

JEE Main Maths Short Notes

Quadratic Equations and Polynomial

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Quadratic Equations and Polynomial

1. Quadratic Equation and Roots

A polynomial equation of second degree i.e an equation of the form $ax^2 + bx + c = 0$ where a,b,c are real numbers and $a \neq 0$, is known as a quadratic equation in x.

A quantity a is known as the root of the quadratic equation $ax^2 + bx + c = 0$ if $a\alpha^2 + b\alpha + c = 0$

A quadratic equation cannot have more than 2 roots.

The quantity $b^2 - 4ac$ is called **the discriminant of the quadratic equation** and denoted by D. **The nature of D will determine the nature of the roots of the equation**.

<u>Case 1: D>0</u> The equation $ax^2 + bx + c = 0$ will have two distinct real roots which

are $-b \pm \sqrt{b^2-4ac}$

<u>Case 2: D=0</u> The equation $ax^2 + bx + c = 0$ has two equal real roots which are

 $-_^{b}$ and $-_^{b}$

<u>Case 3: D<0</u>

The equation $ax^2 + bx + c = 0$ will have no real roots. It will have imaginary roots.

<u>Case 4: D is a square of a rational number</u> The equation $ax^2 + bx + c = 0$ will have rational roots

Case 5: D is not a square of a rational number

The equation $ax^2 + bx + c = 0$ will have non - rational roots i.e., irrational roots and they exist in conjugate pairs

Few important points to keep in mind

- If p+iq is a root, then another root will be p-iq $(i=\sqrt{-1})$
- Imaginary roots always occur in pairs for any polynomial with real coefficients
- **Relation between roots and coefficients for Quadratic Equation:** In a quadratic equation $ax^2 + bx + c = 0$

	b	coefficient of x	
The sum of the roots:	$a + \beta = - = -$	$_2$ The product of the roots a + β =	= -
2			
	а	coefficient of x	
	с	constant term	
	а	coefficient of x	

- The quadratic equation can be wrote using sum of roots and product of the roots as follows: $x^2 - (sum of the roots) x + (product of the roots) = 0$
- If the product of roots of a quadratic equation is negative, then the roots are of opposite sign
- If in a quadratic equation $ax^2 + bx + c = 0$, a=1 and b,c are integers and roots are rational, then the roots are integers.



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2. Symmetric Function of Roots

An expression in a, β is called a symmetric function of a and β if the function is not affected by interchanging a and β . A symmetric function of a and β can always be expressed as a function of $a+\beta$ and $a\beta$.

• $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

•
$$\alpha - \beta = \pm \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$$

3. Common Roots

Let $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ be two quadratic equations

Case 1: When one root α is common

 α^2 α 1

 $b_1c_2 - c_1b_2$ $a_2c_1 - a_1c_2$ $(a_1b_2 - a_2b_1)$

Case 2: When both the roots are common

 $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

4. Nature of roots of simultaneous quadratic equations Let

 D_1 and D_2 be the discriminant of two quadratic equations. If

- $D_1 + D_2 \ge 0$, then at least one of the equations must have real roots
- $D_1 + D_2 < 0$, then at least one of the equations must have non-real roots
- $D_1D_2 > 0$, then either both the equations have real and distinct roots, or both the equations have non-real roots
- $D_1D_2 < 0$, then one of the equations has real and distinct roots while the other has non-real roots
- $D_1D_2 = 0$, the n one equation has equal roots. The other equation can have both real or nonreal roots

5. Sign of roots

Let the roots of $ax^2 + bx + c = 0$ be α and β

- If both roots are positive, then a and c must have same sign
- If both roots are negative, a,b,c have the same sign
- If one root is positive while the other is negative then, a and c must have different signs
- If roots are equal in magnitude but opposite in sign then b=0
- If the roots are reciprocal to each other then a is equal to c
- If c=0, then one of the roots must be 0
- If x is replaced by 1/x, then the new roots of the equation will be 1 and 1
 - α β
- If x is replaced by x^2 , then the new roots of the quadratic equation will be α , $-\alpha$, β , $-\beta$

6. Relation between roots and coefficients

Let $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n = 0$, with $a_0 \neq 0$ be a polynomial equation. Let the roots of the equation be $\alpha_1, \alpha_2, \dots, \alpha_n$.



