# JEE Main Maths Short Notes 

Quadratic Equations and Polynomial

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## Quadratic Equations and Polynomial

## 1. Quadratic Equation and Roots

A polynomial equation of second degree i.e an equation of the form $a x^{2}+b x+c=0$ where $\mathrm{a}, \mathrm{b}, \mathrm{c}$ are real numbers and $a \neq 0$, is known as a quadratic equation in x .

A quantity a is known as the root of the quadratic equation $a x^{2}+b x+c=0$ if $a \alpha^{2}$
$+b \alpha+c=0$
A quadratic equation cannot have more than 2 roots.

The quantity $b^{2}-4 a c$ is called the discriminant of the quadratic equation and denoted by D . The nature of $D$ will determine the nature of the roots of the equation.

Case 1: $\mathrm{D}>0$ The equation $a x^{2}+b x+c=0$ will have two distinct real roots which are $-b \pm \sqrt{ } b 2-4 a c$

Case 2: $\mathrm{D}=0$ The equation $a x^{2}+b x+c=0$ has two equal real roots which are

-     - $^{b}$ and - _-

Case 3: $\mathrm{D}<0$
The equation $a x^{2}+b x+c=0$ will have no real roots. It will have imaginary roots.

## Case 4: D is a square of a rational number

The equation $a x^{2}+b x+c=0$ will have rational roots
Case 5: D is not a square of a rational number
The equation $a x^{2}+b x+c=0$ will have non - rational roots i.e., irrational roots and they exist in conjugate pairs

## Few important points to keep in mind

- If $\mathrm{p}+\mathrm{iq}$ is a root, then another root will be $\mathrm{p}-\mathrm{iq}(\mathrm{i}=\sqrt{-1})$
- Imaginary roots always occur in pairs for any polynomial with real coefficients
- Relation between roots and coefficients for Quadratic Equation: In a quadratic equation $a x^{2}+b x+c=0$
b coefficient of $x$
The sum of the roots: $\alpha+\beta=-=-$
${ }_{2}$ The product of the roots $\mathbf{a}+\boldsymbol{\beta}==-$

| a | $\begin{array}{c}\text { coefficient of } x \\ c\end{array}$ |
| :---: | :---: |
| $a$ | constant term |

- The quadratic equation can be wrote using sum of roots and product of the roots as follows: $x^{2}-$ (sum of the roots) $x+$ (product of the roots) $=0$
- If the product of roots of a quadratic equation is negative, then the roots are of opposite sign
- If in a quadratic equation $a x^{2}+b x+c=0, \mathrm{a}=1$ and $\mathrm{b}, \mathrm{c}$ are integers and roots are rational, then the roots are integers.


## 2. Symmetric Function of Roots

An expression in $a, \beta$ is called a symmetric function of $a$ and $\beta$ if the function is not affected by interchanging $a$ and $\beta$. A symmetric function of $a$ and $\beta$ can always be expressed as a function of $a+\beta$ and $a \beta$.

$$
\begin{array}{ll}
\text { - } & \alpha^{2}+\beta^{2}=(\alpha+\beta)^{2}-2 \alpha \beta \\
\text { - } & \alpha-\beta= \pm \sqrt{(\alpha+\beta)^{2}-4 \alpha \beta}
\end{array}
$$

## 3. Common Roots

Let $a_{1} x^{2}+b_{1} x+c_{1}=0$ and $a_{2} x^{2}+b_{2} x+c_{2}=0$ be two quadratic equations
Case 1: When one root $\alpha$ is common
$\frac{\alpha^{2}}{b_{1} c 2-c_{1} b_{2}} \frac{\alpha}{a_{2 c 1}-a_{1 c 2}} \frac{1}{\left(a_{1} b_{2}-a_{2} b_{1}\right)}=$

Case 2: When both the roots are common
$\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}}$
4. Nature of roots of simultaneous quadratic equations Let
$D_{1}$ and $D_{2}$ be the discriminant of two quadratic equations. If

