2013

Advanced Foundation Engineering

Prof.T.G. Sitharam
Indian Institute of Science, Bangalore

CHAPTER 2: Shallow Foundations

Chapter 2

Shallow Foundations

2.1 Introduction

The foundation of a structure can be defined as that part of the structure in direct contact with the ground and which safely transmits the load of the structure to the ground.

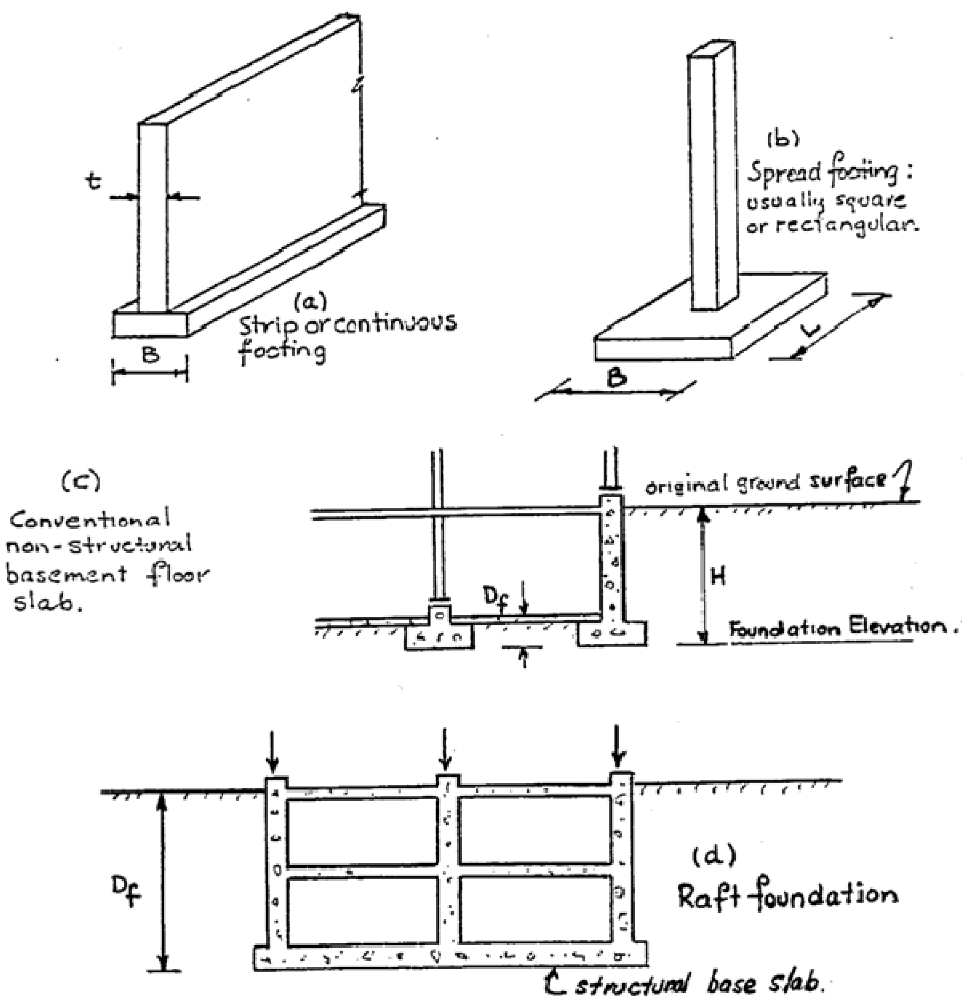


Fig 2.1: Types of footing

In the broadest sense foundation engineering is concerned with both the ability of the soil to support the load and the structural design of the sub-structural element which transmits the load onto the ground. Since the structural behavior of the substructure depends on the characteristics of the supporting soil as well as the possible structural influence of the superstructure the engineer should consider the structure, the foundation and the supporting soil as a whole rather than as independent elements.

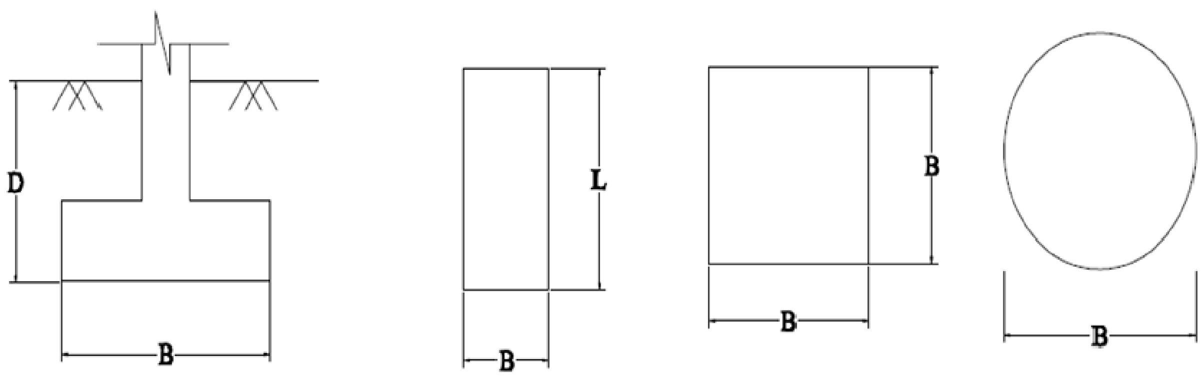


Fig2.2: Simple spread footing.

To introduce the terms depth and breadth of foundation, a simple spread footing is illustrated in Fig 2.2. The depth of foundation, D, is the vertical distance between the ground surface and the base of the foundation. The width or breadth of foundation is the shortest dimension of the foundation in plan and is illustrated in Fig 2.2 for the three usual types: rectangular, square and circular. Foundations are broadly classified under two heads: shallow foundation and deep foundation. According to Terzaghi for a shallow foundation $D \leq B$. However in practice, it is widely accepted that the above criterion may be modified as $D \leq 2B$ for shallow foundations. The various types of shallow foundations provided in practice are:

1. Spread footing, isolated footing or individual footing to support a single column.
2. Combined footing to support two or more columns in a row.
3. Continuous footing or strip footing to support a wall.
4. Mat or raft foundation to support all the columns and walls together.

A brief discussion of the above types of shallow foundations is given at the end of this chapter. The three types of deep foundations one can come across are:

a. Pier foundation
b. Pile foundation and
c. Well foundation.

These have been dealt with in the next chapter.

Prior to the industrial revolution, little attention was given to the design of a foundation. Certain construction practices had been developed and a number of empirical rules had been formulated. The general approach was to employ a form of spread foundation unless soft material was encountered in which case, piles were driven. The procedure proved satisfactory in most instances as the buildings were light and flexible. Towards the end of the 19^{th} century, higher and heavier structures were introduced. Foundation failures became more common and engineers began to seek more reliable procedures.

Structural engineering made rapid progress about this time. The classical theories based on elastic, homogeneous materials were developed and applied to steel and concrete structures. Obviously these theories were not applicable to soil, the most variable and apparently inelastic material available to Civil Engineer. In these circumstances, foundation design remained an art where experience and empirical rules prevailed. It was a little wonder that inadequate foundations were the major cause of structural failure at that time.

The first attempt to rationalize the design of the shallow foundations was the introduction of the "allowable soil pressure" concept. In this method, a table of allowable soil pressures was drawn up for various foundation soil types based upon experience. The method ignored many important factors affecting the behavior of a foundation; hence excessive settlements and failures frequently occurred. This method is still permitted by some Building Regulations and many texts provide tables of estimated allowable bearing pressure for various soil and rock types as illustrated in Table 2.1. However the bearing pressure values listed in such tables should be used only for preliminary design purposes or for minor structures where the cost of soil investigation is not justified.

Table 2.1 Presumed allowable bearing pressure

Group	Types and conditions of rocks and soils	Safe Bearing Pressure (kPa)
Rocks	Rocks (hard) without laminations and defects. For e.g. granite, trap & diorite	3240
	Laminated Rocks. For e.g. Sand stone and Lime stone in sound condition	1620
	Residual deposits of shattered and broken bed rocks and hard shale cemented material	880
	Soft Rock	440
Cohesionless Soils	Gravel, sand and gravel, compact and offering resistance to penetration when excavated by tools	440
	Coarse sand, compact and dry	440
	Medium sand, compact and dry	245
	Fine sand, silt (dry lumps easily pulverized by fingers)	150
	Loose gravel or sand gravel mixture, Loose coarse to medium sand, dry	245
	Fine sand, loose and dry	100
Cohesive Soils	Soft shale, hard or stiff clay in deep bed, dry	440
	Medium clay readily indented with a thumb nail	245
	Moist clay and sand clay mixture which can be indented with strong thumb pressure	150
	Soft clay indented with moderate thumb pressure	100
	Very soft clay which can be penetrated several centimeters with the thumb	50

	Black cotton soil or other shrinkable or expansive clay in dry condition (50 % saturation)	130 - 160

Note:

1. Use γ_d for all cases without water. Use γ_{sat} for calculations with water. If simply density is mentioned use accordingly.
2. Fill all the available data with proper units.
3. Write down the required formula
4. If the given soil is sand, c = 0

Prior to about 1920 attempts to measure the safe bearing pressure consisted of loading to failure in the field a small plate about 0.3m square, as shown in the Fig 2.3(a), and then using the load-settlement curve obtained therefrom to infer the bearing pressure to be used in the design. While this procedure, if correctly performed and interpreted, can provide a satisfactory design, it has been clamied that the plate load test was the greatest single cause of failure in the history of foundations. The reasons for this observation are as follows.

High stresses are produced in the soil below the plate only to a depth equal to about twice the width of the loaded area. Consequently the initial settlement of the plate will be governed by the compressilbity of the soil within the depth of about 0.6m. If the load is increased to failure the plate will usually fail by rotation along some surface such as a b c [Fig 2.3(b)] when shear strength of the soil around the slip surface has been fully mobilised. It follows therefore that compressibility is obtained only for the soil within a depth of about 0.6m and the shear strength inferred from this test relates only to the soil within a distance of about 0.3m below the plate. If the surface soil deposit is underlain by a weaker, more compressible soil stratum (or if the deposit becomes weaker with depth) than a single plate test performed near the ground surface will provide erroneous information for the design of a full size building as shown in Fig 2.3(b). This indeed was the cause of failure of the Transcona grain elevator which has been discussed by Peck & Bryant (1953).

Since 1920 scientific study led by Terzaghi has revolutionized the design of foundations. Today, rational theories are available to predict the bearing capacity and settlement of shallow foundations with confidence. However, soils are not precisely amenable to mathematical solution and the engineer must temper theory with common sense and judgement based upon experience.

The foundation designer must also consider the possible effects that construction techniques may have on the conditions assumed in design.

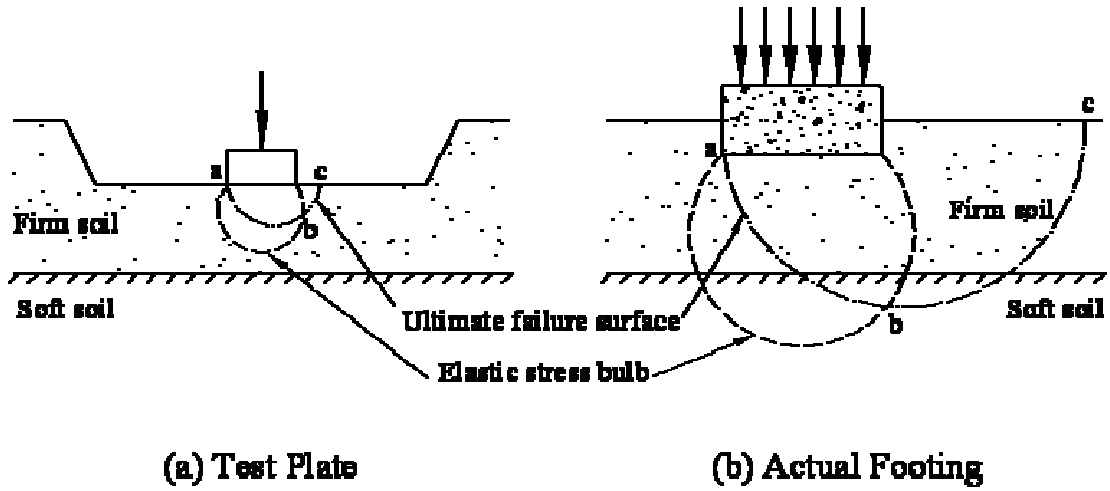


Fig 2.3: Effect of Size of Loaded Area

For instance, it may be necessary to consider such factors as the following:

a) Occurrence during excavation: bottom heave; wetting, swelling and softening of an expansive clay or rock; piping in sands and silts; disturbance of silts and sensitive clays.
b) Adjacent construction activities: ground water lowering; excavation; blasting.
c) Other effects during or following construction: scour and erosion; frost action; flooding.

In addition it is the responsibility of the foundation designer to ensure that the foundation design allows for any vertical and horizontal extensions of the structure, that the client may be contemplating.

2.1.1 Requirements of a Good Foundation

Basically a satisfactory foundation must satisfy three criteria:

1) It must be sufficiently deep to be free from seasonal climatic effects such as frost damage including possible thawing in permafrost areas, damage from adjacent

construction or possible scour from water flow. The foundation must also be located below any topsoil, other organic material, or any unconsolidated soil such as filled in areas, abandoned garbage dumps, etc.

2) It must be safe from breaking into the ground (bearing capacity failure) and,
3) The settlement of the structure must be kept within tolerable limits to minimize the angular distortion of the parts of the structure, to minimize the possibility of excessive tilting, particularly of buildings with a high aspect ratio and to prevent damage to adjacent buildings or attached services, etc.

The first condition varies of course with each individual case but generally in cold regions a minimum foundation depth of about 1.0m to 1.5m is used to place exterior footings below the frost line. In the hot regions, where frost is not a problem, minimum depth of foundation is governed by the depth of erosion due to surface water runoff to prevent possible loss of support. This in practice is about 1 m. The last two requirements are studied in subsequent sections.

2.1.2 Basic Definitions:

1. Gross pressure intensity (q) is the intensity of pressure at the base of foundation due to load from super structure, self weight of foundation and overburden, if any.
2. Net pressure intensity (q_n) is gross pressure intensity minus the over burden pressure at the level of base of foundation prior to excavation.

$$q_n = q - \gamma D$$

3. Ultimate bearing capacity (q_f) is the minimum gross pressure intensity at which the soil at the base of foundation fails by shear.
4. Net ultimate bearing capacity (q_{nf}) is the minimum net pressure intensity at which the soil at the base of foundation fails by shear.

$$q_{nf} = q_f - \gamma D$$

5. Net safe bearing capacity (q_{ns}) is the maximum net pressure intensity to which the soil at the base of foundation can be subjected without risk of shear failure.

$$q_{ns} = \frac{q_{nf}}{F}$$

where F= factor of safety against shear failure.

6. Safe bearing capacity (q_s) is the maximum gross preasure intensity to which the soil at the base of foundation can be subjected without risk of shear failure.

$$q_s = \frac{q_f}{F}$$

or more appropriately, $q_s = q_{ns} + \gamma D$

$$\text{i.e. } q_s = \frac{q_{nf}}{F} + \gamma D$$

7. Allowable bearing pressure (q_a) is the maximum gross pressure intensity to which the soil at the base of foundation can be subjected without risk of shear failure and excessive settlement detrimental to the structure.

$$q_a = q_{na} + \gamma D$$

where q_{na} = net allowable bearing pressure.

The term bearing capacity qualitatively refers to the supporting power of a soil or rock. But to define it or quantify it one should pay attention to the prefixes introduced earlier.

2.1.3 Design Loading and General Philosophy

Every foundation element must be able to support with an adequate margin of safety the maximum loading to which it may be subjected even if this loading may act only briefly over the lifetime of the structure. That is to say an overload or a misjudgment of the soil properties should result only in an increase of settlement and not in the complete failure of the sub-soil.

The design live loads of buildings is specified according to the type of occupancy by building codes such as Bureau of Indian Standards. The structural designer must compute the contribution of dead and live loads to be supported by each column on the basis of these specifications and the structural action of the superstructure. Allowances must also be made for variations in column loading due to all possible combinations of dead loads, live loads, wind, earthquake, thermal expansion, etc. Reduction factors are usually specified according to the probability of the maximum effect of all of these occurring simultaneously. Each footing must be able to support safely the maximum load calculated on this basis. However, depending upon the type of foundation soil these maximum column loads may not always be the most appropriate for design of footing with respect to settlement. The reasons for this are as follows.

1) The settlement of footings on coarse grained cohesionless soils, such as sands and gravels, occurs most rapidly. Consequently, much of the settlement due to dead loads will have occurred by the time the structure is completed. The settlement due to live loads will also occur as soon as the live load is in place even if the live load exists for a relatively short duration of time.
2) In contrast to sands, the consolidation settlement of structures founded on saturated clays occurs very slowly and is essentially unaffected by short duration applications of live load (provided of course a bearing capacity failure is not approached). Consequently the long-term settlement of structures on saturated clay should be computed using dead loads plus the best possible estimate of the long-term average live load. The immediate settlement may occur when clays are first loaded but usually such settlement is not significant.

The philosophy used in the design of foundations is to consider bearing capacity and settlement separately. A factor of safety of at least two is required against a bearing capacity failure even if the maximum loading can be computed accurately and the soil properties have been reliably determined. In practice a FS=2 is usually used for foundations on cohesionless soils and a FS=3 is required for foundations on cohesive soils.

