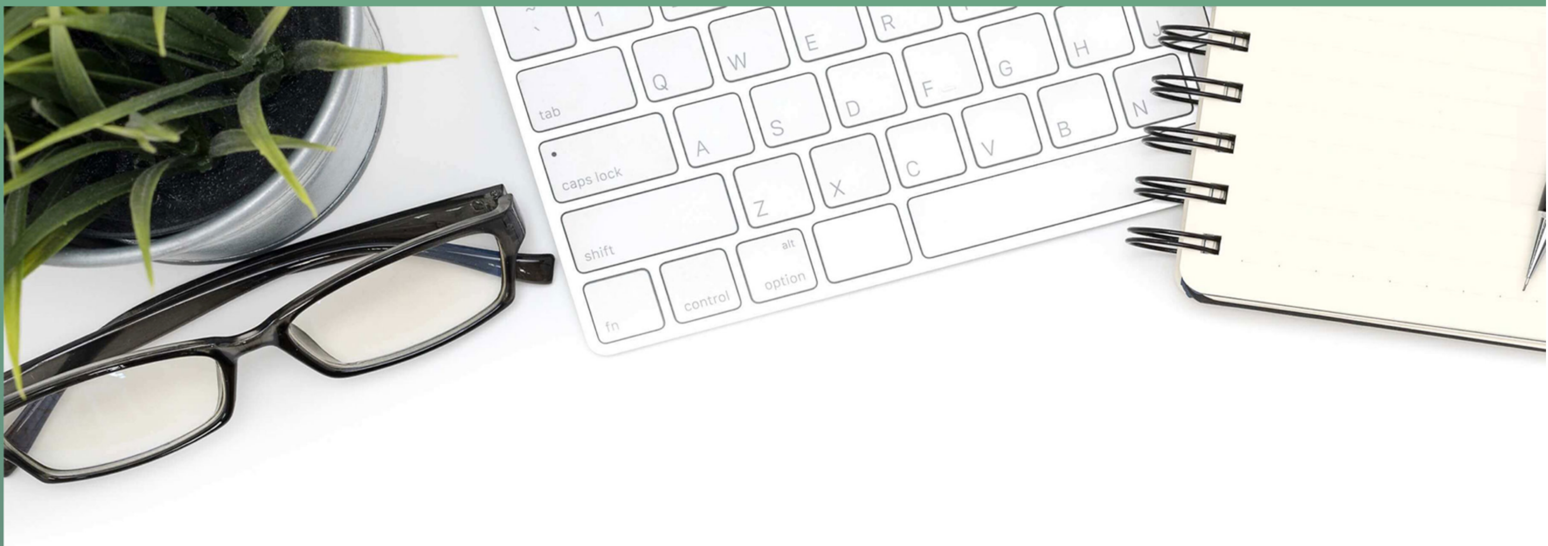

JEE Main Maths

Short Notes

Complex Numbers

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Complex Number and Quadratic Equation is an important topic from the JEE Main exam point of view. Every year 2-3 questions are asked. Further the concept of complex numbers, iota, quadratic equation and other included topics are used very often in different topics of JEE Main Syllabus. This short notes on Complex Number and Quadratic equation will help you in revising the topic before the JEE Main/JEE Advanced Exam.

Complex Numbers

1.Introduction to complex numbers

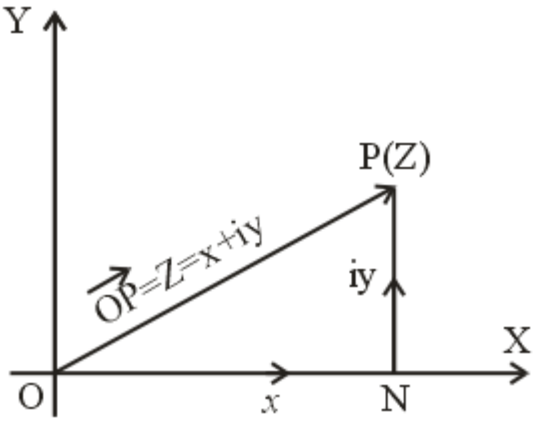
A number of the form x+ iy where x and y belong to the set of real numbers and $i=\sqrt{-1}$ is called a complex number. Here i(iota) is defined as the Fundamental Imaginary Unit.

Iota is nothing but the root of the equation $x^2+1=0$. The role of iota in a complex number is to keep the real part and imaginary part separate. Here x is called the real part of a complex number z and denoted by Re(z) while the imaginary part y is denoted by Im(z).

