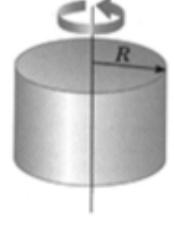
The radius of gyration of a body about an axis of rotation is defined as the root mean square distance of its particles from the axis of rotation.

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$$k = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + r_4^2 + \dots r_n^2}{n}}$$

### Moment of Inertia for Simple Geometrical Objects

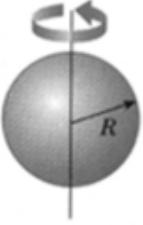
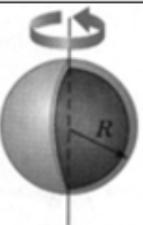
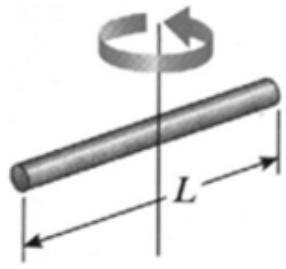
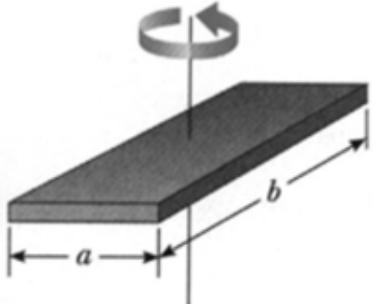
Object Name	Moment of Inertia	Geometrical Object
<b>Circular Ring</b>	$I = MR^2$	 <p>A diagram of a circular ring rotating about its central vertical axis. The ring is shown as an ellipse with a dashed vertical line through its center representing the axis of rotation. A radius vector R is drawn from the center to the outer edge of the ring.</p>
<b>Hollow Cylinder</b>	$I = MR^2$	 <p>A diagram of a hollow cylinder rotating about its central vertical axis. The cylinder is shown with a shaded surface and a dashed vertical line through its center representing the axis of rotation. A radius vector R is drawn from the center to the outer edge of the cylinder.</p>
<b>Solid Cylinder</b>	$I = \frac{MR^2}{2}$	 <p>A diagram of a solid cylinder rotating about its central vertical axis. The cylinder is shown with a shaded surface and a dashed vertical line through its center representing the axis of rotation. A radius vector R is drawn from the center to the outer edge of the cylinder.</p>



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<b>Solid Sphere</b>	$I = \frac{2}{5}MR^2$	
<b>Spherical Shell</b>	$I = \frac{2}{3}MR^2$	
<b>Thin Rod</b>	$I = \frac{Ml^2}{12}$	
<b>Rectangular Plate</b>	$I = \frac{M}{12}(a^2 + b^2)$	

## 6. Parallel and Perpendicular Axis Theorems

### Theorem of Perpendicular Axes

It states that the moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its plane and intersecting each other at the point, where the perpendicular axes pass through the lamina.

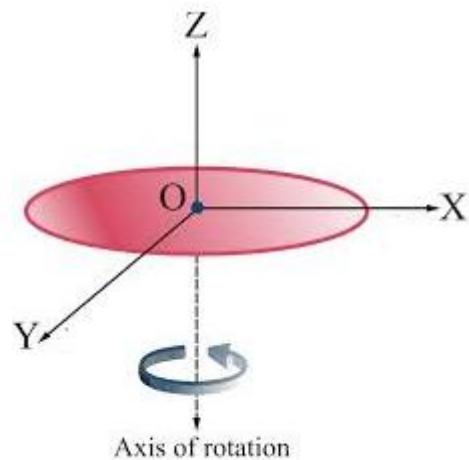


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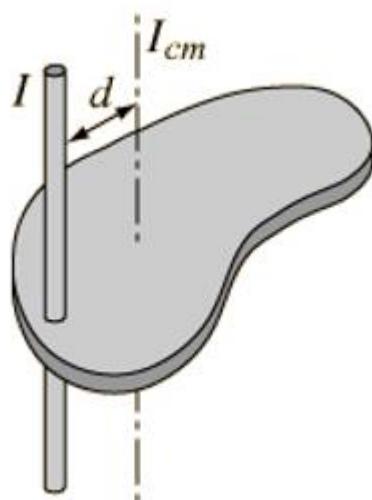
$$I_Z = I_X + I_Y$$



### Theorem of Parallel Axes

It states that the moment of inertia of a rigid body about any axis is equal to the moment of inertia of the body about a parallel axis through its centre of mass plus the product of mass of the body and the square of the perpendicular distance between the parallel axes.