On the other hand, no load factor or margin of safety is used when estimating settlements; rather, the anticipated settlements are calculated from the actual design loading and the foundation is proportioned to keep these calculated settlements within tolerable limits. Footing sizes are usually selected to try and achieve equal settlements to minimize the differential settlements.

2.2 Ultimate Bearing Capacity of Axially Loaded Continuous Footing

When a load Q is applied to a soil in gradually increasing amounts by a rigid footing, as shown in Fig 2.4(a), the footing settles and a pressure-settlement curve similar to that shown in Fig 2.4(b) can be obtained. Both the shape of the curve and the ultimate maximum value (Q_{ult}) of the load vary, in general, with the strength and compressibility of the soil and the size and shape of the footing. The general nature of the soil response to loading will be explained in the following discussion for the somewhat simplified case of a continuous rigid footing placed on the surface of a uniform deposit of saturated clay.

Generally, for small load increments, the response of the clay will be linear and the settlement of the footing could be computed using equation

$$S = \mu_0 \mu_1 q \frac{B}{E}$$

…….Eq 2.1

As the load on the footing is increased the ultimate strength of the clay is reached locally at the edges of the footing and the clay yields plastically. The progression of these zones of yielding with increasing load is also shown in Fig 2.4(a). It can be seen that first yield occurs at a load approximately equal to 27% of the ultimate maximum.

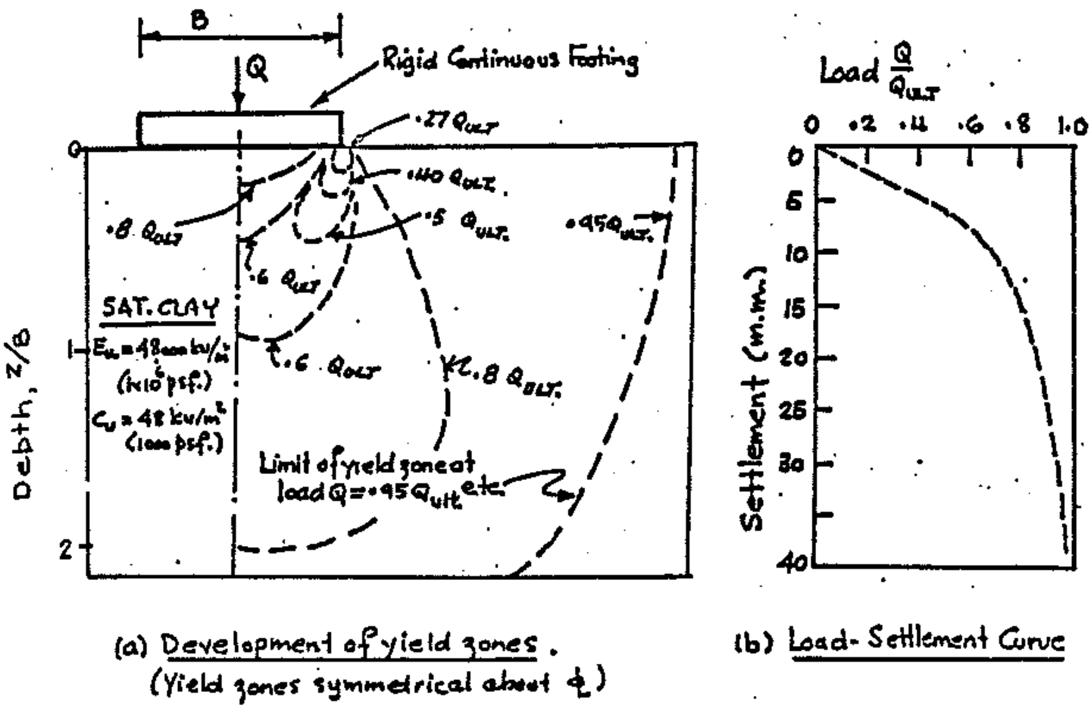


Fig 2.4: Typical Footing Response to Load

Ultimately the footing is surrounded by soil whose maximum strength has been reached and it is possible for the footing to fail either by excessive vertical displacement or by rotation if rotation of the footing is not prevented by the superstructure.

We now require to be able to compute the ultimate bearing pressure which may be applied to any soil deposit. A rigorous analysis such as that described above is quite difficult because, in general, stress-strain relationships for soils are not reliably known and in any event some form of computer aided analysis is required. It is possible, however, to obtain reasonably accurate approximate solutions by studying possible failure mechanisms in much the same manner as is used in structural analysis to obtain statically admissible (lower-bound) and kinematically admissible (upper-bound) solutions for the collapse load of structural frames.

2.2.1 Determination of Ultimate Bearing Capacity

The ultimate bearing capacity of a foundation is determined by the methods listed below:

1. By the use of theoretical analyses,
   such as Terzaghi's analysis, Skempton analysis, Meyerhof analysis, etc.
2. By the use of plate load test results
3. By the use of penetration test results
4. By the use of building codes

Attempts to obtain equations for evaluating ultimate beating capacity of foundations dates back to the middle of 19th century, with Rankine's analysis and Pauker's analysis being the earliest. Both the analyses are based on classical earth pressure theory. In the beginning of 20th century Bell (1915) proposed an equation for ultimate bearingcapacity of footing, again based on classical earth pressure theory. Prandtl (1921) and Fellenius (1939) presented their analysis based on theory of plastic equilibrium. The equations obtained from the above cited analyses are not used in practice because of serious limitations.

Significant contributions to the subject of bearing capacity were later made by Terzaghi (1943), Meyerhof (1951), Skempton (1951), Brinch Hansen (1961) and Balla (1962). In the following discussion Terzaghi's analysis and those following it as indicated in the above list are reviewed. It is indeed of interest to go through the derivations which will help in fully appreciating the limitations of each analysis. For this purpose the student is advised to go through Appendix I.

2.2.1.1 Prandtl's Analysis

Prandtl's analysis is based on a study of plastic failure in metals when punched by hard metal punchers (Prandtl, 1920); Prandtl (1921) adapted the above study to soil loaded to shear failure under a relatively rigid foundation. Prandtl's equation for ultimate bearing capacity is

$$q_f = \mathrm{c} \cos \Phi \, (N_\Phi \,.\, e^{\pi tan\Phi} - 1) \qquad \text{. . .Eq 2.2(i)}$$

where$N_\Phi = tan^2\left(45^0 + \frac{\Phi}{2}\right) = \frac{1+\sin\Phi}{1-\sin\Phi}$

It is applicable for $c - \Phi$ soil. But for a cohesionless soil for which$c = 0$, Eq 2.2(i) gives $q_f = 0$, which is ridiculous. This anamoly which is due to the assumption that the soil is weight less was removed by Taylor (1948). Prandtl's equation with Taylor's correction is

$$q_f = \left(c\cot\Phi + \frac{1}{2}\gamma B\sqrt{N_\Phi}\right)(N_\Phi\,.\,e^{\pi tan\Phi} - 1) \quad \text{. . . . Eq 2.2(ii)}$$

Taylor also attempted to include the effect of overburden pressure in the case of a footing founded at depth D below the ground surface, resulting in the following equation.

$$q_f = \left(c\cot\Phi + \frac{1}{2}\gamma B\sqrt{N_\Phi}\right)(N_\Phi\,.\,e^{\pi tan\Phi} - 1) + + \gamma N_\Phi\,.\,e^{\pi tan\Phi} \quad \text{. . . .Eq 2.2(iii)}$$

Assumptions made in Prandtl's Analysis

The following assumptions were made in Prandtl's analysis.

1) The soil is homogeneous and isotropic.
2) The soil mass is weight less.
3) The shear strength of soil can be expressed by Mohr-coulomb equation.
4) Prandtl assumed the failure zones to be formed as shown in Fig 2.5.

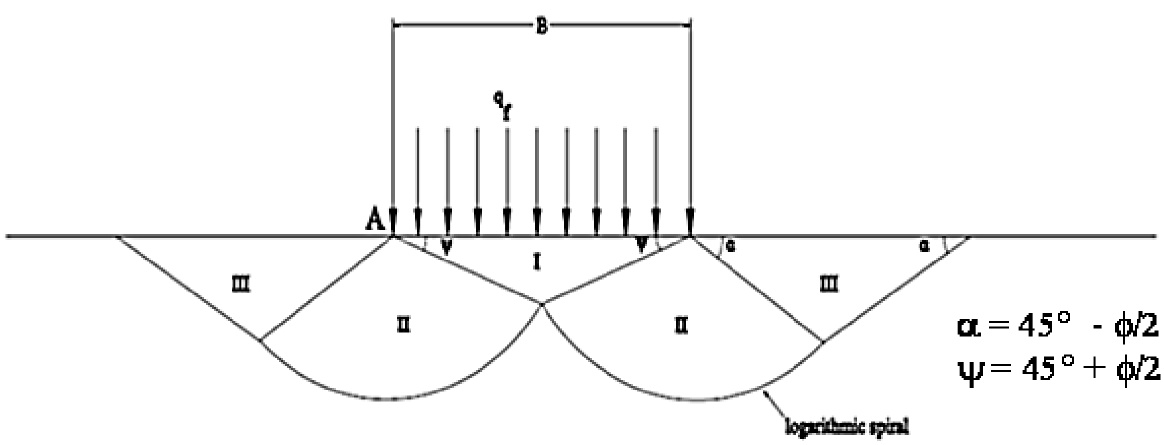


Fig2.5: Failure Zones Assumed in Prandtl's Analysis

Zone I is bound by two planes inclined at $\left(45^0 + \frac{\Phi}{2}\right)$ to the horizontal and acts as a rigid body.Zone II is bound by two planes inclined at $\left(45^0 + \frac{\Phi}{2}\right)$ and $\left(45^0 - \frac{\Phi}{2}\right)$ to the horizontal. The base of this zone is a logarithmic spiral in section. All radial sectors in this zone are failure planes.Zone III is bound by two planes inclined at $\left(45^0 - \frac{\Phi}{2}\right)$to horizontal and also acts as a rigid body.

5) The problem is essentially two dimensional, i.e., the equation is derived for a long strip footing.
6) The base of the footing is smooth.

The Limitations of Prandtl's Analysis are

1) In the original Prandtl's equation, the ultimate bearing capacity reduces to zero for cohesionless soil.
2) The original Prandtl's equation is applicable only for a footing resting on surface. Attempts have been made by Taylor to overcome the anomalies arising due to assumptions (1) and (2) to some extent.
3) In the case of a footing resting on purely cohesive soil, Prandtl's equation leads to an indeterminate quantity. Only by applying L' Hospital's rule the limiting value $\Phi \rightarrow 0$ is obtained as q_u =5.148.
4) In the original Prandtl's equation, the size of the footing is not considered.

2.2.1.2 Terzaghi's Analysis

Terzaghi derived equation for ultimate bearing capacity of strip footing as:

$q_f = cN_c + \gamma DN_q + 0.5\gamma BN_\gamma$ . . . Eq2.3(i)

where, c = unit cohesion of soil

γ = unit weight of soil

D = depth of foundation

B = width of foundation

N_c,N_q, N_γ are Terzaghi's bearing capacity factors for strip footing. These factors are dimensionless and depend only on angle of shearing resistance Φ of soil. It is to be noted that values of γ in the second and third terms of Eq 2.3(i) depend on position of water table and will be discussed in a later section.

Assumptions made in Terzaghi's Analysis

Terzaghi while deriving equation for ultimate bearing capacity of strip footing made the following assumptions.

1) The soil mass is homogeneous and isotropic.
2) The shear strength of soil can be represented by Coulomb's equation.
3) The problem is two dimensional.
4) The footing has rough base.
5) The ground surface is horizontal.
6) The loading is vertical and symmetrical.
7) Terzaghi assumed the failure zones to be formed as shown in Fig 2.6.

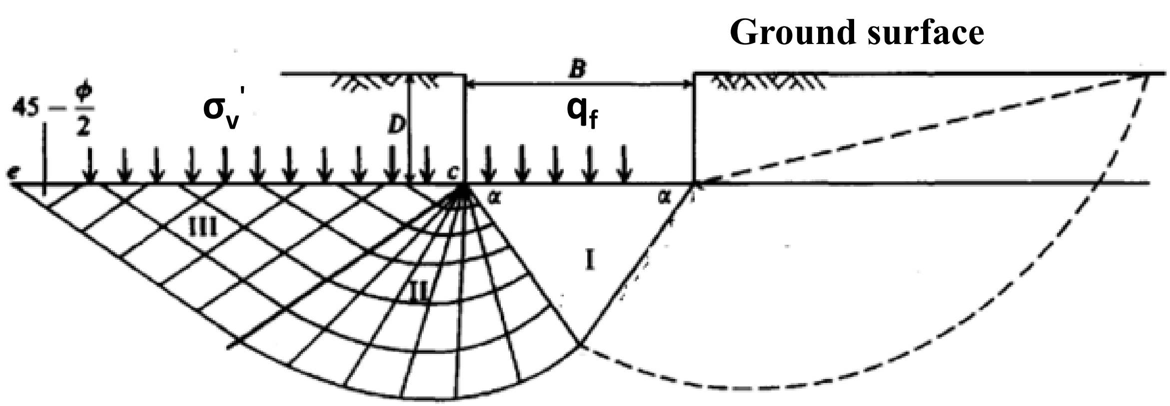


Fig2.6: Failure zones according to Terzaghi.

Zone I is elastic zone. When footing moves downward during failure, this zone moves downward along with footing. It behaves as though it is a part of the footing. Zone II is radial shear zone bound by two planes inclined at Φ and $\left(45^0-\frac{\Phi}{2}\right)$ to the horizontal, and the base being a logarithmic spiral in section. One set of planes in this zone radiate from a corner of the footing. Zone III is linear shear zone or Rankine passive zone with failure planes inclined at $\left(45^0-\frac{\Phi}{2}\right)$to the horizontal.

8) Failures zones are assumed to be formed fully.
9) The principle of superposition is applicable.
10) The failure zones do not extend above the base level of the footing, the effect of soil surrounding the footing above its base level is considered equivalent to a surcharge $\sigma=\gamma D$.

Limitations in Terzaghi's analysis

1) Terzaghi's analysis assumes the plastic zones develop fully before failure occurs. This is true only in the case of dense cohesionless soils and stiff cohesive soils.
2) The value of Φ is assumed to remain constant. But Φ can change as soil gets compressed.
3) The failure zones are assumed not to extend above the base level of footing. Thus the shearing resistance of soil surrounding it above its base level is neglected. The error due to this assumption increases as the depth of footing is increased.
4) The load is assumed to be vertical and acting concentrically with uniform pressure distribution at the base.

Terzaghi's Bearing Capacity Factors

Terzaghi's bearing capacity factors for strip footing assuming general shear failure have been obtained as:

$$N_c = (N_q - 1)\cot\Phi$$

$$N_q = \frac{\mathrm{a}^2}{2cos^2\left(45^0 + \frac{\Phi}{2}\right)}$$

$$N_\gamma = \frac{1}{2}\left(\frac{K_{p\gamma}}{cos\Phi^2} - 1\right)\tan\Phi$$

with $$\mathrm{a} = \mathrm{Exp}\left[\left(\frac{3\pi}{4} - \frac{\Phi}{2}\right)\tan\Phi\right]$$

To compute N_γ the value of earth pressure coefficient $K_{p\gamma}$ is required but as to how $K_{p\gamma}$ can be obtained is a point which was not made clear by Terzaghi. However, Terzaghi provided values of N_γ along with N_c, N_q without the need for specific values of $K_{p\gamma}$ .The bearing capacity factors can be obtained from chart in Fig 2.7(a).