- $D_{1}+D_{2} \geq 0$, then at least one of the equations must have real roots
- $\quad D_{1}+D_{2}<0$, then at least one of the equations must have non-real roots
- $D_{1} D_{2}>0$, then either both the equations have real and distinct roots, or both the equations have non-real roots
- $D_{1} D_{2}<0$, then one of the equations has real and distinct roots while the other has non-real roots
- $D_{1} D_{2}=0$, the $n$ one equation has equal roots. The other equation can have both real or nonreal roots

5. Sign of roots

Let the roots of $a x^{2}+b x+c=0$ be $\alpha$ and $\beta$

- If both roots are positive, then a and c must have same sign
- If both roots are negative, $a, b, c$ have the same sign
- If one root is positive while the other is negative then, a and c must have different signs
- If roots are equal in magnitude but opposite in sign then $b=0$
- If the roots are reciprocal to each other then a is equal to $c$
- If $c=0$, then one of the roots must be 0
- If $x$ is replaced by $1 / x$, then the new roots of the equation will be ${ }^{1}$ and ${ }^{1}$
- If $\mathbf{x}$ is replaced by $x^{2}$, then the new roots of the quadratic equation will be $\alpha,-\alpha, \beta,-\beta$


## 6. Relation between roots and coefficients

Let $a_{0} x^{n}+a_{1} x^{n-1}+a_{2} x^{n-2}+\cdots+a_{n-1} x+a_{n}=0$, with $a_{0} \neq 0$ be a polynomial equation. Let the roots of the equation be $\alpha_{1}, \alpha_{2}, \ldots . \alpha_{n}$.

Sum of the roots taken one at a time: $\alpha_{1}+\alpha_{2}+\cdots \alpha_{n}={ }_{a 0}$
Sum of the roots taken two at a time: $\alpha_{1} \alpha_{2}+\alpha_{1} \alpha_{3}+\cdots+\alpha_{1} \alpha_{n}+\alpha_{2} \alpha_{3}+\cdots+\alpha_{n-1} \alpha_{n}={ }_{a 0}$

$$
a^{3}
$$

Sum of the roots taken three at a time: $\alpha_{1} \alpha_{2} \alpha_{3}+\alpha_{1} \alpha_{2} \alpha_{4}+\cdots \alpha_{n-2} \alpha_{n-1} \alpha_{n}=-{ }_{a 0}$
$\square$
Product of the roots: $\alpha_{1} \alpha_{2} \alpha_{3} \ldots . \alpha_{n}=$ $\qquad$

## 7. Remainder Theorem, Factor Theorem, Divisibility Theorem

Remainder theorem states that if a polynomial is $f(x)$ is divided by $(x-a)$ where a is independent of $x$, then the remainder will be $f(a)$.

Factor theorem states that if $f(a)=0$, then ( $x-a$ ) will be a factor of $f(x)$
Divisibility theorem states that if $f(a)=0$, then $f(x)$ will be divisible by ( $x-a$ )

Let $\mathrm{f}(\mathrm{x})$ be divided by ( $\mathrm{x}-\mathrm{a}$ )
$f(x)=(x-a) p(x)+R$ where R is the remainder
Put $x=a$
$f(a)=R$
This proves the remainder theorem.
If $f(a)=0$, then $R=0$ so $(x-a)$ is a factor of $f(x)$. This proves the factor theorem and divisibility theorem.

## 8. Inequalities Using Wavy Curve Method

$$
f^{(x)}=\frac{\left(\left(x-a_{1}\right)^{n_{1}}\left(x-a_{2}\right)^{n_{2}} \ldots . .\left(x-a_{k}\right)^{n_{k}}\right)}{\left(x-b_{1}\right)^{m_{1}}\left(x-b_{2}\right)^{m_{2}} \ldots .\left(x-b_{p}\right)^{m_{p}}}>0(\text { or }<0 \text { or } \leq 0 \text { or } \geq 0)
$$

Here $n_{1}, n_{2}, \ldots n_{k}, m_{1}, m_{2}, \ldots m_{p}$ are all natural numbers and $a_{1}, a_{2}, ., a_{k}, b_{1}, b_{2}, \ldots b_{p}$ are all real numbers and none of them are equal to each other.