Therefore z=x+ iy=Re(z)+i Im(z)

A complex is often defined as an ordered pair of real numbers x and y, and is denoted by (x, y).

A complex number x +iy may also be defined as a 2-dimensional vector in xy plane with point of initiation as origin and point of termination as (x,y). In such a case the unit vector along positive x axis is 1 while that along positive y axis is i. When complex numbers are plotted as vectors on such a plane they must follow all the properties of vector. Such a plane is called the Argand Plane



- Two complex numbers z_1 and z_2 are said to be equal if and only if their real and imaginary parts are equal separately.
- Two unequal complex numbers do not possess order property i.e $x_1+iy_1>x_2+iy_2$ or $x_1+iy_1<x_2+iy_2$ does not make any sense.
- A real number can be a complex number with imaginary part 0
- A complex number z is said to be purely real number if Im(z)=0 and it lies on the x axis while it is said to be a purely imaginary number if Re(z)=0 and it lies on the y axis.

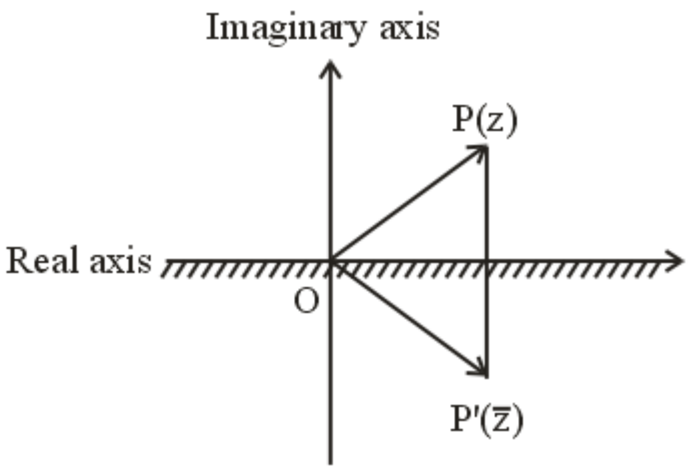
2. Algebra of complex numbers

- Addition: $(x_1+iy_1)+(x_2+iy_2)=(x_1+x_2)+(iy_1+iy_2)$
- Subtraction: $(x_1+iy_1)-(x_2+iy_2)=(x_1-x_2)+(iy_1-iy_2)$
- Multiplication: $(x_1+iy_1).(x_2+iy_2)=(x_1x_2-y_1y_2).i(x_1y_2-x_2y_1)$
- Division: $\frac{x_1+iy_1}{x_2+iy_2}=\frac{(x_1+iy_1)(x_2-iy_2)}{(x_2+iy_2)(x_2-iy_2)}=\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}+i.\frac{(x_2y_1-x_1y_2)}{x_2^2+y_2^2}$
- Equality: If $(x_1+iy_1)=(x_2+iy_2)$, then $x_1=x_2$ and $y_1=y_2$

- $i^2 = -1\,, i^3 = -i\,, i^4 = 1, i^5 = i \ldots .etc$

3. Conjugate of a complex number

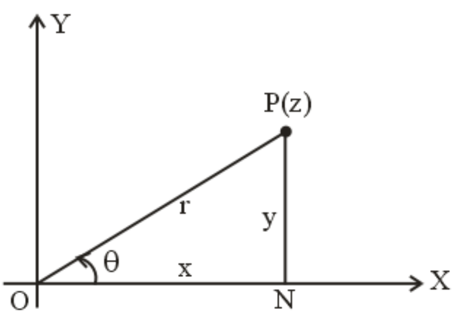
The mirror image of the complex number z=x+iy on the real axis i.e z'=x-iy is called the conjugate of the complex number z. Conversely z is the conjugate of z'.



Properties of conjugate

- $(z')' = z$
- $(z_1 + z_2)' = z_1' + z_2'$
- $(z_1 - z_2)' = z_1' - z_2'$
- $(z_1 z_2)' = z_1' z_2'$
- $\left(\frac{z_1}{z_2}\right)' = \frac{z_1'}{z_2'}$
- $z_1 z_2' + z_1' z_2 = 2Re(z_1' z_2) = 2Re(z_1 z_2')$
- $(z^n)' = (z')^n$
- $If\ z = f(z_1) then\ z' = f(z_1')$
- $Re(z) = \frac{z+z'}{2}$
- $Im(z) = \frac{z-z'}{2i}$

4. Modulus and argument of complex number



The distance r of the point P from origin is called the modulus of the complex number z and is denoted by |z|. $r = |z| = \sqrt{x^2 + y^2}$. If θ is the angle made by the vector OP with the positive real axis then it is called the argument of the complex number and is denoted by arg(z) where $-\pi < \theta \le \pi$.