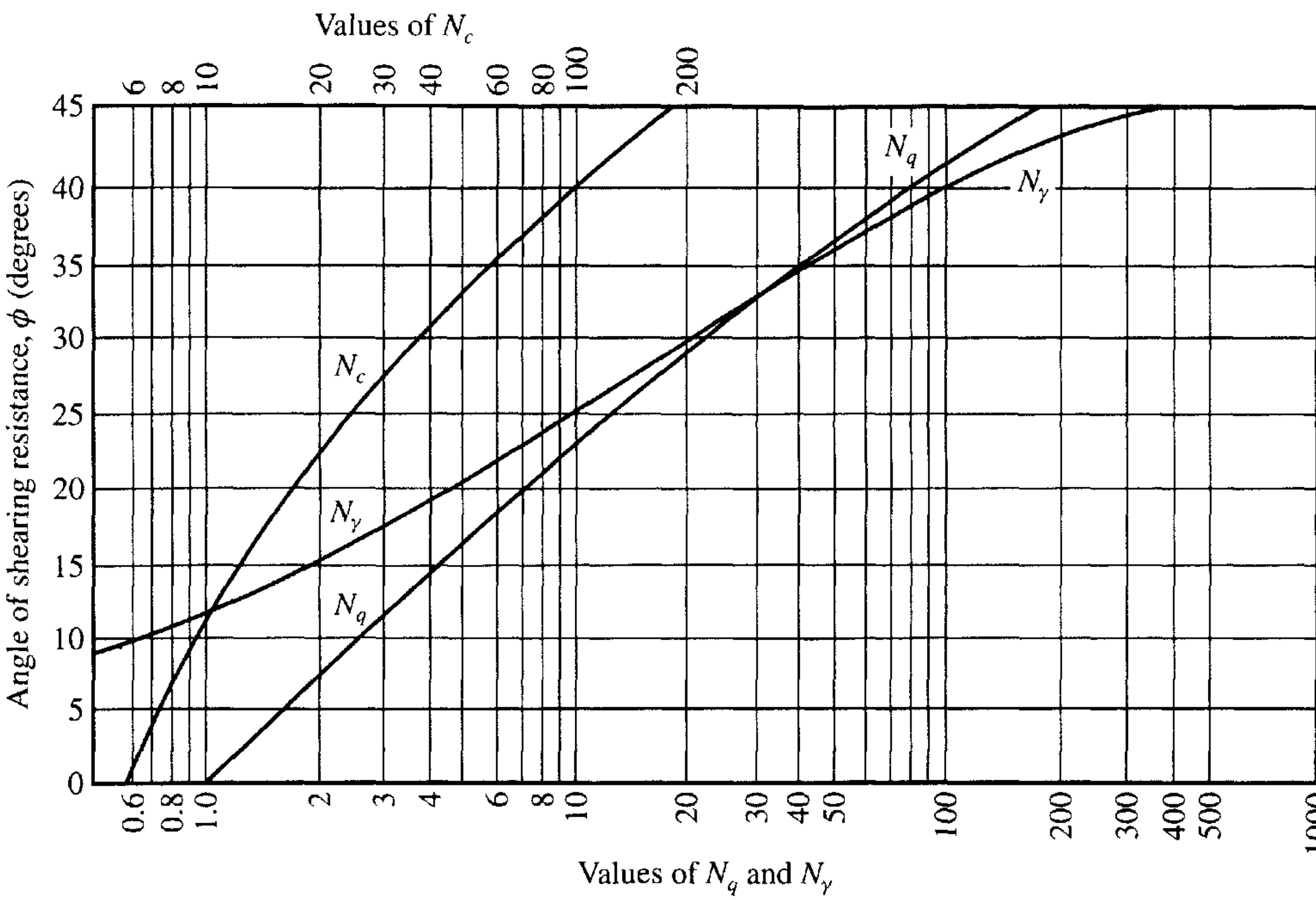


Fig 2.7(a): Terzaghi's bearing capacity factors

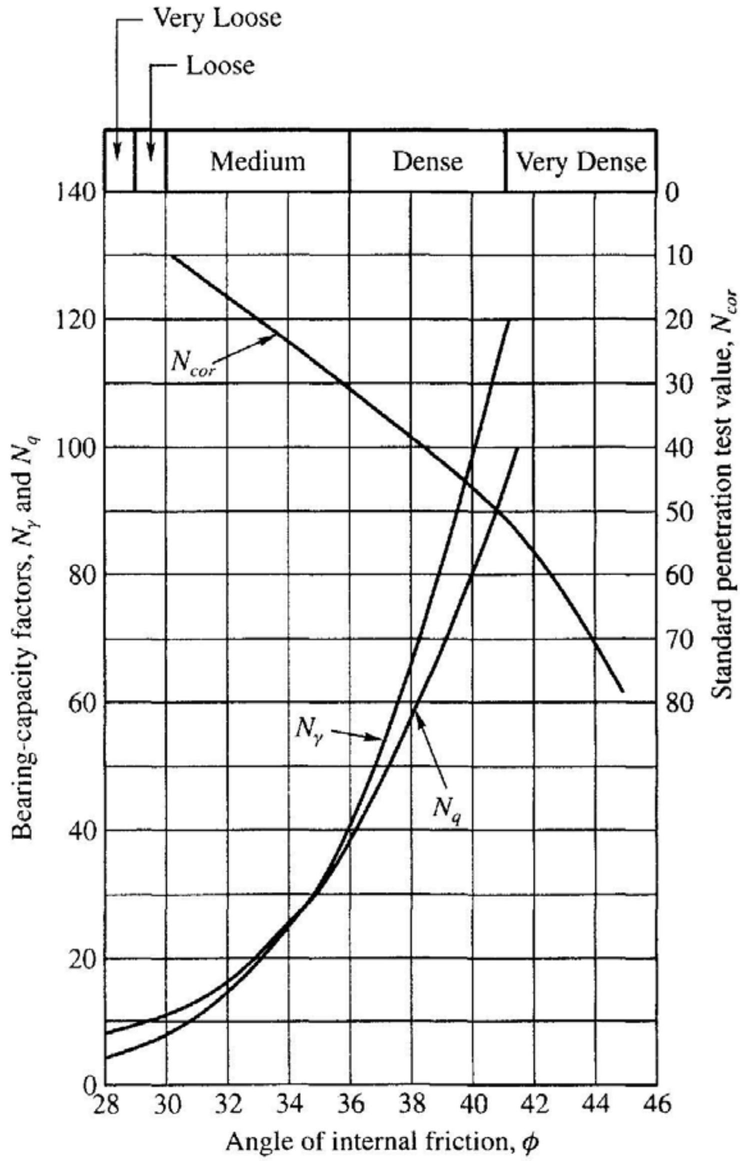


Fig 2.7(b): Terzaghi's bearing capacity factors for transitional state

Terzaghi suggested that in the case of local shear failure reduced shear strength parameters C_m and Φ_m given by the following equations, be used instead of c and Φ

$$C_m = \frac{2}{3}c$$

$$\Phi_m = \tan^{-1}\left(\frac{2}{3}\tan\Phi)\right)$$

The bearing capacity factors for local shear failure condition are usually denoted by N_c' ,N_q' and N_γ'. They are obtained corresponding to Φ_m using the same chart provided (Fig 2.7) for general shear failure condition. However Fig 2.7(b) can be used for transitional state in the case of sands.

Prediction of the Type of Failure Condition

It is difficult to predict exactly the type of failure condition that may occur in the general case of a C soil. The laboratory stress-strain curve or the loading intensity-settlement curve obtained from field load test serves as a guide line.

Referring to Fig 2.8 curves (1) are typical of soils in which general shear failure can be expected and the curves (2) of soils in which local shear failure can be expected.

For curves of type 1, failure point will be observed at less than about 5% strain and in the case of curves of type 2, no failure will be eminent even at 10% or 20% strain.

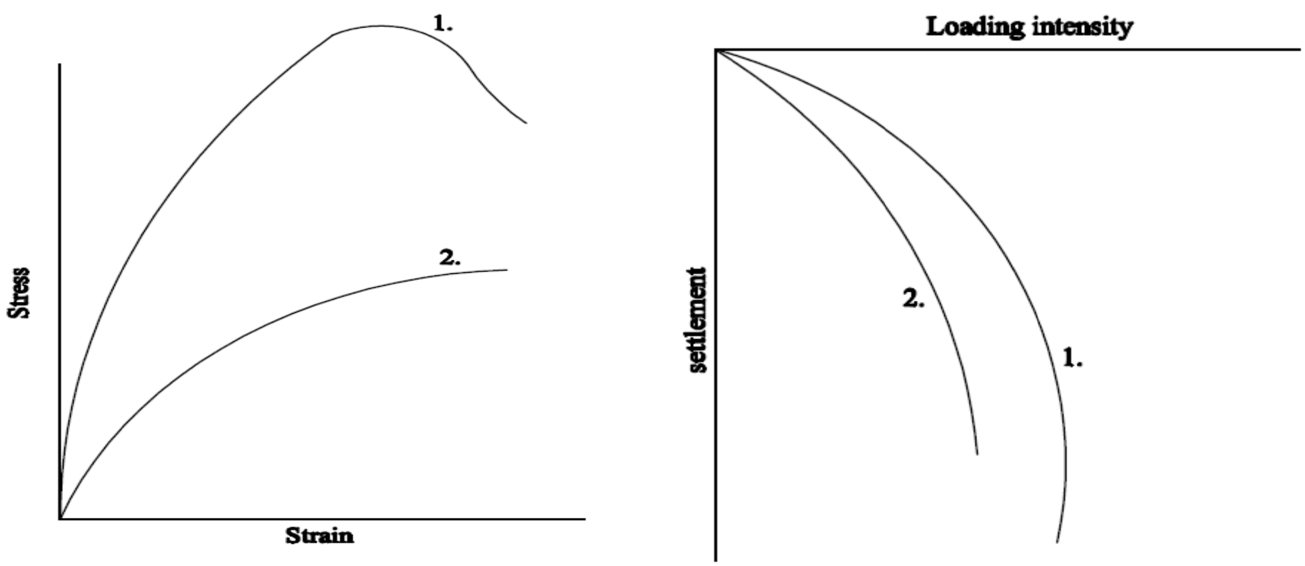


Fig 2.8: Typical curves for predicting type of failure

In the case of cohesionless soils general shear failure can be expected if density index I_d is greater than 70% and local shear failure if I_d is less than 30%. Based on Φ, general shear failure may be expected when Φ ≥ and local shear failure Φ ≤

For intermediate values, we have transition from local shear failure to general shear failure for which the bearing capacity factors may be obtained by interpolation. Peck, Hansen and Thornburn (1974) have presented Fig 2.7(b) in which the transition state is also incorporated.

In the case of purely cohesive soil, local shear failure may be expected when unconfined compressive strength, $q_u \leq 100 kN/m^2$

2.2.1.2a Effect of Shape on Ultimate Bearing Capacity of Footing

Terzaghi derived the equation for ultimate bearing capacity of strip footing in which the case the problem is essentially two-dimensional. But in the case of square or circular footing the problem becomes three-dimensional and more complicated from mathematical point of view. In the absence of rigorous theoretical analysis, Terzaghi suggested that the following equations may be used

For square footing

$$q_f = 1.3cN_c + q_o N_q + 0.4\gamma B N_\gamma$$

For circular footing

$$q_f = 1.3cN_c + q_o N_q + 0.3\gamma B N_\gamma$$

For quite some time the equation obtained for strip footing was used in the case of rectangular footing. Later rectangular footing was distinguished from strip footing as one for which L≤5B and the following equation was suggested.

For rectangular footing

$$q_f = \left(1 + 0.3\frac{B}{L}\right)CN_c + q_o N_q + \left(1 - 0.2\frac{B}{L}\right)0.5\gamma B N_\gamma$$

In all the three equations q_o denotes the effective overburden pressure at the base level of foundation.

2.2.1.2b Effect of Size on Ultimate Bearing Capacity of Footing

Case (1) Footing on cohesive soil (c−Φ soil)

$$q_f = cN_c + \gamma D N_q + 0.5\gamma B N_\gamma \qquad \text{...Eq 2.4a}$$

From equation (2.4a) it is clear that in this case the ultimate bearing capacity depends on size of footing. It increases as the width of the footing is increased keeping depth constant.

Case (2) Footing on cohesionless soil (c=0)

When c=0, equation (2.4a) reduces to the following form

$$q_f = \gamma D N_q + 0.5\gamma B N_\gamma \qquad \text{... Eq 2.4b}$$

We notice that in this case also the ultimate bearing capacity depends on size of footing and increases as the width is increased keeping depth constant.

For Φ=0, Terzaghi's bearing capacity factors are

$N_c = 5.7$, $N_q = 1$ and $N_\gamma = 0$

Equation (2.4a) will then reduce to the following form

$$q_f = 5.7c + \gamma D \quad \text{. . . Eq 2.4c}$$

From equation (2.4c) it is clear that for footing on purely cohesive soil, the ultimate bearing capacity is independent of size of footing.

2.2.1.2c Effect of Water Table on Ultimate Bearing Capacity of Footing

Method 1: Reduction factor method

The submerged density of a soil is nearly half of its saturated density. Based on this fact water-table reduction factors have been proposed to consider the effect of rise in water table.

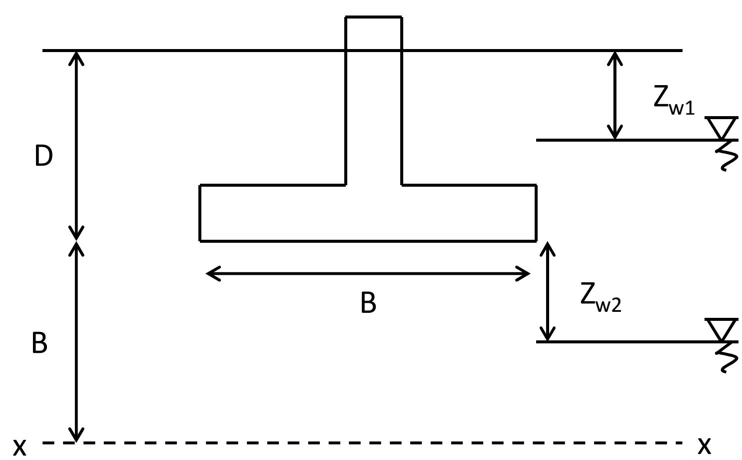


Fig 2.9: Fluctuation of water table.

When the water table lies at a depth, equal to or greater than width B of foundation, below the base of foundation, it has no effect on the ultimate bearing capacity. When the water table rises above level X-X marked in Fig 2.9 the effect of rise in water table is to reduce the ultimate bearing capacity. Consider, for example, Terzaghi's equation for ultimate bearing capacity of strip footing

$$q_f = cN_c + \gamma DN_q + 0.5\gamma BN_\gamma \quad \text{....Eq 2.5a}$$

To take into account the effect of rise in water table, the second and third terms of equation (2.5a) are to be multiplied by factors R_{w_1} and R_{w_2} respectively.

R_{w_1} and R_{w_2} are known as water table reduction factors and are expressed as

$$R_{w_1} = 0.5\left(1 + \frac{Z_{w_1}}{D}\right)$$

$$R_{w_2} = 0.5\left(1 + \frac{Z_{w_2}}{B}\right)$$

where Z_{w_1} is the depth of water table measured below ground surface and Z_{w_2} the depth of water table measured below base of footing. The limiting values of Z_{w_1} and Z_{w_2} are as indicated below

Z_{w_1}=0 when water table is at or above ground level

Z_{w_1}=D when water table is at or below base level of footing

Z_{w_2}=0 when water table is at or above base level of footing

Z_{w_2}=0 when water table is at depth equal to or greater than width B, below base of footing.

Both R_{w_1} and R_{w_2} can have 0.5 as minimum value and 1 as maximum value ,that is

$0.5 \leq R_{w_1} \leq 1$

$0.5 \leq R_{w_2} \leq 1$

Equation (2.5a) can then be written as

$$q_f = cN_c + \gamma D N_q R_{w_1} + 0.5\gamma B N_\gamma R_{w_2} \qquad \text{. . . Eq 2.5b}$$

The water table effect on the bearing capacity can also be evaluated without using the water table reduction factors. The other approach is based on the effective unit weight and is known as effective unit weight method.

Method 2: Equivalent effective unit weight method

The bearing capacity equation of the strip footing can be expressed as,

$$q_f = cN_c + \gamma_{e1} D N_q + 0.5\gamma_{e2} B N_\gamma \qquad \text{.......Eq 2.5c}$$

where γ_{e1} = weighted effective unit weight of soil lying above the base level of the foundation,

γ_{e2} = weighted effective unit weight of soil lying within the depth B below the base level of the foundation,

γ_m= moist or saturated unit weight of soil lying above WT (case I or case 2)

γ_{sat}= saturated unit weight of soil below the WT (case 1 or case 2)

γ_b= submerged unit weight of soil = γ_{sat}-γ_w

Case I: An equation for γ_{e1} may be written as,

$$\gamma_{e1} = \gamma_b + \frac{D_{w1}}{D_f}(\gamma_m - \gamma_b) \qquad \text{Eq. 2.5d}$$

Case:II

$$\gamma_{e2} = \gamma_b$$

$$\gamma_{e1} = \gamma_m$$

$$\gamma_{e2} = \gamma_b + \frac{D_{w2}}{D_f}(\gamma_m - \gamma_b) \qquad \text{Eq. 2.5e}$$

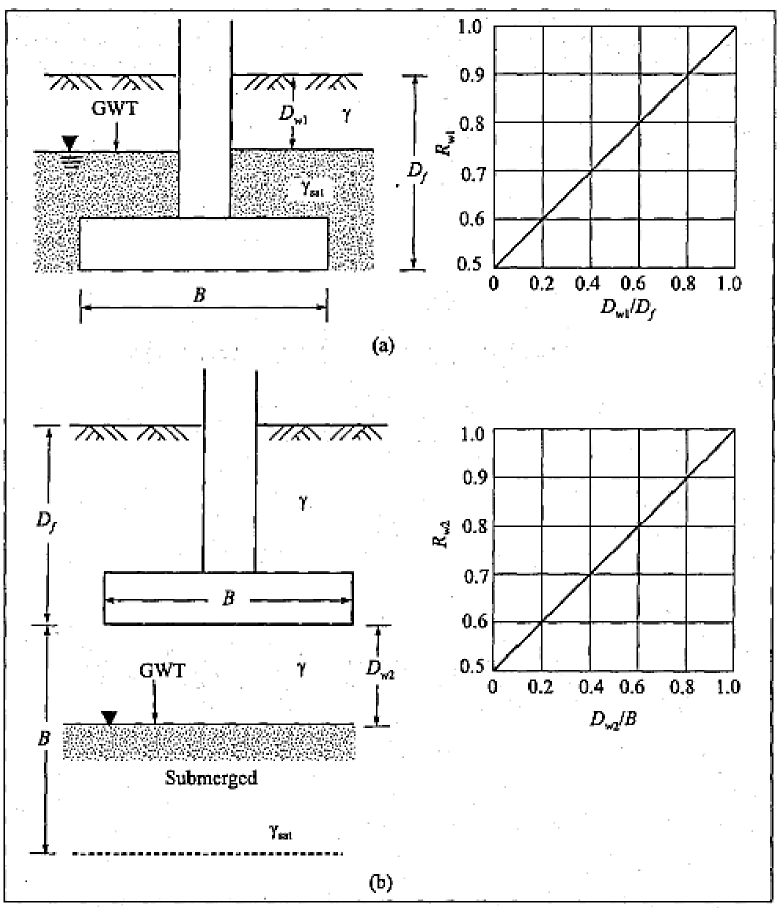


Fig. 2.10. Effect of WT on bearing capacity: (a) Water table above base level of foundation, (b) Water table below base level of foundation

2.2.1.2d Effect of Foundation Depth on Bearing Capacity

Some researchers have studied the additional contribution to bearing capacity provided by the shear strength of the surcharge soil and have expressed in this contribution in the form of depth factors to be included in the bearing capacity equation.

$$q_{ult} = cN_c s_c d_c + \gamma D_f N_q s_q d_q + 0.5\gamma B N_\gamma s_r d_r \qquad \text{....... Eq 2.6}$$

Recommendations for such depth factors are given in Hansen (1974), Bowles (1982) and other texts. It is sometimes convenient to include the depth effect for special cases but for most design situations it is better to be conservative and ignore the depth factor.