 $a^{\underline{1}}$



	Sum of the roots taken one at a time: $\alpha_1 + \alpha_2 + \cdots + \alpha_n = -a_0$			
	Sum of the roots taken two at a time: $\alpha_1\alpha_2 + \alpha_1\alpha_3 + \cdots + \alpha_1\alpha_n + \alpha_2\alpha_3 + \cdots + \alpha_{n-1}\alpha_n = \alpha_0$	a <u>2</u>		
Sum of the roots taken three at a time: $\alpha_1 \alpha_2 \alpha_3 + \alpha_1 \alpha_2 \alpha_4 + \cdots + \alpha_{n-2} \alpha_{n-1} \alpha_n = - \alpha_0^{a^3}$				
	$(-1)^n a_n$			

Product of the roots: $\alpha_1 \alpha_2 \alpha_3 \dots \alpha_n =$ _____

7. Remainder Theorem, Factor Theorem, Divisibility Theorem

Remainder theorem states that if a polynomial is f(x) is divided by (x-a) where a is independent of x, then the remainder will be f(a).

Factor theorem states that if f(a)=0, then (x-a) will be a factor of f(x)

Divisibility theorem states that if f(a)=0, then f(x) will be divisible by (x-a)

Let f(x) be divided by (x-a)f(x) = (x - a)p(x) + R where R is the remainder Put x = af(a) = RThis proves the remainder theorem.

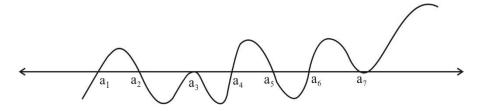
If f(a)=0, then R=0 so (x-a) is a factor of f(x). This proves the factor theorem and divisibility theorem.

8. Inequalities Using Wavy Curve Method

$$f(x) = \frac{((x - a_1)^{n_1}(x - a_2)^{n_2} \dots (x - a_k)^{n_k})}{(x - b_1)^{m_1}(x - b_2)^{m_2} \dots (x - b_p)} > 0 \ (or < 0 \ or \le 0 \ or \ge 0)$$

Here $n_1, n_2, ..., n_k, m_1, m_2, ..., m_p$ are all natural numbers and $a_1, a_2, ..., a_k, b_1, b_2, ..., b_p$ are all real numbers and none of them are equal to each other.

- Function Zero: A point x=a is called a function zero if f(a)=0
- Point of discontinuity: A point x=b, is called a point of discontinuity if f(b) does not exist, that is the denominator becomes 0
- Single Point: Consider $(x a_k)^{nk}$. If n_k is an odd integer then $x = a_k$ is called a single point. The function changes sign on either side of a_k
- Double Point: Consider $(x a_p)^{n_p}$. If n_p is even integer then $x = a_p$ is called a double point. The function has the same sign on either side of a_p



