JEE Main Maths

Short Notes

Sequence and Series

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SEQUENCE AND SERIES

1. Arithmetic Progression

Arithmetic Progression (AP) is defined as a series in which difference between any two consecutive terms is constant throughout the series. This constant difference is called the common difference. If a is the first term and d is the common difference then an AP can be written as a, (a +d), (a+2d) ...

The nth term of an AP is given by T_n= a + (n-1) d
And the sum up to n terms is given by S_n = (n/2) [2a + (n-1) d] =(n/2) [a + l] where l is the last or the nth term of the A.P.

Properties of A.P. Series:

- If $a_1, a_2 \ldots, a_n$ are in AP with common difference d, then $a_1 + k$, $a_2 + k$, ..., $a_n + k$ will also be in AP with same common difference.
- If a_1 , a_2,........, a_n are in AP with common difference d, then $a_1 - k$, $a_2 - k$, ..., $a_n - k$ will also be in AP with same common difference.
- If a_1 , a_2,........, a_n are in AP with common difference d, then ka_1 , ka_2,........, ka_n will also be in AP with same common difference kd.
- If a_1 , a_2,........, a_n are in AP with common difference d, then (a_1 / k), (a_2 / k),........,
- (a_n / k),........, will also be in AP with same common difference d/k
- If a_1 , a_2,........, a_n are in AP with common difference d_1 and b_1 , b_2,........, b_n are in AP with common difference d_2 then $a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3, \ldots . . a_n \pm b_n$ is in AP with common difference $d_1 \pm d_2$
- In a finite AP, the sum of terms equidistant from the beginning and end is always same i.e. $a_1 + a_n = a_2 + a_{n-1}$ = ...
- Three numbers a, b, c is in AP implies 2b = a + c
- If a, A_1, A_2 ...., A_n , b are in AP, then A_1 ,A_2 ...., A_n are called n Arithmetic Means between a and b.

Selection of terms in AP:

- 3 terms: a - d, a, a + d
- 4 terms: a-3d, a-d, a + d, a+3d
- 5 terms: a-2d, a-d, a, a + d, a+2d
- 6 terms: a-5d, a-3d, a-d, a + d, a+3d, a+5d

Sum of first n natural numbers: 1+2+3+…n = [(n(n+1)) /2]

Sum of squares of 1st n natural numbers: $1^2 + 2^2 + 3^2 + \ldots\ldots n^2$ = [(n(n+1) (2n+1)) /6]

Sum of cubes of 1st n natural numbers: $1^3 + 2^3 + 3^3 + \ldots\ldots n^3$ = $[(n(n+1)) /2]^2$

2. Geometric Progression

GP is defined as a series in which ration between any two consecutive terms is constant throughout the series. This constant is called the common ratio. If a is the first term and r is the common ratio, then a GP can be written as a, ar, ar^2 , ar^3 ,………ar^n

The nth term is given by $T_n = ar^{(n-1)}$

Sum up to n terms is given by $S_n = \frac{a(1-r^n)}{1-r} \quad if\ r \neq 1$

$$= na \quad if\ r = 1$$

Properties of GP:

- If $a_1, a_2......, a_n$ are in GP with common ratio r then ka_1 , ka_2,…….., ka_n will also be in G.P. with the same common ratio provided k is non-zero
- If $a_1, a_2......, a_n$ are in GP with common ratio r then (a_1 / k), (a_2 / k)…..,(a_n / k) will also be in GP with the same common ratio provided k is non-zero
- If $a_1, a_2......, a_n$ are in GP then a_1^k, a_2^k……, a_n^k will also be in GP with common ratio r^k
- If $a_1, a_2......, a_n$ be in GP with all terms positive and common ration r then log a_1, log a_2, ……., log a_n will be in AP with common difference log r .
- Product of two individual GP's will have common ration as the product of two common ratios
- In a finite GP, the product of the terms quidistant from the beginning and the end is always the same i.e. $a_1a_n = a_2a_{n-1}$ = ………
- Three numbers a, b, c are in GP if $b^2 = ac$
- If a, G_1, G_2 ……….., G_n , b are in GP, then G_1 ,G_2 ………., G_n are called the n geometric means between a and b

Selection of terms in GP:

- 3 terms: (a / r), a, ar
- 4 terms: (a / r^3), (a / r) , ar, ar^3
- 5 terms: (a / r^2), (a / r) ,a, ar, ar^2
- 6 terms: (a / r^5), (a / r^3) , a / r, ar, ar^3, ar^5

Infinite GP Series:

If number of terms of a GP is very large i.e. n tends to infinite, then such a GP series is known as infinite GP series. An infinite GP will have finite sum if and only if |r|<1

Sum of infinite series = a / (1 - r)

3. Arithmetic-Geometric Progression

If each term of a progression is the product of the corresponding terms of an AP and a GP, then it is called a AGP.
i.e. a, (a + d)r, (a + 2d)r^2 , (a + 3d) ar^3 ,………(a + (n-1)d) r^{n-1}

The general term is T_n= (a + (n-1) d) $r^{(n-1)}$
To find the sum of n terms we suppose its sum is ,S_n .Multiply both sides by the common ratio of GP.