2.2.1.3 Meyerhof's Analysis

Meyerhof (1951, 1963) proposed an equation for ultimate bearing capacity of strip footing which is similar in form to that of Terzaghi but includes shape factors, depth factors and inclination factors. Meyerhof's equation is

$$q_f = cN_c s_c d_c i_c + q_o N_q s_q d_q i_q + 0.5\,\gamma B N_\gamma s_\gamma d_\gamma i_\gamma \qquad \text{. . . Eq 2.7}$$

Meyerhof's bearing capacity factors are expressed as

$$N_q = e^{\pi tan\Phi} tan^2\left(45^0 + \frac{\Phi}{2}\right)$$

$$N_c = (N_q - 1)\cot\Phi$$

$$N_\gamma = (N_q - 1)\tan(1.4\Phi)$$

The shape factors are given by

$$s_c = 1 + 0.2K_p \frac{B}{L} \text{ for any } \Phi$$

$$s_q = s_\gamma = 1.0 \text{ for } \Phi = 0^0$$

$$s_q = s_\gamma = 1 + 0.1K_p \frac{B}{L} \text{ for } \Phi \geq 10^0$$

$$d_c = 1 + 0.2\sqrt{K_p}\frac{D}{B} \text{ for any } \Phi$$

$$d_q = d_\gamma = 1.0 \text{ for } \Phi = 0^0$$

$$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\frac{D}{B} \text{ for any } \Phi \geq 10^0$$

The inclination factors are given by

$$i_c = i_q = \left(1 - \frac{\theta}{90^0}\right)^2 \text{ for any } \Phi$$

i_γ=1 for Φ=0^0

i_γ=$\left(1-\frac{\theta}{\Phi}\right)^2$ for Φ≥ 10^0

where K_p=$tan^2\left(45^0+\frac{\Phi}{2}\right)$ and θ =angle of inclination of load with respect to vertical.

It is further suggested that the value of Φ for the plane strain condition expected in long rectangular footings can be obtained from Φ_{triaxial} as

Φ_{ps}=$\left(1.1-0.1\frac{B}{L}\right)\Phi_{\text{triaxial}}$

The Meyerhof's bearing capacity factors can be obtained from chart in Fig 2.11.

2.2.1.3.1 Brief Comparison between Meyerhof's Analysis and Terzaghi's Analysis

As already explained, Terzaghi assumed the failure zones to be formed as shown in Fig 2.6. Meyerhof assumed the failure zones to be formed as shown in Fig 2.12.

Zone I is elastic zone with its sides inclined at φ to the horizontal where, $\varphi=\left(45^0+\frac{\Phi}{2}\right)$. It may be recalled that $\varphi=\Phi$in Terzaghi’s analysis. Zone II is radial shear zone with one set of radial planes radiating from corner of footing, as in Terzaghi’s analysis.

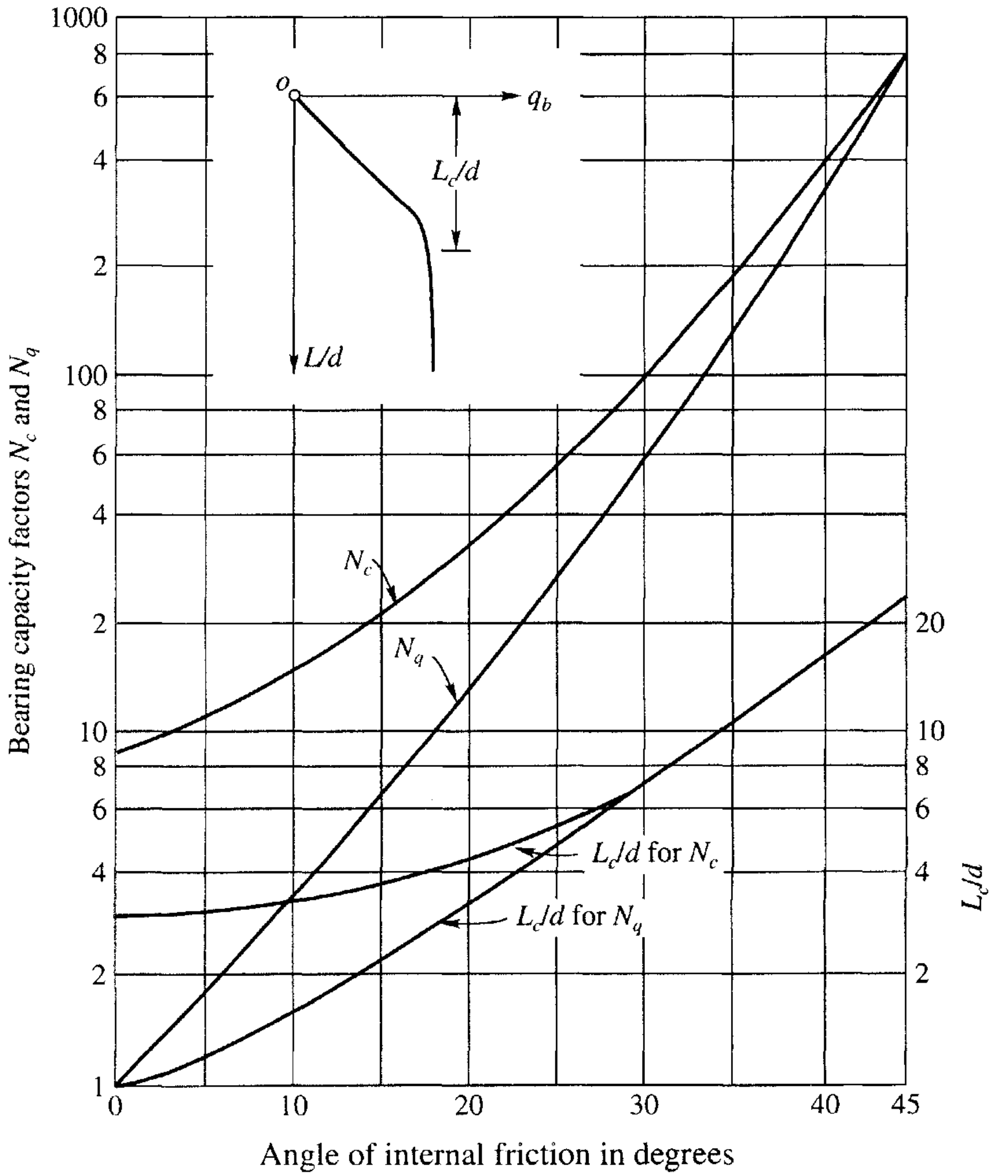


Fig 2.11: Bearing capacity factors and critical depth ratios L_c/dfor driven piles (after Meyerhof, 1976)

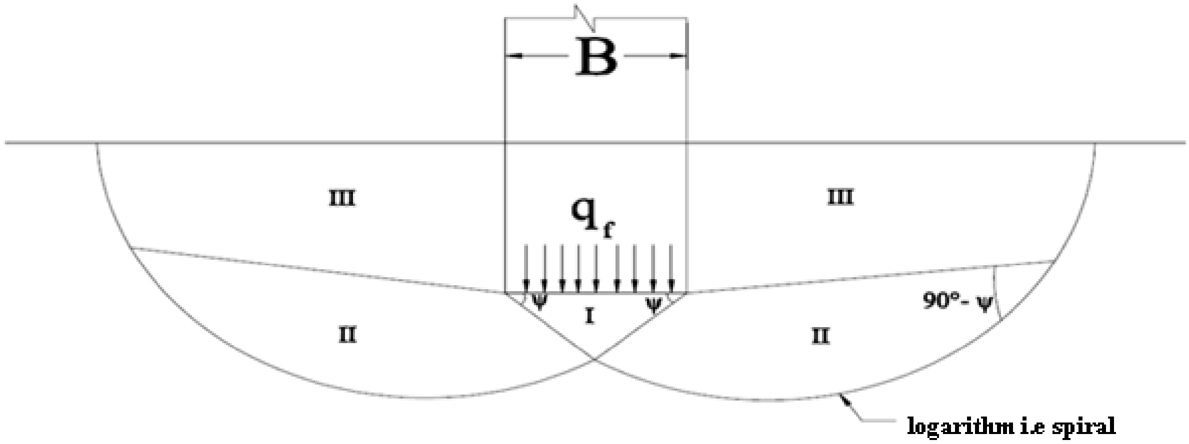


Fig 2.12: Failure Zones Assumed in Meyerhof's Analysis

Zone III is mixed shear zone. Unlike in Terzaghi's analysis the logarithmic spiral extends right up to the ground surface. It should be noted that Meyerhof assumed the failure zones to extend above base level of the footing, whereas in Terzaghi's analysis the failure zones are assumed not to extend above the base level of footing. Where as in Terzaghi's analysis the shearing resistance of soil above base level of footing is neglected, in Meyerhof's analysis it is considered. Thus Meyerhof's analysis is preferred when the depth of foundation is large. In Meyerhof's analysis the bearing capacity factors depend on four quantities

1) Angle of shearing resistance Φ of soil,
2) Shape of footing,
3) Depth of footing and
4) Roughness of the base of footing. In Terzaghi's analysis they depend only on Φ.

2.2.1.4 Hansen's Analysis

J. Brinch Hansen (1970) proposed what is referred to as general bearing capacity equation.

$$q_f = cN_cS_cd_ci_cb_cg_c + q_oN_qS_qd_qi_qb_qg_q + 0.5\,\gamma BN_\gamma S_\gamma d_\gamma i_\gamma b_\gamma g_\gamma \ldots \text{Eq 2.7a}$$

where, q_o is the effective overburden pressure at the base level of foundation.

All the factors used in Hansen's equation are expressed as follows.

1) Bearing capacity factors

$$N_q = e^{\pi \tan \Phi} \left(45^0 + \frac{\Phi}{2}\right)$$

$$N_c = (N_q - 1) \cot \Phi$$

$$N_\gamma = (N_q - 1) \tan \Phi$$

2) Shape factors

$$S_c = 0.2 \frac{B}{L} \text{ for } \Phi = 0$$

$$S_c = \left(1 + \frac{N_q}{N_c} \frac{B}{L}\right) \text{ for } \Phi > 0$$

However, for strip footing S_c=1 for any Φ.

$$S_q = 1 + \frac{B}{L} \tan \Phi$$

$$S_\gamma = 1 - 0.4 \frac{B}{L}$$

3) Depth factors

For $\Phi = 0$, $d_c = 0.4 \frac{D}{B}$ when $\frac{D}{B} \leq 1$

$d_c = 0.4 \tan^{-1} \frac{D}{B}$ when $\frac{D}{B} > 1$

For $\Phi > 0$, $d_c = 1 + 0.4 \frac{D}{B}$ when $\frac{D}{B} \leq 1$

$d_c = 1 + 0.4 \tan^{-1} \frac{D}{B}$ when $\frac{D}{B} > 1$

For all Φ, $d_q = 1 + 2 \tan \Phi (1 - \sin \Phi) \frac{D}{B}$ when $\frac{D}{B} \leq 1$

$d_q = 1 + 2 \tan \Phi (1 - \sin \Phi) \tan^{-1} \frac{D}{B}$ when $\frac{D}{B} > 1$

$$d_\gamma = 1$$

4) Load inclination factors

$i_c = 0.5 - 0.5 \sqrt{1 - \frac{\mathrm{Q_h}}{AC_a}}$ for $\Phi = 0$

$i_c = i_q - \frac{1 - i_q}{N_{q-1}}$ for $\Phi > 0$

$$i_q = \left(1 - \frac{0.5\mathrm{Q_h}}{\mathrm{Q_v} + \mathrm{AC_a} \cot \Phi}\right)^5$$

$$i_\gamma = \left(1 - \frac{0.7\mathrm{Q_h}}{\mathrm{Q_v} + \mathrm{AC_a} \cot \Phi}\right)^5$$

where Q_h = horizontal component of load Q

Q_v = vertical component of load Q

A = contact area of footing

C_a= unit adhesion on base of footing.

5) Base inclination factors

$$b_c = \frac{\alpha}{147} \quad \text{for } \Phi = 0$$

$$b_c = 1 - \frac{\alpha}{147} \quad \text{for } \Phi > 0$$

$$b_q = e^{-2\alpha} \tan \Phi$$

$$b_\gamma = e^{-2.7\alpha} \tan \Phi$$

where, α = angle in degrees made by base with horizontal line

$$g_c = \frac{\beta}{147} \quad \text{for } \Phi = 0$$

$$g_c = \left(1 - \frac{\beta}{147}\right) \quad \text{for } \Phi > 0$$

$$g_q = g_\gamma = (1 - 0.5 \tan \beta)^2$$

where, β = angle in degrees made by ground surface with horizontal

To a great extent, Hansen's work is an extension of Meyerhof's analysis, as is evident from comparison between the two equations. To include conditions for footing on slope Hansen has introduced two additional factors viz., the ground factors and base factors.

2.2.1.5Skempton's Analysis

Skempton (1951) based on his investigations of footings on saturated clays observed that the bearing capacity factor N_c is a function of ratio D/B in the case of strip footing and square or circular footings, for $\Phi = 0$ condition. He presented the chart in Fig 2.13 which gives N_c for different values of D/B. The value of N_c obtained from the chart can be used to compute the net ultimate bearing capacity, q_{nf}.

$$q_{nf} = cN_c$$

where, c= unit cohesion of soil which can be obtained from unconfined compression test.

He suggested that N_c for a rectangular footing can be obtained from Nc of square footing with same D/B ratio using the following relationship.

$$N_{c(rectangle)} = \left(0.84 + 0.16\frac{B}{L}\right) \times N_{c(square)}$$

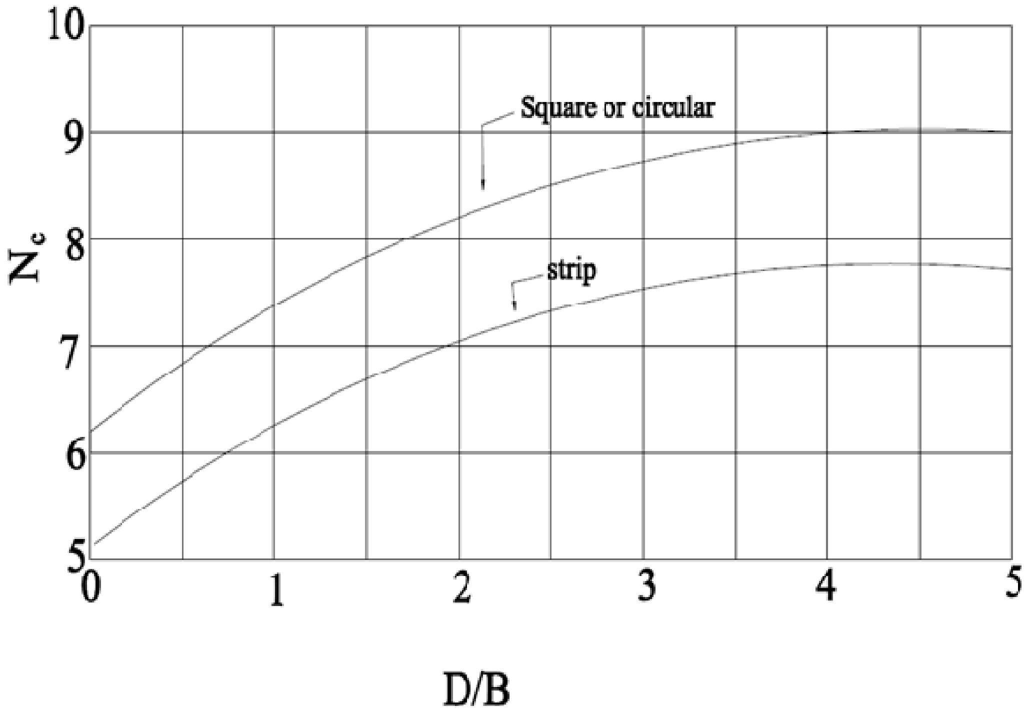


Fig 2.13: Skempton's bearing capacity factor, N_c

It is clear from Fig 2.13that for a footing resting on surface (D=0),

N_c = 5.14 for strip footing

N_c = 6.2 for square or circular footing

The maximum values of N_c are 7.5 for strip footing and 9 for square or circular footing.