$I = I_{CM} + Md^2$ , where  $I_{CM}$  is the moment of inertia of the body about an axis passing through its centre of mass and  $d$  is the perpendicular distance between the two parallel axes.



### 7. Rigid Body Rotation and its Equations

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In rotation about a fixed axis, every particle of the rigid body moves in a circle which lies in a plane perpendicular to the axis and has its centre on the axis. Every point in the rotating rigid body has the same angular velocity but different linear velocities at any instant of time.

### A. Kinematics and Dynamics of Rotational Motion about a Fixed Axis

#### Equation of Rotational Motion

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 - \omega_0^2 = 2\alpha\theta$$

- The linear velocity of a particle of a rigid body rotating about a fixed axis is given by  $\vec{v} = \vec{\omega} \times \vec{r}$
- Newton's second law of rotational motion states that the angular acceleration during rotational motion of a rigid body is directly proportional to the applied torque and inversely proportional to the moment of Inertia of that body.

$$\alpha \propto \frac{\tau}{I}$$

$$\tau = I\alpha$$

## 8. Rolling Motion

### A. Pure Rolling

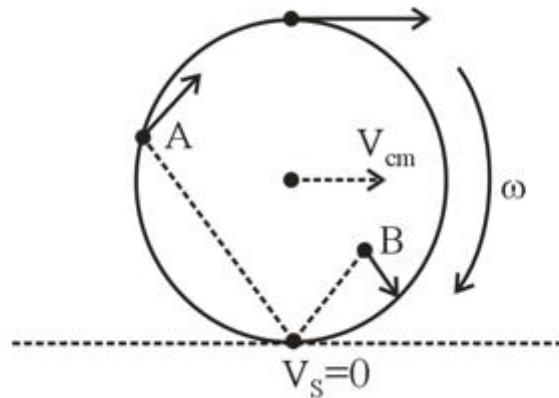
Rolling is a type of motion that combines rotation and translation of that object with respect to a surface. When an object experiences pure translational motion, all of its points move with the same velocity as the center of mass; that is in the same direction and with the same speed. But in pure rolling there is no translation motion body only rotate about a point.



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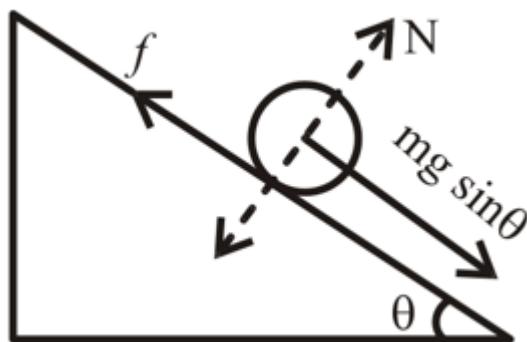
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### Rolling Motion of on fixed incline surface

Let us assume that a body of mass  $m$  and radius  $R$  rolls without slipping (combination of translation and rotational motion) on an incline surface.



$$a = \frac{g \sin \theta}{1 + \frac{k^2}{R^2}}$$

The acceleration of the body is , where  $k$  is the radius of gyration of body

$$f = \frac{mg \sin \theta}{1 + \frac{R^2}{k^2}}$$

Friction force



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$$\mu_{\min} = \frac{\tan \theta}{1 + \frac{R}{k^2}}$$

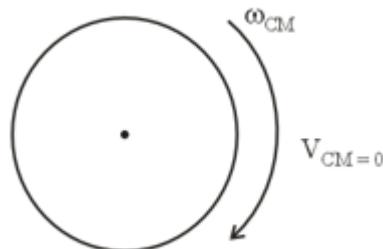
Minimum coefficient of friction is,

### Kinetic Energy of Rolling Motion

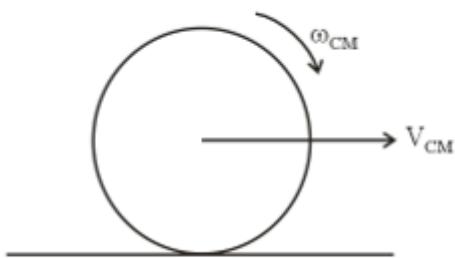
If an object is in pure rotation, then there is no translation motion of the body and the kinetic energy of

$$K = \frac{1}{2} I_{cm} \omega^2$$

the body in rotation is given by



If an object is rolling without slipping (combination of translation and rotation), then its kinetic energy can be expressed as the sum of the translational kinetic energy of its center of mass plus the rotational kinetic energy about the center of mass.



$$KE = \frac{1}{2} I_{cm} \omega^2 + \frac{1}{2} I_{cm} v_{cm}^2$$



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