S_n = a + (a + d)r + (a + 2d)r^2 + (a + 3d) ar^3 + ……..+ (a + (n-1)d)r^{n-1}
rS_n = ar + (a + d)r^2 + (a + 2d)r^3 + (a + 3d) ar^4 + ……..+ (a + (n-1)d)r^n

So,
S_n (1 – r) = a + rd + r^2d + …… + [a + (n - 1) d]r^n
S_n = (a/(1-r)) + [rd (1 – r^{n-1}) / (1 – r)^2] - [a + (n - 1) d]r^n / (1 – r)

4. Harmonic Progression

If a_1 , a_2,…….., a_n are in AP such that none of them is 0, then (1/a_1), (1/a_2), …….., (1/a_n) are said to be in HP

- If a, b, c are in HP, then (1/a), (1/b), (1/c) are in AP
- If a, H_1, H_2 ...., H_n, b are in HP, then H_1 ,H_2 ...., H_n are called n harmonic means between a and b.

5. Inequalities

Arithmetic mean(A) = $\frac{x_1+x_2+\cdots x_n}{n}$

Geometric mean(G)= $(x_1 x_2 \ldots x_n)^{\frac{1}{n}}$

Harmonic mean(H)=$\frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\ldots\frac{1}{x_n}}$

$$A \geq G \geq H$$

Equality holds when $x_1 = x_2 = \cdots = x_n$

Weighted arithmetic mean(A*) = $\frac{m_1x_1+m_2x_2+\cdots m_nx_n}{m_1+m_2+\cdots m_n}$

Weighted geometric mean(G*) = $\left(x_1^{m_1} x_2^{m_2} \ldots x_n^{m_n}\right)^{\frac{1}{m_1 m_2 \ldots m_n}}$

Weighted harmonic mean(H*) = $\frac{m_1+m_2+m_3+\cdots m_n}{\frac{m_1}{x_1}+\frac{m_2}{x_2}+\cdots+\frac{m_n}{x_n}}$

$$A^* \geq G^* \geq H^*$$

Equality holds when all the entries are equal

Let $x_1, x_2, \ldots x_n$ be n positive real numbers and let m be a real number. Then

$\frac{x_1^m+x_2^m+..x_n^m}{n} \geq \left[\frac{x_1+x_2+\cdots x_n}{n}\right]$if m belongs to all real numbers outside the range [0,1]

$\frac{x_1^m+x_2^m+..x_n^m}{n} \leq \left[\frac{x_1+x_2+\cdots x_n}{n}\right]$ if m lies in between 0 and 1

$\frac{x_1^m+x_2^m+..x_n^m}{n} = \left[\frac{x_1+x_2+\cdots x_n}{n}\right]$ if m is equal to 0 or 1

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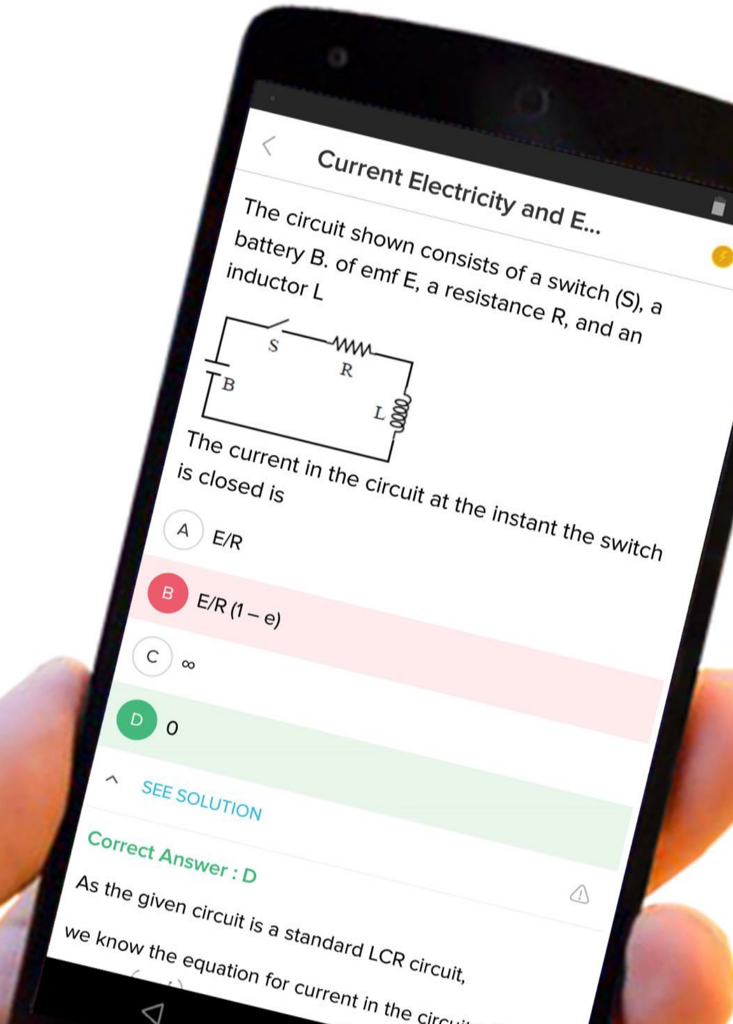

Current Electricity and E...
The circuit shown consists of a switch (S), a battery B. of emf E, a resistance R, and an inductor L
S
R
B
L
The current in the circuit at the instant the switch is closed is
A E/R
B E/R (1 – e)
C ∞
D 0
SEE SOLUTION
Correct Answer : D
As the given circuit is a standard LCR circuit,