2.2.1.6 Upper Bound Solutions to the Bearing Capacity of a Footing on Saturated Clay

Limit analysis is a powerful method for stability analysis and limit bearing capacity of engineering structures. Ingeotechnical engineering, upper bound limit analysis is widely used to analyze the slope stability. Drucker (1952) firstly presented limit analysis based on plastic limit theorem, and then Chen (1975) introduced limit analysis into the geotechnical engineering for analyzing the bearing capacity, earth pressure on retaining wall and slope stability. It takes advantage of the lower and upper theorems of classical plasticity to bracket the true solution from a lower bound to an upper bound. However, it is difficult to obtain analytical solution for practical engineering, and numerical approaches are often required for limit analysis. In the past three decades, many studies have been devoted to developing numerical methods of limit analysis.

The shear strength of a saturated clay under undrained condition may be assumed as:

$$S = c_u$$

$$\text{i.e., } \varphi = 0°$$

To find the upper bound solutions for the footing and slip surfaces refer Fig 2.14

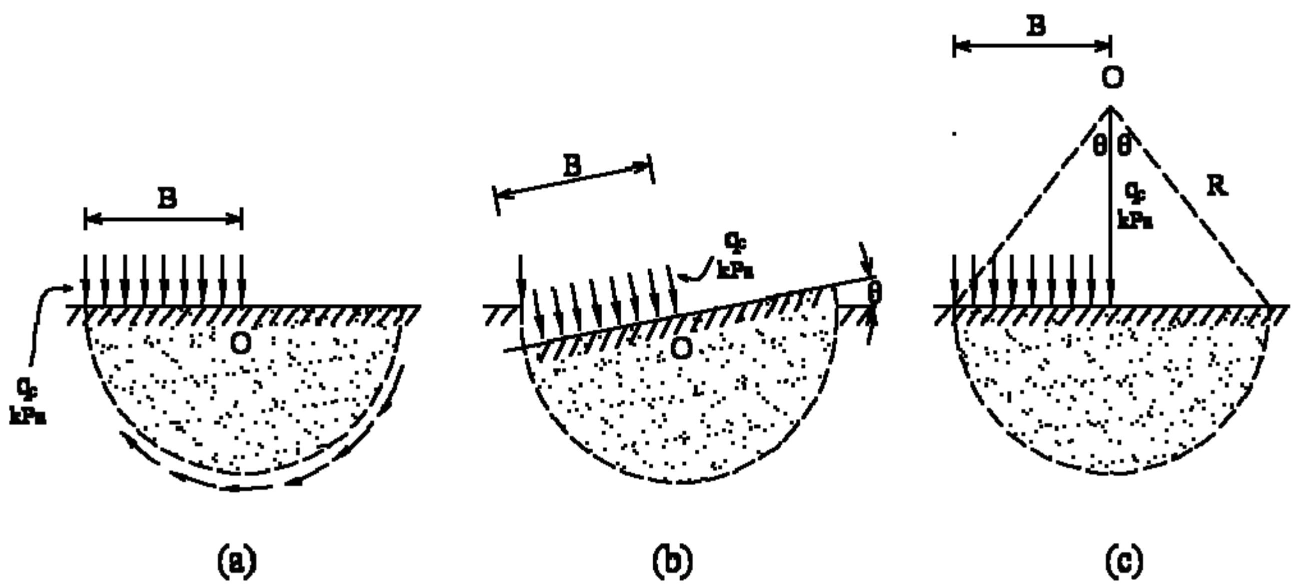


Fig 2.14: Kinematically Admissible Mechanisms

In case (a) the slip surface is assumed to be a semi-circle with centre at O, the edge of the footing.

Considering a section 1m thick and constant shear resistance of c_u around the failure surface consider the work done by the applied load q_u and soil resistance c_u. Thus referring to diagram (b) and equating:

External work = Internal work

It follows that:

$$(q_c B)\left(B\frac{\theta}{2}\right) = (\pi B c_u)(B\theta)$$

$$q_c = 2\pi c_u$$

$$\text{Or } q_c = 6.3c_u$$

This is an upper bound solution of the collapse pressure for the mechanism shown in Fig 2.14(b)

The previous result could have also been achieved by equating the sum of the moments about O so that,

$$(q_c B)\left(\frac{B}{2}\right) = (c_u)(\pi B)B$$

$$\text{Or } q_c = 2\pi c_u$$

If the slip surface is assumed to be a part of a circle with centre at O, vertically above the edge of the footing as shown in Fig 2.9c, then,

$$B = R\sin\theta$$

And arc length = $2R\theta$ (θ in radians)

So that, for $\sum M_o = 0$,

$$q_c(R\sin\theta)\left(\frac{R\sin\theta}{2}\right) = c_u(2R\theta)(R)$$

Or

$$q_c = \frac{c_u\theta}{sin^2\theta}$$

Now,

$$\frac{dq_c}{d\theta} = 4c_u\left(\frac{1}{sin^2\theta} - \frac{2\theta cos\theta}{sin^3\theta}\right)$$

$$= 0 \text{ (for minimum value of } q_c)$$

Therefore, minimum value occurs when $n\theta = 2\theta$, i.e., when $\theta \approx 66° \ (1.15\ radians)$

Therefore,

$$q_{c\ min} = \frac{c_u 1.15}{(0.913)^2}$$

$$= 5.5c_u$$

This is the minimum value of the upper-bound estimate of the collapse pressure which can be obtained with a circular slip surface and with O located at the edge of the footing. Thus it can be seen that by the use of these simple bounded solutions it can be inferred that the value of the true collapse load, for this short-term analysis, lies between $5.0c_u$ and $5.5c_u$

2.2.1.7 The Standard Penetration Test (SPT)

When a cohesionless soil is loaded as, for example, in a consolidation test the void ratio-applied vertical stress curve obtained is dependent on the initial relative density of the soil. i.e., there is no unique void ratio-pressure curve as there is for clay. Because of this fact, it is essential to know the in-situ relative density values of any cohesionless soils that occur at a particular site. It is extremely difficult however to sample cohesionless soils and to prepare samples for laboratory tests without necessarily disturbing them in process.

Consequently, it is usual to infer soil properties such as relative-density, angle of internal friction, and compressibility, indirectly from the results of Standard Penetration Test or Static Cone Penetration.

Other, more sophisticated methods of measuring in-situ soil properties exist, such as Pressuremeter tests for use on important jobs but only SPT will be discussed here.

Site investigation data provide values of the S.P.T blows per 30cm (blows per foot) required to drive a standard split-spoon sampling tube into the cohesionless soil using a standard energy input. These SPT values are termed the N_{field} values.

The field N values are affected by the magnitude of the effective vertical overburden pressure at the level of the split-spoon in the bore-hole at the time of the test. Consequently, it is necessary to standardize the N values to a particular vertical stress value.

The approach suggested by Peck Hansen & Thornburn (1974) was to use a value of σ'_v = 100 kPa as the standard and to correct the field values N values according to Eq 2.8

$$N_{corr} = N_{field} \times C_N \quad \text{...... Eq 2.8}$$

where the C_n values are given in Fig 2.4

It can be seen from Fig 2.15 that C_N = 1.0 for σ'_v = 100 kPa. It should also be noted that to calculate the effective overburden pressures it is necessary to know both the ground surface and ground-water elevations for each borehole.

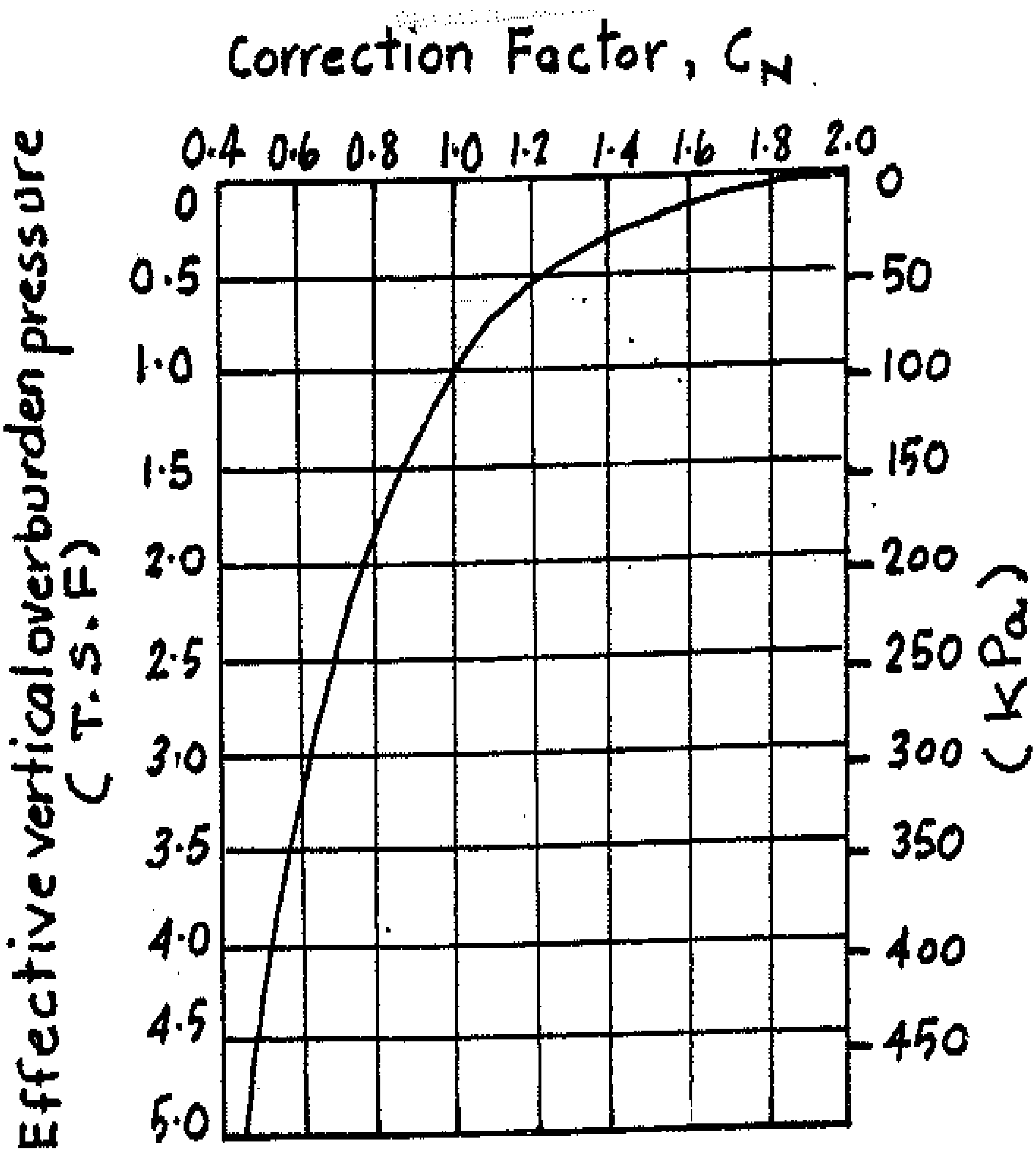


Fig 2.15: Chart for correction on N-values in sand for influence of overburden pressure [reference value of effective overburden pressure 1 ton/sq.ft (100 kPa) After Peck, Hansen & Thornburn (1974)]

2.3 Bearing capacity of footings on Layered Soils

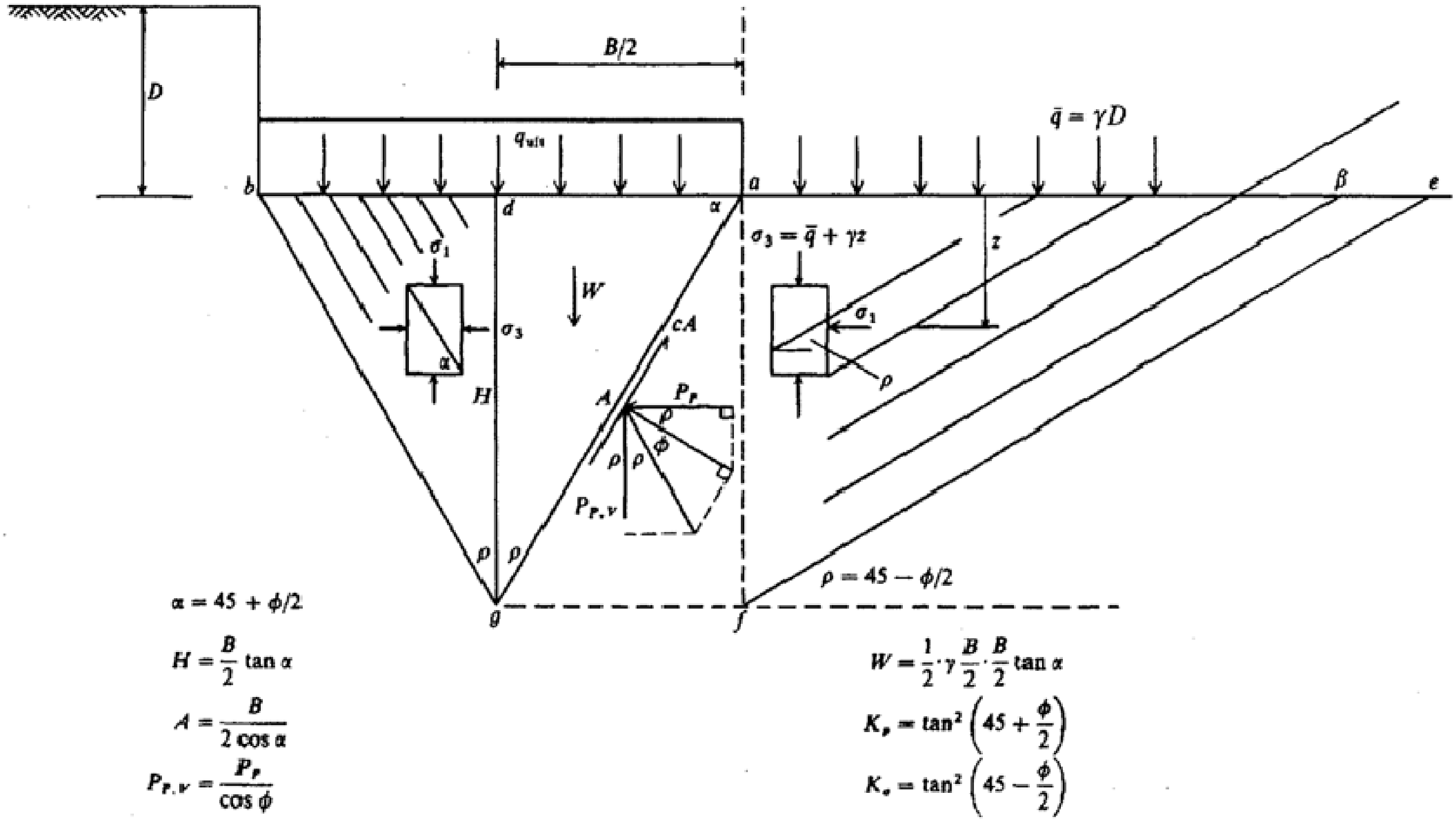


Fig 2.16: Simplified bearing capacity for a $\varphi - c$ soil.

It may be necessary to place footings on stratified deposits where the thickness of the top stratum from the base of the footing $d_1$is less than the *H* distance computed as in Fig 2.16. In this case the rupture zone will extend into the lower layer(s) depending on their thickness and require some modification of q_{ult}. There are three general cases of the footing on a layered soil as follows:

Case 1. Footing on layered clays (all $\varphi = 0$) as in Fig 2.17(a).

a. Top layer weaker than lower layer *($c_1 < c_2$)*

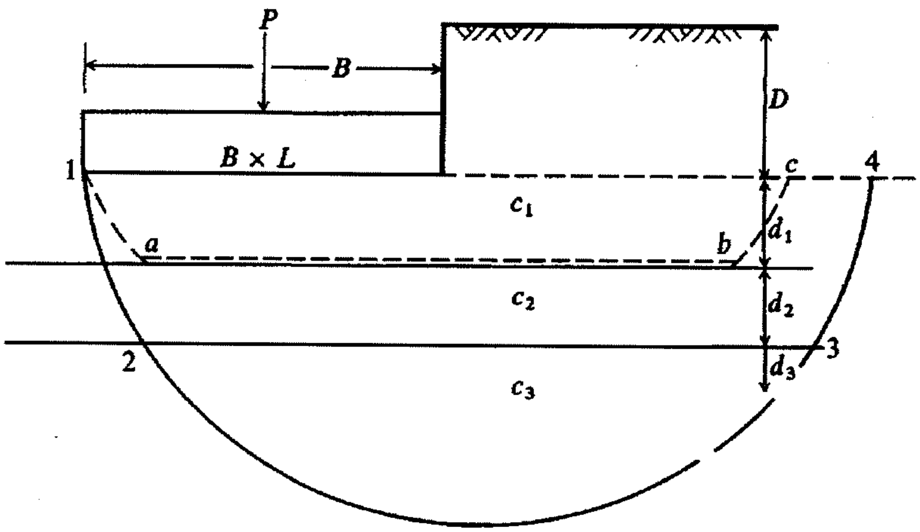
b. Top layer stronger than lower layer *($c_1 > c_2$)*

Case 2. Footing on layered φ -c soils with *a, b* same as case 1.