Hence $x + iy = (r\cos\theta) + i(r\sin\theta) = r(\cos\theta + i.\sin\theta) = re^{i\theta} = |z|e^{i.\arg(z)}$

5. Representation of a complex number

Cartesian Representation

Any complex number z can be represented in terms of the Cartesian coordinates (x,y) as z=x+iy where x,y belong to the set of real numbers.

Polar Representation

Any complex number z can be represented by its modulus r and argument θ as

$$z = (r\cos\theta) + i(r\sin\theta)$$

Euler Representation

Any complex number z can be represented by its modulus r and argument θ as

$$z = re^{i\theta}$$

Properties of modulus

- $|z| \geq 0$
- $zz' = |z|^2$
- $|z_1 z_2| = |z_1||z_2|$
- $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$
- $|z_1 + z_2| \leq |z_1| + |z_2|$ This inequality is also called the triangular inequality of complex numbers
- $|z_1 - z_2| \geq ||z_1| - |z_2||$
- $|z^n| = |z|^n$
- $z_1 z_2' + z_1' z_2 = 2|z_1||z_2|\cos(\theta_1 - \theta_2)$ *where* $\theta_1 = \arg(z_1)$ *and* $\theta_2 = \arg(z_2)$

Properties of argument

- $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$
- $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
- $z = re^{i\theta}$ *implies* $z' = re^{-i\theta}$

Argument of complex numbers in various quadrants

Quadrant	Sign of x and y	Argument
I	x>0,y>0	$\tan^{-1}\left(\frac{y}{x}\right)$
II	x<0,y>0	$\pi + \tan^{-1}\left(\frac{y}{x}\right)$
III	x<0,y<0	$-\pi + \tan^{-1}\left(\frac{y}{x}\right)$
IV	x>0,y<0	$\tan^{-1}\left(\frac{y}{x}\right)$

For purely real numbers $\arg(z) = 0$ *or* π
For purely imaginary numbers $\arg(z) = \frac{\pi}{2}$ *or* $-\frac{\pi}{2}$

6. Cube roots of unity

$\sqrt[3]{x} = 1$

$x^3 = 1$
$(x-1)(x^2+x+1) = 0$
$either\ x-1=0\ or\ x=1$
$or\ x^2+x+1=0$
$x = \dfrac{-1 \pm i\sqrt{3}}{2}$
$There\ are\ three\ roots\ of\ the\ equation, one\ real\ and\ two\ complex$

The complex roots are represented by w and w^2. It does not matter which of the complex number is represented by w. The other one will be its square. Thus $1, w, w^2$ are the three roots of unity.

Properties of cube roots of unity

- $1 + w + w^2 = 0$
- $w^3 = 1 \ \ but\ w \neq 1$
- $w^{3n} = 1, w^{3n+1} = w\ , w^{3n+2} = w^2 \ \ where\ n\ is\ an\ integer$
- $w' = w^2 \ \ and\ (w^2)' = w$

Important Factorisations

- $x^2 + y^2 = (x+iy)(x-iy)$
- $x^2 + x + 1 = (x-w)(x-w^2)$
- $x^2 - x + 1 = (x+w)(x+w^2)$
- $x^2 + xy + y^2 = (x-yw)(x-yw^2)$
- $x^2 - xy + y^2 = (x+yw)(x+yw^2)$
- $x^3 + y^3 = (x+y)(x+yw)(x+yw^2)$
- $x^3 - y^3 = (x-y)(x-y^2)(x-y^2)$
- $x^2 + y^2 + z^2 - xy - yz - zx = (x+yw+zw^2)(x+yw^2+zw)$
- $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x+yw+zw^2)(x+yw^2+zw)$