Here a1, a2, a3, a4, a5, a6, a7 are function zeroes

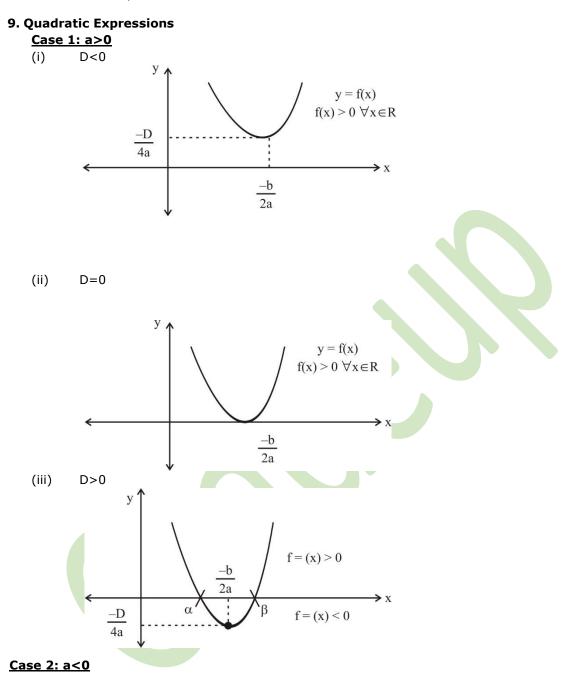
*a*₁, *a*₂, *a*₄, *a*₅, *a*₆are single points because the function changes sign







*a*₃, *a*₇are double points

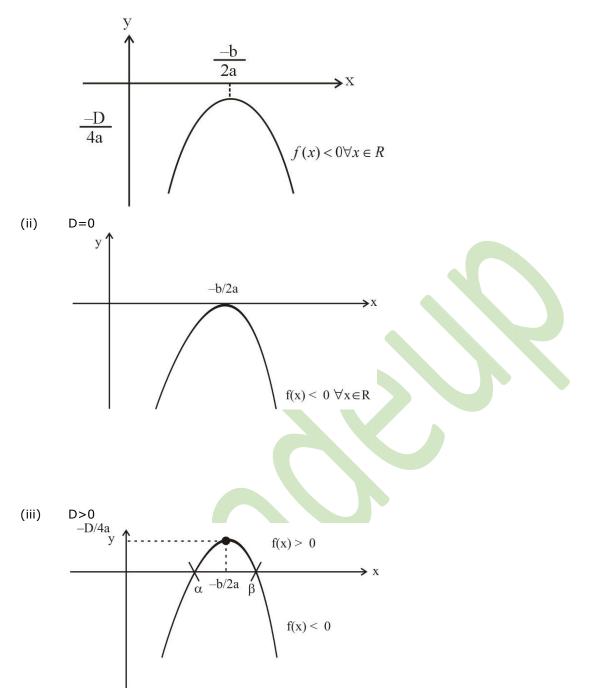


(i) D<0









10. Sign of quadratic expression

Let $f(x) = ax^2 + bx + c$ be a quadratic expression, with a not equal to 0. Let D be the discriminant of the corresponding quadratic equation and α and β be its roots.

- If D<0, the sign of f(x) is same as that of a for all values of x either positive or negative
- If D=0, the sign of f(x) is same as that of a for all values of x
- If D>0, the sign of f(x) is same as that of a for x < α and x > β. The sign of f(x) is opposite to that of a for α < x < β

11. Location of roots • Root lies in (0, p) if and only if -b > 0 and c > 0 when

p>0



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- Root lies in (-p,0) if and only if -b < 0 and c < 0 when p>0
- Both roots are greater than a given number k if the following three conditions are satisfied, $D \ge 0, -\frac{b}{2} > k \text{ and } a. f(k) > 0$
- Both roots are less than a given number k if the following three conditions are satisfied, $D \ge 0, -\frac{b}{2} < k$ and a. f(k) > 0
- Both the roots will lie in the interval (k_1, k_2) if $D \ge 0$, $k_1 < -2 \leq b_a < k_2$, and $a. f(k_1) > 0$ and $a. f(k_2) > 0$
- Exactly one root will lie in (k_1, k_2) if $f(k_1)$. $f(k_2) < 0$
- A given number k will lie in between the roots if a. f(k) < 0

12. Highest and Least Values of a quadratic expression If a<0, then the

highest value of $f(x) = ax^2 + bx + c$ is $-\underline{b}$ and it is obtained at $x = -\underline{b}$

If a>0, then the least value of f(x) is - and it is obtained at x = -

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