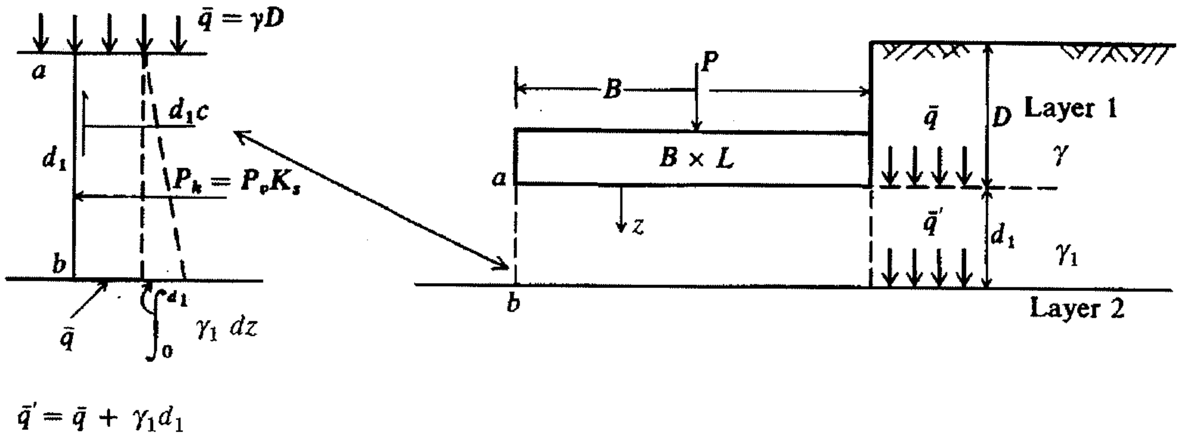
Case 3. Footing on layered sand and clay soils as in Fig. 2.17(b).

a. Sand overlying clay

b. Clay overlying sand



(*a*) **Footing on layered clay soil. For very soft c_1 failure may occur along sliding block 1*abc* and not a circular arc and reduce N_c to a value less than 5.14.**



(b)

Fig 2.17: Footings on layered soil.

Experimental work to establish methods to obtain q_{ult} for these three cases seems to be based mostly on models—often with B <75 mm. Several analytical methods exist as well, and apparently the first was that of Button (1953), who used a circular arc to search for an approximate minimum, which was found (for the trial circles all in the top layer) to give $N_c = 5.5$

The use of trial circular arcs can be readily programmed for a computer for two or three layers using s_u for the layers. Note that in most cases the layer s_u will be determined form q_u tests,

so the circle method will give reasonably reliable results. It is suggested that circular arcs be limited to cases where the strength ratio $C_R = c_2/c_1$ of the
top two layers is on the order of

$$0.6 < C_R \leq 1.3$$

Where C_R is much out of this range there is a large difference in the shear strengths of the two layers, and one might obtain N_c using a method given by Brown and Meyerhof (1969) based on model tests as follows:

For $C_R \leq 1$

$$N_{c,s} = \frac{1.5d_1}{B} + 5.14C_R \leq 5.14 \quad \text{(for strip footing)}$$

……Eq 2.9

For a circular base with B = diameter

$$N_{c,r} = \frac{3.0d_1}{B} + 6.05C_R \leq 6.05 \quad \text{(for round base)}$$

……Eq 2.9a

When $C_R > 0.7$ reduce the foregoing $N_{c,i}$ by 10 percent.

For $C_R > 1$ compute:

$$N_{1,s} = 4.14 + \frac{0.5B}{d_1} \text{(strip)}$$

……Eq 2.9b

$$N_{2,s} = 4.14 + \frac{1.1B}{d_1}$$

……Eq 2.9c

$$N_{1,r} = 5.05 + \frac{0.33B}{d_1} \text{(round base)}$$

……Eq 2.9d

$$N_{2,r} = 5.05 + \frac{0.66B}{d_1}$$

……Eq 2.9e

In the case of $C_R > 1$ we compute both N_1,i and N_2,i depending on whether the base is rectangular or round and then compute an averaged value of $N_{c,i}$ as

$$N_{c,i} = \frac{N_{1,i} \times N'_{2,i}}{N_{1,i} + N_{2,i}} \times 2$$

……Eq 2.9f

When the top layer is very soft with a small d_1/B ratio, one should give consideration either to placing the footing deeper onto the stiff clay or to using some kind of soil improvement method. Model tests indicate that when the top layer is very soft it tends to squeeze out from beneath the base and when it is stiff it tends to "punch" into the lower softer layer [Meyerhof and Brown (1967)].

2.4 Eccentric and Inclined loading

It has so far been assumed that the loads on a foundation were vertical. However, footings may be subjected to inclined loads and/or eccentric loading. In this event symmetrical bearing capacity mechanisms are not appropriate and the bearing capacity is reduced. The effect of both load eccentricity and inclination is to reduce the allowable bearing pressure quite significantly; therefore, efforts should be made, if possible, to change the structural layout to avoid or minimize these effects.

2.4.1 Eccentric Loading

In some situations it may be possible to:

1) Revise the structural layout to avoid eccentric loading, or
2) If the load condition is not variable, it may be possible to make the centre of area of the footing coincident with the load.

If these remedies are not possible then the reduced bearing capacity must be determined. One commonly used technique is presented in the following discussion.

2.4.1.1 Contact Pressure Distribution

If a rectangular rigid footing of width, B, and length, L, as shown in Fig 2.18(a), is subjected to a vertical load, P, located with eccentricities e_b and e_L from the centre line in the x and y directions, respectively, the contact pressure distribution can be determined as:

$$p = \frac{P}{A} + \frac{M_x\, y}{I_x} + \frac{M_y x}{I_y}$$ …Eq2.10

where, M_x =Pe$_L$

M_y =Pe$_B$

$I_x = \frac{BL^3}{12}$ for contact over the full area

$I_y = \frac{LB^3}{12}$ for contact over the full area

and '*x* ' and '*y* ' are distances from x and y axis to the point for which the pressure is required.

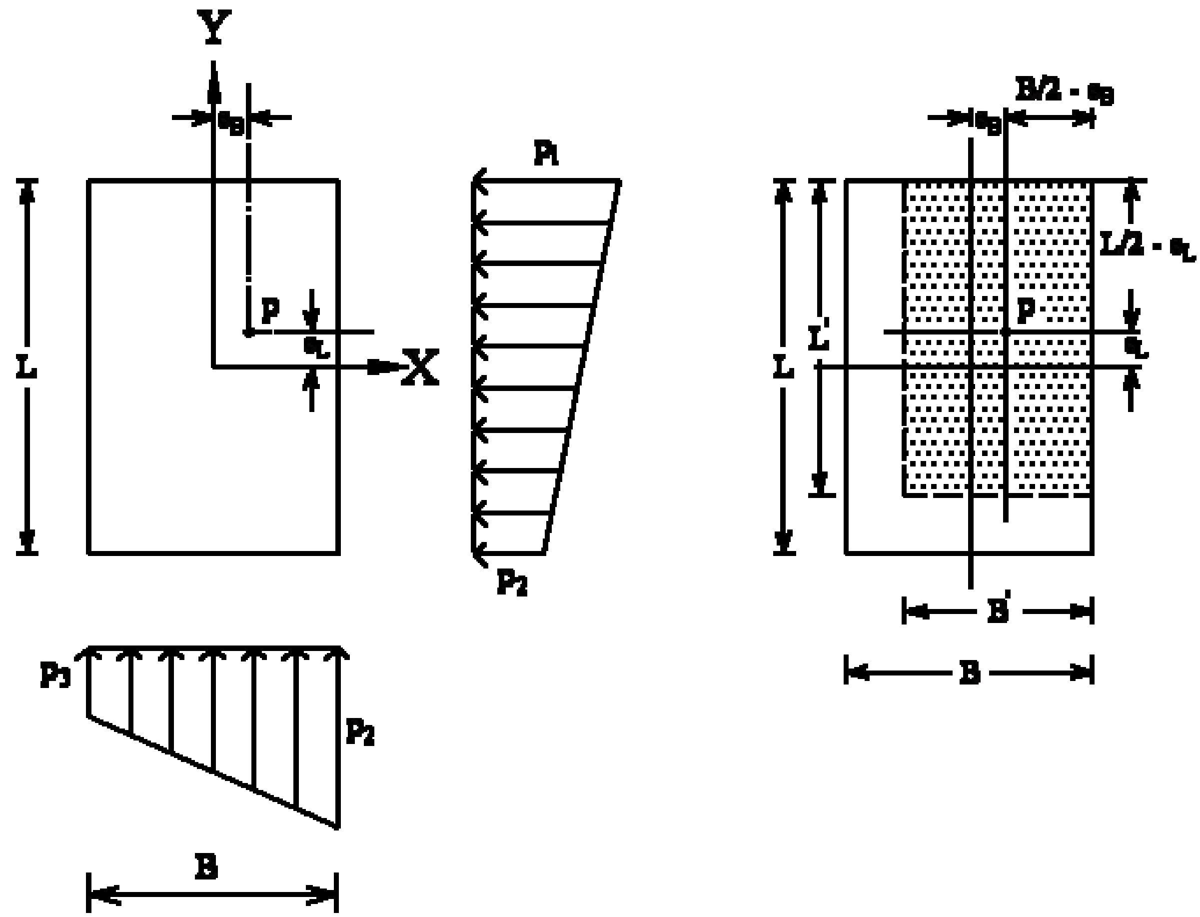
Provided $e_b < \frac{B}{6}$ and $e_L < \frac{L}{6}$ contact over full area will be maintained and footings should normally be proportioned to ensure that this is the case. In this event Eq.2.10 can be expressed as,

$$p = \frac{P}{A}\left[1 + \frac{6e_L}{L} + \frac{6e_B}{B}\right]$$ ……Eq2.11

The pressure distribution obtained from Eq 2.11 is shown qualitatively on Fig 2.18(a)

2.4.1.2 Concept of Useful Width

To determine the ultimate or allowable bearing capacity of an eccentrically loaded footing, the concept of useful width was introduced by Meyerhof (1953) and Hansen (1970). By this concept, the rectangular portion of the footing which is symmetrical about the load is considered to be useful and the other portion is simply assumed superfluous for the convenience of computation. If the eccentricities are e_b and e_L as shown if Fig 2.18(b), the useful widths are (B - 2 e_b) and (L – 2 e_L). The equivalent area (B - 2 e_b) x (L – 2 e_L) is considered to be subjected to a central load for determination of bearing capacity.



(a) Contact pressure distribution where e_L < L/6 and e_B < B/6

(b) Useful widths for determination of bearing capacity of eccentrically loaded footing

Fig 2.18: Eccentrically loaded footing

Thus net ultimate bearing capacity of this hypothetical footing can be expressed using the equivalent width B′ as,

$q_{net\,ult} = C'N_c + \gamma D_f(N_q - 1) + 0.5\gamma B'N_\gamma$ (strip) …….Eq 2.12

$q_{net\,ult} = C'N_c + \gamma D_f(N_q - 1) + 0.4\gamma B'N_\gamma$ (rectangular) …….Eq 2.13

For particular case of short-term conditions of saturated clay (C_u , φ = 0)

$q_{net\,ult} = C_u N_c$ …….Eq 2.14

where, N_c must be determined from Fig 2.18 using $\frac{D_f}{B'}$ and $\frac{B'}{L}$

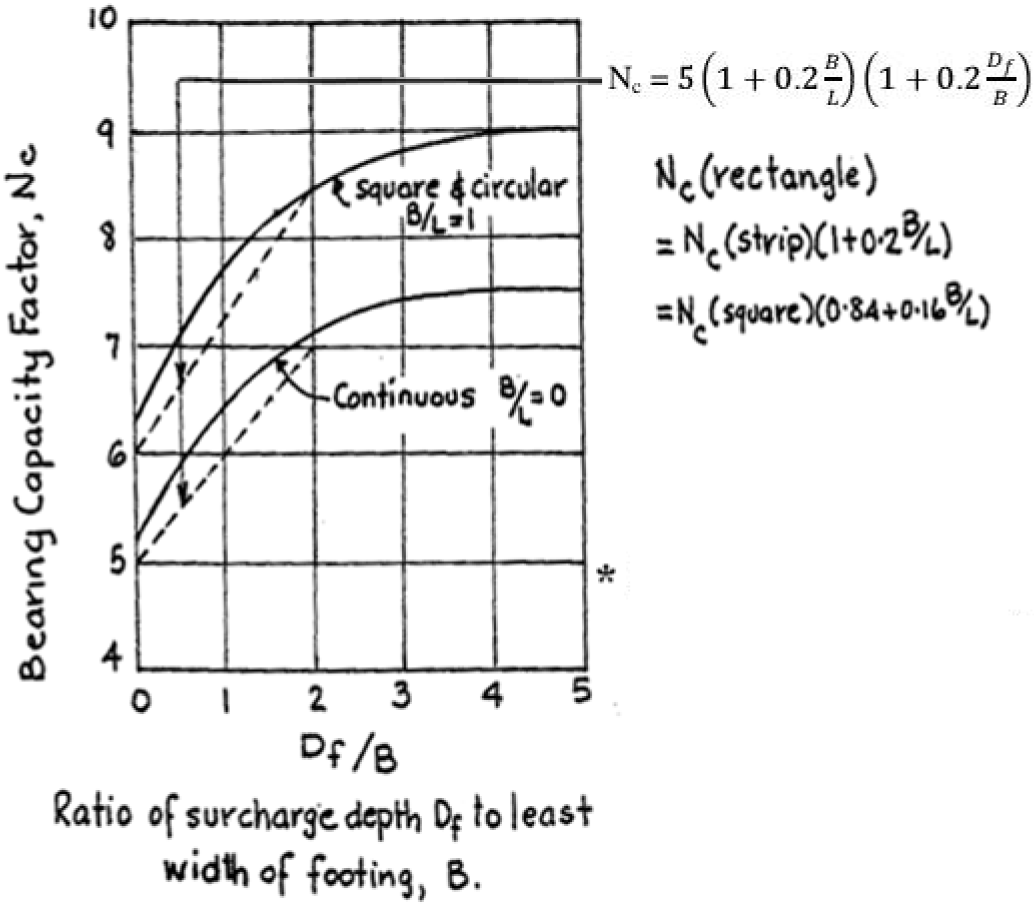


Fig 2.19: Bearing capacity factors for foundations on clay under φ = 0 conditions (after Skempton, 1951)

2.4.2 Inclined Loading

The effect of inclined loading on bearing capacity may be taken into account by means of inclination factors.

Recommendations regarding these factors have been given by Meyerhof (1953) and Hansen (1970) and others and these are summarized in Bowles (1974). The following values are those due to Meyerhof (1953). If the angle of inclination of the resultant load, P, to the vertical is 'α', then the bearing capacity factors N_γ , N_c and N_q should be multiplied respectively by the following factors.

$$i_\gamma = \left(1 - \frac{\alpha}{\varphi}\right)^2$$ …..Eq 2.15a

$$i_c = i_q = \left(1 - \frac{\alpha}{90}\right)^2$$ …..Eq 2.15b

The bearing capacity equations for a continuous footing on cohesionless and cohesive soils become, respectively:

$$q_{net\ ult} = 0.5\gamma B N_\gamma i_\gamma + \gamma D_f\left(N_q i_q - 1\right)$$ …….Eq 2.15c

$$q_{net\ ult} = C_u N_c i_c$$ …….Eq 2.15d

For footing shapes other than a continuous strip, appropriate shape factors may be introduced.

Eq 2.15c and Fig 2.15d show that for a given value of α the i_γ term may become significantly less than the i_q term. In this event the ultimate bearing capacity of footings (with inclined loading) may be mostly due to the effect of the surcharge. Consequently the designer should be careful to assess the effective surcharge pressure conservatively.

2.4.3 Combined Eccentric and Inclined Loading

When a footing is subjected to a load Q which are both eccentric and inclined, corrections must be applied for both of these conditions. Eq 2.16a and Eq 2.16b are to be used for cohesionless and cohesive soils respectively.

$$q_{net\ ult} = 0.5\gamma B' N_\gamma i_\gamma + \gamma D_f\left(N_q i_q - 1\right)$$ …….Eq 2.16a

$$q_{net\ ult} = C_u N_c i_c$$ …….Eq 2.16b

In Eq. 2.16b the bearing capacity factor, N_c , is determined using the useful width, B' , and if appropriate, the useful length, L'.

2.4.4 Settlement under Eccentric and Inclined Loading

Some indication of the average settlement of a footing under eccentric and/or inclined loading may be gained by assuming that the vertical component of the load, Q, acts uniformly over the equivalent area, (B - 2 e_b) x (L - 2 e_L). Settlement calculations should then proceed. This approach will not, however, provide any estimate of the rotation of the footing induced by the eccentricity and the horizontal component of the load, and should therefore be used with caution.

2.5 Bearing Capacity of Footings on Slopes

There are occasions where structures are required to be built on slopes or near the edges of slopes. Since full formations of shear zones under ultimate loading conditions are not possible on the sides close to the slopes or edges, the supporting capacity of soil on that side get considerably reduced. Meyerhof (1957) extended his theories to include the effect of slopes on the stability of foundations.