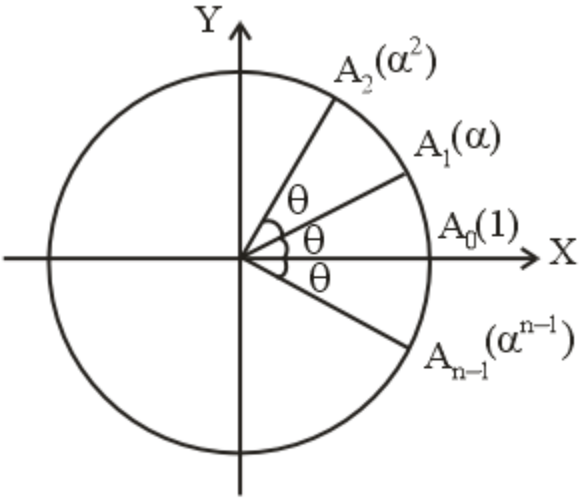
7. n^{th} roots of unity

The equation $x^n = 1$ has n roots and are called the n^{th} roots of unity
$x^n = 1 = \cos 0 + i.\sin 0$
$= \cos 2k\pi + i.\sin 2k\pi$,
Where k is an integer
$x = (\cos 2k\pi + i.\sin 2k\pi)^{\frac{1}{n}}$
$x = \cos\frac{2k\pi}{n} + i.\sin\frac{2k\pi}{n}$ where k=0,1,2,3....n-1

Let $\alpha = \cos\frac{2k\pi}{n} + i.\sin\frac{2k\pi}{n}$. Then every n^{th} root is of the form α^k where k=0,1,2,3...n-1 or k=1,2,3...n i.e, the n^{th} roots of unity are 1, α, α^2, α^3,... α^{n-1} which are in G.P

From theory of equations, we can say that **sum of n^{th} roots of unity is 0**
Product of n^{th} roots of unity=$(-1)^{n+1}$

These roots are located at the vertices of an n sided regular polygon inscribed in a unit circle having centre at origin and one vertex on positive real axis.

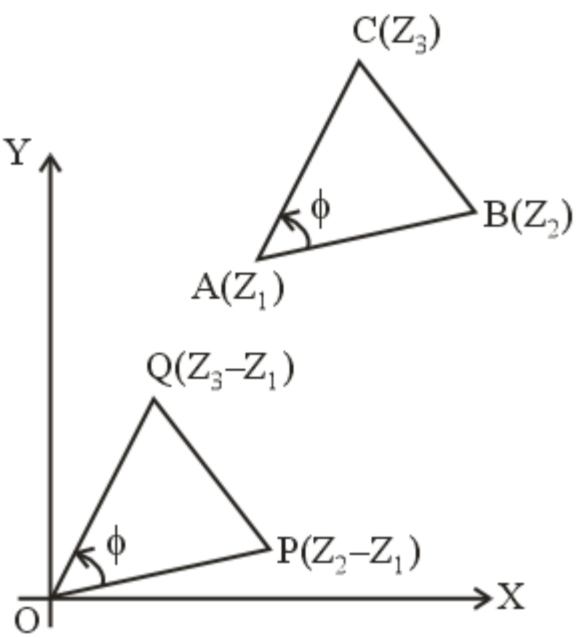


Application in trigonometric series summation

Since sum of roots=0

- $\sum_{k=1}^{n} \alpha^k = 0$
- $\sum_{k=1}^{n} \cos\frac{2k\pi}{n} + \sum_{k=1}^{n} \sin\frac{2k\pi}{n} = 0$
- $\sum_{k=1}^{n} \cos\frac{2k\pi}{n} = 0$
- $\sum_{k=1}^{n} \sin\frac{2k\pi}{n} = 0$

8. Rotation of complex numbers



If z_1, z_2, z_3 be the complex numbers of the vertices of a triangle ABC described in counter clock wise sense then,

$\frac{(z_3 - z_1)}{z_2 - z_1} = \frac{OQ}{OP}(\cos ø + i.\sin ø)$
$= \frac{CA}{BA} e^{iø}$
$= \left|\frac{z_3-z_1}{z_2-z_1}\right| e^{iø}$
So, $\arg\left(\frac{z_3-z_1}{z_2-z_1}\right) = ø$

9. Complex number geometry

Section formula:
The complex number of a point P which divides the line segment joint the point $A(z_1)$ and $B(z_2)$ internally in the ration m: n is $\frac{mz_1+nz_2}{m+n}$

This is similar to the section formula in Cartesian geometry

Thus, the complex number of the midpoint M of a line joint z_1 and z_2 is $\frac{z_1+z_2}{2}$

Complex number of centroids of a triangle with vertices z_1, z_2 and z_3 is $\frac{z_1+z_2+z_3}{3}$

Equation of straight lines
The equation of straight line passing through z_1 and z_2 is given by

$$\begin{vmatrix} z & z' & 1 \\ z_1 & z_1' & 1 \\ z_2 & z_2' & 1 \end{vmatrix} = 0$$

The general equation of a straight line is given by
$a'z + az' + b = 0$ *where b is a purely real number*

Equation of circles
The equation of circle with centre at origin and radius a is given by $|z| = a$