- Function Zero: A point $x=a$ is called a function zero if $f(a)=0$
- Point of discontinuity: A point $x=b$, is called a point of discontinuity if $f(b)$ does not exist, that is the denominator becomes 0
- Single Point: Consider $\left(x-a_{k}\right)^{n k}$. If $n_{k}$ is an odd integer then $x=a_{k}$ is called a single point. The function changes sign on either side of $a_{k}$
- Double Point: Consider $\left(x-a_{p}\right)^{n_{p}}$. If $n_{p}$ is even integer then $x=a_{p}$ is called a double point. The function has the same sign on either side of $a_{p}$


Here $a_{1}, a_{2}, a_{3}, a_{4}, a_{5}, a_{6}, a_{7}$ are function zeroes
$a_{1}, a_{2}, a_{4}, a_{5}, a_{6}$ are single points because the function changes sign
No. 1 site \& app
$a_{3}, a_{7}$ are double points
9. Quadratic Expressions

## Case 1: $a>0$

(i) $\quad \mathrm{D}<0$

(ii) $\mathrm{D}=0$

(iii) $D>0$


Case 2: $\mathbf{a}<0$
(i) $\mathrm{D}<0$

(ii)

(iii) $D>0$

10. Sign of quadratic expression

Let $f(x)=a x^{2}+b x+c$ be a quadratic expression, with a not equal to 0 . Let D be the discriminant of the corresponding quadratic equation and $\alpha$ and $\beta$ be its roots.

- If $D<0$, the sign of $f(x)$ is same as that of a for all values of $x$ - either positive or negative
- If $D=0$, the sign of $f(x)$ is same as that of a for all values of $x$
- If $\mathrm{D}>0$, the sign of $\mathrm{f}(\mathrm{x})$ is same as that of a for $x<\alpha$ and $x>\beta$. The sign of $\mathrm{f}(\mathrm{x})$ is opposite to that of a for $\alpha<x<\beta$

11. Location of roots - Root lies in ( $0, \mathrm{p}$ ) if and only if $-b>0$ and $c>0$ when $p>0$

- Root lies in $(-p, 0)$ if and only if $-b<0$ and $c<0$ when $p>0$
- Both roots are greater than a given number $k$ if the following three conditions are satisfied,

$$
D \geq 0,-{ }_{\overline{2 a}}^{b}>k \text { and } a . f(k)>0
$$

- Both roots are less than a given number k if the following three conditions are satisfied, $D \geq$

$$
0,-{ }_{2 a}^{\underline{h}}<k \text { and } a . f(k)>0
$$

- Both the roots will lie in the interval ( $k_{1}, k_{2}$ ) if $D \geq 0, k_{1}<-{ }_{2}{ }^{b_{a}}<k_{2}$, and $a . f\left(k_{1}\right)>0$ and $a . f\left(k_{2}\right)>0$
- Exactly one root will lie in ( $k_{1}, k_{2}$ ) if $f\left(k_{1}\right) . f\left(k_{2}\right)<0$
- A given number k will lie in between the roots if $a . f(k)<0$

12. Highest and Least Values of a quadratic expression If $a<0$, then the highest value of $f(x)=a x^{2}+b x+c$ is $-ـ^{D}$ and it is obtained at $x=-ـ^{b}$

If $\mathrm{a}>0$, then the least value of $\mathrm{f}(\mathrm{x})$ is $-\frac{\square}{4 a}$ and it is obtained at $x=-\frac{-}{2 a}$

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