Fig 2.20 shows a section of a foundation with the failure surfaces under ultimate loading condition. The stability of the foundation depends on the distance $\bar{b}$of the top edge of the slope from the face of the foundation.

The ultimate bearing capacity equation for a strip footing may be expressed as (Meyerhof, 1957)

$$q_u = cN_{cq} + \frac{1}{2}\gamma B N_{\gamma q}$$

…….Eg 2.17a

The upper limit of the bearing capacity of a foundation in a purely cohesive soil may be estimated from

$$q_u = cN_{cq} + \gamma D_f$$

…….Eg 2.17b

The resultant bearing capacity factors Ncq and $N_{\gamma q}$depend on the distance $\bar{b}$, β, φ and the D_f / B ratio. These bearing capacity factors are given in Fig 2.20(a) and Fig 2.20(b) for strip foundation in purely cohesive and cohesionless soils respectively. It can be seen from the figures 2.20 (a) and 2.20 (b) that the bearing capacity factors increase with an increase of the distance $\bar{b}$ .Beyond a distance of about 2 to 6 times the foundation width *B,* the bearing capacity is independent of the inclination of the slope, and becomes the same as that of a foundation on an extensive horizontal surface.

For a surcharge over the entire horizontal top surface of a slope, a solution of the slope stability has been obtained on the basis of dimensionless parameters called the stability number N_s , expressed as

$$N_s = \frac{c}{\gamma H}$$

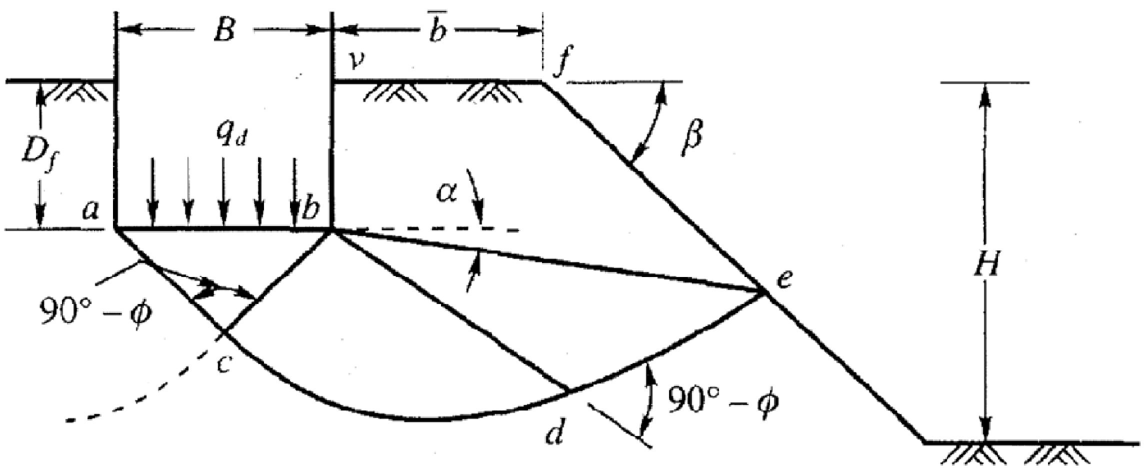


Fig 2.20: Bearing capacity of a strip footing on top of a slope (Meyerhof, 1957)

The bearing capacity of a foundation on purely cohesive soil of great depth can be represented by Eq. 2.17b where the N_{cq} factor depends on $\bar{b}$as well as β, and the stability number N_s. This bearing capacity factor, which is given in the lower parts of Fig 2.17a, decrease considerably with greater height and to a smaller extent with the inclination of the slope. For a given height and slope angle, the bearing capacity factor increases with an increase in $\bar{b}$, and beyond a distance of about 2 to 4 times the height of the slope, the bearing capacity is independent of the slope angle. Figure 2.21(a) shows that the bearing capacity of foundations on top of a slope is governed by foundation failure for small slope height (N_s approaching infinity) and by overall slope failure for greater heights.

The influence of ground water and tension cracks (in purely cohesive soils) should also be taken into account in the study of the overall stability of the foundation. Meyerhof (1957) has not supported his theory with any practical examples of failure as any published data were not available for this purpose.

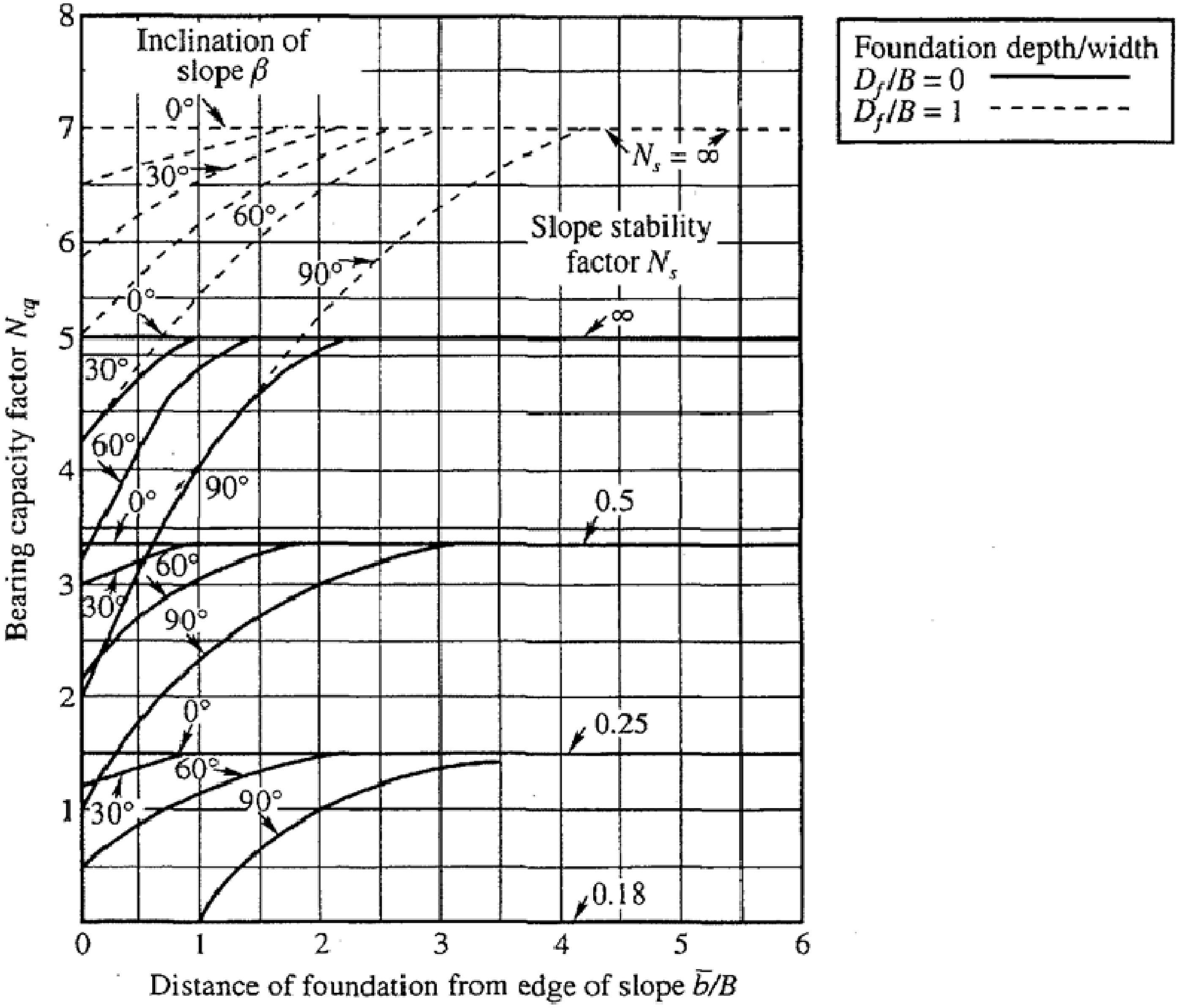


Fig 2.21(a): Bearing capacity factors for strip foundation on top of slope of purely cohesive material (Meyerhof, 1957)

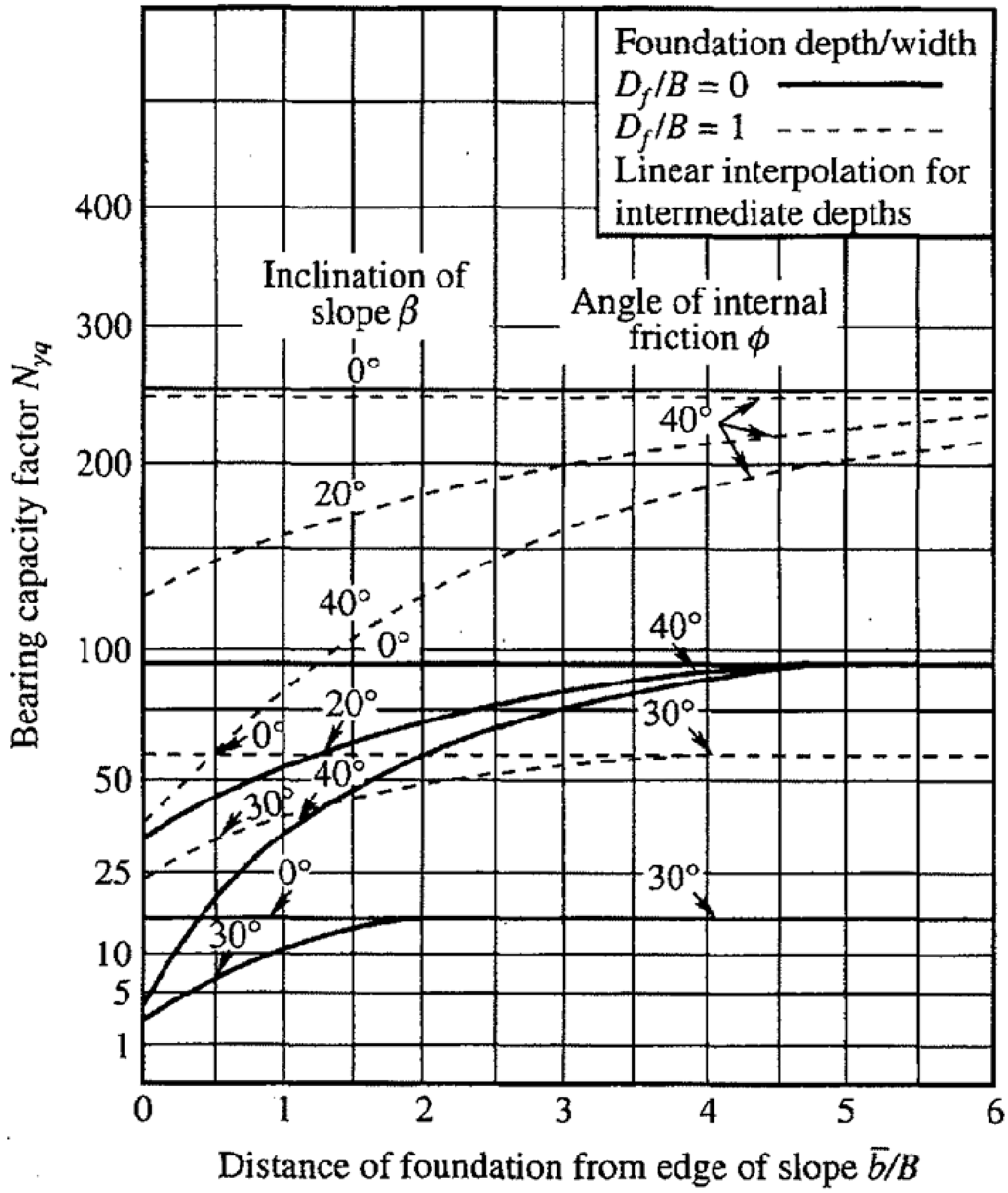


Fig 2.21(b): Bearing capacity factors for strip foundation on top of slope of cohesionless material (Meyerhof, 1957)

2.6 Foundation Settlement

2.6.1 Calculation of Settlement: General comments

The settlement of a structure is the result of the deformation of the supporting soil, and may result from:

(1) Elastic deformation of the foundation soil,
(2) Volume changes in the soil due to reduction of the water content (consolidation), or the air content (compaction)
(3) Plastic deformation of the soil due to loading at relatively high stress levels,
(4) Other factors such as long-term creep effects in cohesive soils, effect of vibrations on cohesionless soils, sink-hole formation or mining subsidence.

2.6.2 Permissible Settlement

The maximum settlement of a structure is of great concern because appearance, access and services attached to the building may be affected. However, if one part of the building settles more than another, the structural frame can be distorted and the effects are likely to be more serious than if the settlements were relatively uniform. For conventional foundations using isolated footings some differential settlement will usually occur because of the natural variability of the soil compressibility across the site even if the total settlements are calculated to be uniform. If a raft foundation is provided then the structural rigidity of the foundation assists in minimizing these differential settlements.

It is difficult to provide definite criteria for the allowable settlement of structures since, in some countries, structural settlements of several meters have occurred and been tolerated by the structure. However these occurrences should be considered as special cases. Special care should also be taken with structures with high aspect ratio such as towers or chimneys since non-uniform settlement of the base may result in excessive tilting.

Differential settlements are frequently controlled indirectly by limiting the design total settlement. For conventional buildings it is usual to limit the total settlement so as not to exceed the following approximate listed in Table 2.2.

Table 2.2 Permissible differential settlements and tilt for shallow foundations (reproduced with permission from the Bureau of Indian Standards)

TABLE 1 PERMISSIBLE DIFFERENTIAL SETTLEMENTS AND TILT (ANGULAR DISTORTION) FOR SHALLOW FOUNDATION IN SOILS

(*Clause* 16.3.4)

Sl No.	Type of Structure	Isolated Foundations: Sand and Hard Clay			Isolated Foundations: Plastic Clay			Raft Foundations: Sand and Hard Clay			Raft Foundations: Plastic Clay		
		Maximum settlement	Differential settlement	Angular distortion	Maximum settlement	Differential settlement	Angular distortion	Maximum settlement	Differential settlement	Angular distortion	Maximum settlement	Differential settlement	Angular distortion
		mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
i)	For steel structure	50	·003 3L	1/300	50	·003 3L	1/300	75	·003 3L	1/300	100	·003 3L	1/300
ii)	For reinforced concrete structures	50	·001 5L	1/666	75	·001 5L	1/666	75	·002 1L	1/500	100	·002L	1/500
iii)	For multistoreyed buildings												
	a) RC or steel framed buildings with panel walls	60	·002L	1/500	75	·002L	1/500	75	0·002 5L	1/400	125	0·003 3L	1/300
	b) For load bearing walls												
	1) L/H = 2+	60	·000 2L	1/5 000	60	·000 2L	1/5 000	Not likely to be encountered					
	2) L/H = 7+	60	·000 4L	1/2 500	60	·000 4L	1/2 500	Not likely to be encountered					
iv)	For water towers and silos	50	·001 5L	1/666	75	·001 5L	1/666	100	·002 5L	1/400	125	·002 5L	1/400

NOTE — The values given in the table may be taken only as a guide and the permissible total settlement/different settlement and tilt (angular distortion) in each case should be decided as per requirements of the designer.

L denotes the length of deflected part of wall/raft or centre-to-centre distance between columns.

H denotes the height of wall from foundation footing.

+For intermediate ratios of L/H, the values can be interpolated.

19

These values are only a rough guide to maximum acceptable settlement values. Normally footings on sand would be restricted to design value of 25 mm (1 inch).

The allowable differential settlement is equally difficult to specify since it is influenced by such factors as:

1) The flexibility of the structural frame and architectural façade.
2) The ductility of the construction materials.
3) The time interval during which settlement occurs. If the rate of settlement is slow, most structures can themselves deform plastically and better accommodate to the deformation caused by differential settlement.

Approximate limitations to the magnitude of angular distortion due to differential settlements commonly quoted for various classes of structures are provided in Table 2.3 and Fig 2.22.

Table 2.3: Angular distortion limits (reproduced with permission from the Bureau of Indian Standards)

Rotation limits for structure	
Relative rotation	Type of limit and structure
$\frac{1}{740}$	• Limit where difficulties with machinery sensitive to settlements are to be feared.
$\frac{1}{600}$	• Limit of danger for frames with diagonals.
$\frac{1}{500}$	• Safe limit for buildings where cracking is not permissible.
$\frac{1}{300}$	• Limit where first cracking in panel walls is to be expected. • Limit where difficulties with overhead cranes are to be expected.
$\frac{1}{250}$	• Limit where tilting of high, rigid buildings might become visible.
$\frac{1}{100}$	• Considerable cracking in panel walls and brick walls. • Safe limit for flexible brick walls. $h/l < \frac{1}{4}$ • Limit where structural damage of general buildings is to be feared.

Design limits on differential settlement are frequently set in totally unrealistic terms. In fact, each structure must be considered individually, and the values given above should be used only as a guide.