The equation of a circle with centre at z_0 and radius a is given by $|z - z_0| = a$

The general equation of a circle is given by $zz' + az' + a'z + b = 0$ where b is a purely real number. The centre of this circle is at -a and its radius is $\sqrt{aa' - b}$

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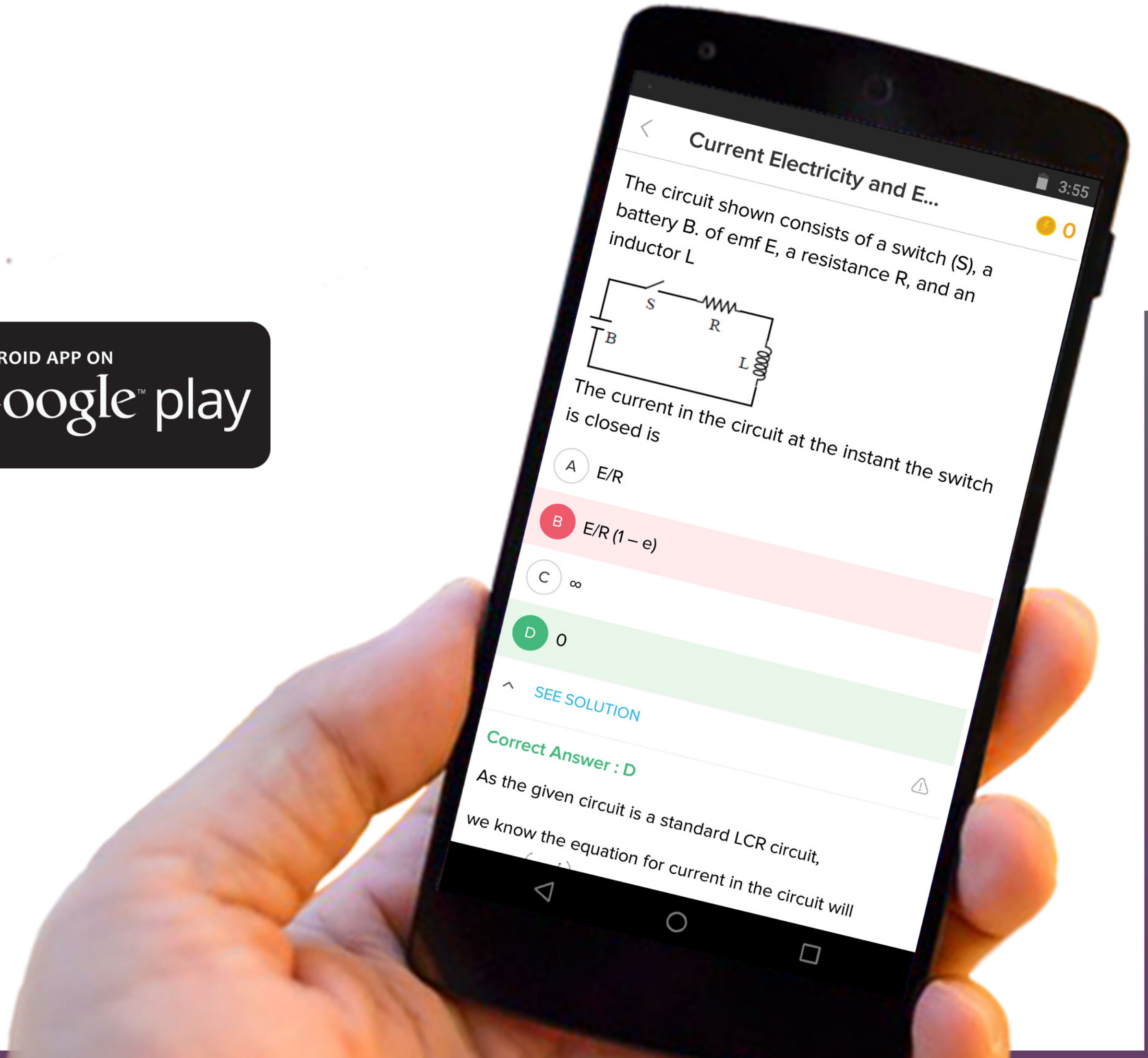

3:55
Current Electricity and E...
0
The circuit shown consists of a switch (S), a battery B. of emf E, a resistance R, and an inductor L
S
R
B
L
The current in the circuit at the instant the switch is closed is
A E/R
B E/R (1 – e)
C ∞
D 0
SEE SOLUTION
Correct Answer : D
As the given circuit is a standard LCR circuit,
we know the equation for current in the circuit will