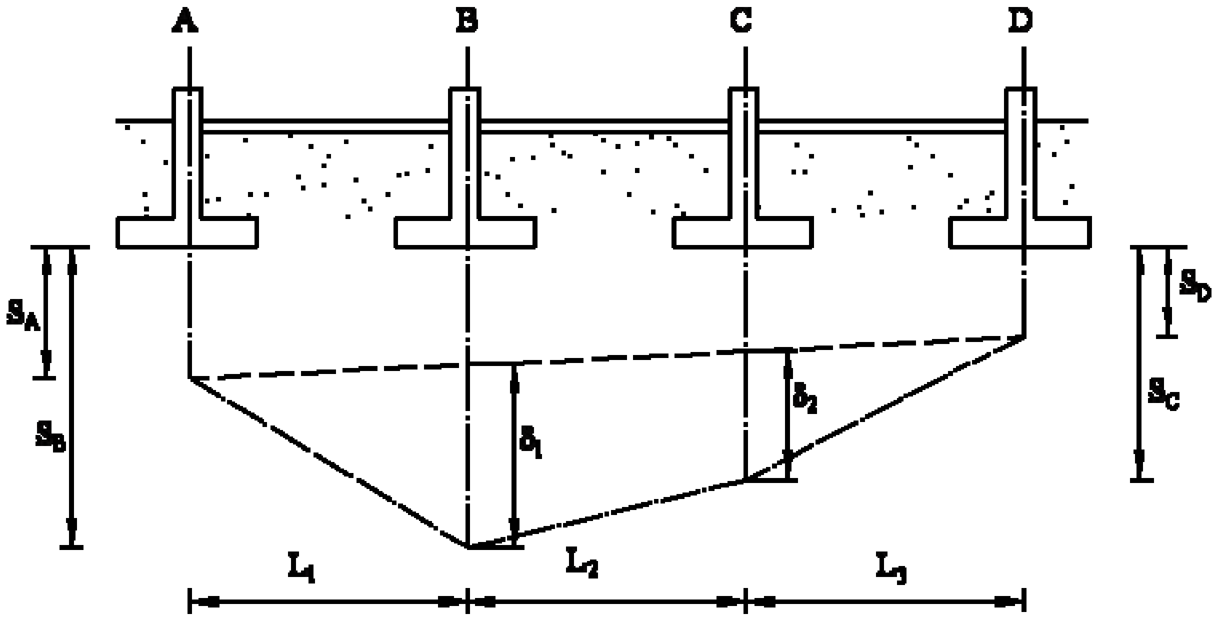


Fig 2.22: Typical section through structure for Differential Settlement and Angular Distortion Related to Building Performance

2.6.3 Shallow Foundations on Clay: Settlement

In addition to bearing capacity, the consolidation settlement of footings on clay should be evaluated. The settlement estimates are based on one-dimensional consolidation theory and oedometer test data.

In practice, footings are dimensionally finite, and therefore, some lateral strains occur during loading and the estimated 'oedometer' settlement may be in error. It is common procedure to apply a correction which makes an allowance for footing geometry and the geological history of the clay deposits. Hence,

$$\rho(field) = \mu \times \rho(oedometer)$$

where, μ is a correction factor which may be taken from the Fig 2.23.

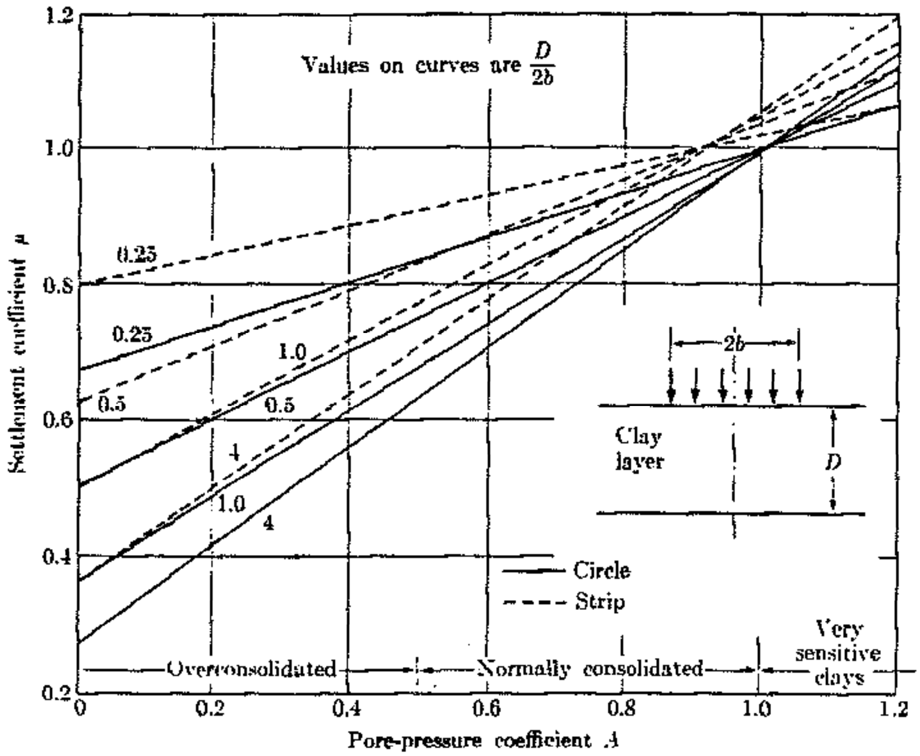


Fig 2.23: Settlement coefficient versus pore-pressure coefficient for circular and strip footings. [Reproduced with permission from Scott (1963)]

Long term creep settlement or secondary compression may occur after excess pore-water pressures have dissipated. The relative importance of this settlement varies with the type of soil, the ratio of the load increment to initial stress and the thickness of the soil deposit.

2.6.4 Components of Total Settlement

The total settlement of a foundation comprises three parts as follows

$$S = Se + Sc + Ss$$

where,

S = total settlement

S_e = elastic or immediate settlement

S_c = consolidation settlement

S_s = secondary settlement

Immediate settlement, *Se,* is that part of the total settlement, S, which is supposed to take place during the application of loading. The consolidation settlement is that part which is due to the expulsion of pore water from the voids and is time-dependent settlement. Secondary settlement normally starts with the completion of the consolidation. It means, during the stage of this settlement, the pore water pressure is zero and the settlement is only due to the distortion of the soil skeleton. Footings founded in cohesionless soils reach almost the final settlement, S, during the construction stage itself due to the high permeability of soil. The water in the voids is expelled simultaneously with the application of load and as such the immediate and consolidation settlements in such soils are rolled into one. In cohesive soils under saturated conditions, there is no change in the water content during the stage of immediate settlement. The soil mass is deformed without any change in volume soon after the application of the load. This is due to the low permeability of the soil. With the advancement of time there will be gradual expulsion of water under the imposed excess load. The time required for the complete expulsion of water and to reach zero water pressure may be several years depending upon the permeability of the soil. Consolidation settlement may take many years to reach its final stage. Secondary settlement is supposed to take place after the completion of the consolidation settlement, though in some of the organic soils there will be overlapping of the two settlements to a certain extent.

Immediate settlements of cohesive soils and the total settlement of cohesionless soils may be estimated from elastic theory. The stresses and displacements depend on the stress-strain characteristics of the underlying soil. A realistic analysis is difficult because these characteristics are nonlinear. Results from the theory of elasticity are generally used in practice, it being assumed that the soil is homogeneous and isotropic and there is a linear relationship between stress and strain. A linear stress-strain relationship is approximately true when the stress levels are low relative to the failure values. The use of elastic theory clearly involves considerable simplification of the real soil. Some of the results from elastic theory require knowledge of Young's modulus (E_s), here called the compression or deformation modulus, E_d, and Poisson's ratio, μ, for the soil.

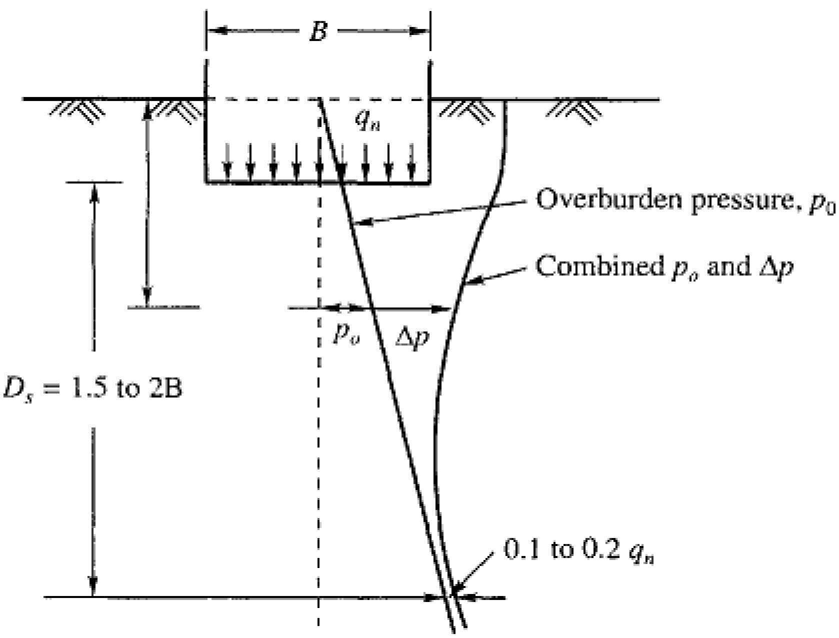


Figure 2.24Overburden pressure and vertical stress distribution

2.6.5 Seat of Settlement

Footings founded at a depth D_f below the surface settle under the imposed loads due to the compressibility characteristics of the subsoil. The depth through which the soil is compressed depends upon the distribution of effective vertical pressure p'_o of the overburden and the vertical induced stress Δp resulting from the net foundation pressure q_n as shown in Fig. 2.24.

In the case of deep compressible soils, the lowest level considered in the settlement analysis is the point where the vertical induced stress Δp is of the order of 0.1 to $0.2q_n$, where q_n is the net pressure at the base of foundation. This depth works out to about 1.5 to 2 times the width of the footing. The soil lying within this depth gets compressed due to the imposed foundation pressure and causes more than 80 percent of the settlement of the structure. This depth D_S is called the *zone of significant stress.* If the thickness of this zone is more than 3 m, the steps to be followed in the settlement analysis are

1) Divide the zone of significant stress into layers of thickness not exceeding 3 m,
2) Determine the effective overburden pressure p'_o at the center of each layer,
3) Determine the increase in vertical stress Δp due to foundation pressure q at the center of each layer along the center line of the footing by the theory of elasticity,

4) Determine the average modulus of elasticity and other soil parameters for each of the layers.
5) Compute settlement of each layer and add to get total settlement.

2.6.6 Settlement of Foundations on Cohesionless Soils

The techniques developed by Schmertmann (1970) and provided in Craig (1983), are available for situations where the designer wishes to assess the foundation settlement as accurately as possible; however, in this section only two methods of calculating settlement of foundations on sand will be discussed as listed below.

(1) For isolated footings, empirical design relationships or charts are available which have been developed from field observations of actual footings. The relative density of the in-situ deposits is required to be known and in practice this is inferred indirectly from the results of Standard Penetration Test or Static Cone Penetration Test.

(2) For those cases where the influence of the applied loads extends to a significant depth in compressible granular deposits a modified form of the theory of consolidation is frequently employed, using various modifications of the equation

$$S = \frac{H\ C_c}{1 +\ e_0} \log \frac{(\sigma'_{vo} +\ \Delta\sigma_v)}{\sigma'_{vo}}$$

in which the in-situ compressibility of the soil, C_c, is inferred either from the standard penetration test or preferably from static cone penetration tests.

For the approach given as item (1) above, Peck et al. (1974) indicated that typical load-settlement relationships exist for footings of different widths on the surface of a homogeneous sand deposit as illustrated in Fig 2.25

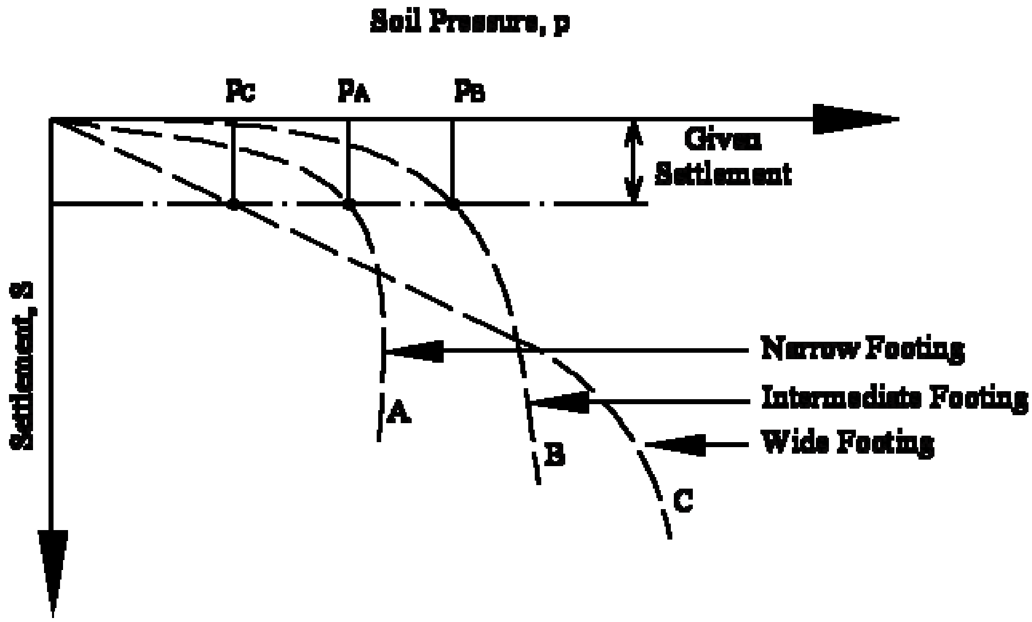


Fig 2.25: Load-settlement curves for footings of increasing widths A, B and C and constant $\frac{D_f}{B}$ ratio on sand of uniform relative density.

These curves show that at low stress levels there is a linear relationship between pressure and settlement. However as the applied pressure is increased, part of the soil below the footing starts to yield (see figure 2.25), the load settlement relation becomes curved and if the soil pressure becomes sufficiently high, complete failure of the underlying soil can occur.

The maximum pressure that a given cohesionless soil can withstand increases with the size of the footing. Consequently if, as shown in Fig 2.24, three footings of increasing size A, B and C, are loaded so as to achieve the same settlement then the required pressures p_a, p_b, p_c will vary as shown. The large footing will have produced only elastic stress levels in the soil and the settlement will be essentially recoverable. On the other hand, for the small footing, p_a is close to the ultimate failure pressure and the settlement in this case will be predominantly plastic or irrecoverable. Footing (A) will also be on the verge of complete collapse. The intermediate size footing (B) will have some plastic yielding of the underlying soil but will not yet have reached complete failure.

The foregoing discussion depends, as well, on the amount of settlement being permitted. For these reasons the relationship between design settlement, footing size and allowable bearing pressure is quite complex. For a design settlement value of 25mm (1 in.) Peck et al (1974) concluded from analysis of actual footings that the pressure-settlement relationship for various footing widths, could be approximately expressed independently of footing size as,

$$p = 11.0 \times N_{corr}(kPa) \cong q_{net\ allow} \qquad \text{.....Eq 2.18}$$

Where,

p = net vertical pressure applied by the footing for which the settlement should not exceed 25mm provided the ground water table is located below a depth B below the base of the footing (B being the least width of the footing)

N_{corr} = design value of the corrected SPT blow count.

Equation 2.18 is purely empirical and based largely on field observations. Consequently its use should be limited to standard design situations similar to the field examples from which the original data was obtained. With this restriction and despite the fact that the SPT is subject to many errors, Eq. 2.18 provides a satisfactory design tool when used with sound judgment.

For those design situations where the ground water table is located above a depth B below the base of the footing the compressibility of the soil is increased by the reduction in effective stresses due to water. It is not easy to quantify the increase in settlement due to a rise in ground water level. It is usual, however, to simply multiply the pressure 'p' given by the Eq.2.18 by a water table correction factor 'C_w'. Peck et al. (1974) suggested the following empirical relationship for C_w.

$$C_w = 0.5\left(1 + \frac{D_w}{D_f + B}\right) \qquad \text{......Eq. 2.19}$$

where, D_w is the depth of water table below the ground surface

B is the least width of the footing and,

D_f is depth of the foundation.

For this general case Eq. 2.18 then becomes,

$$p = 11.0 \times C_w \times N_{corr}(kPa) \qquad \text{.....Eq. 2.20}$$

It should be emphasized that the settlement criterion of Eq.2.20 was obtained from field measurements of actual footings. A considerable amount of scatter of field data can always be expected and the relationship given by Eq.2.20 was selected to ensure that the footing settlement would not exceed 25mm. The settlement of any one footing could be much less.

On this basis, then, it can be expected that the differential settlement between any two adjacent columns would be limited to about 20mm (0.75 ins.). This value would provide an angular distortion of 0.0033 if the columns were spaced approximately 6m apart. This is a commonly used design value.

If a footing is to be designed for an allowable settlement greater than the standard value of 25mm (1 in.) then the allowable bearing pressure may be increased assuming a linear relationship between pressure and settlement. However, the F.S on bearing capacity may govern the design and should be checked. This procedure is based on the premise that provided a FS of at least 2 is maintained on bearing capacity, the stresses below the footing remain essentially elastic and provide a linear relationship between settlement and applied pressure.

2.6.7 Settlement of Foundations on Saturated Cohesive Soil

Most of the settlement of foundations on saturated cohesive soils is usually due to consolidation and associated dissipation of excess pore-water pressure. The method of calculation of such settlement using the results of consolidation tests is always used for the design of important structures.

Provided the vertical stress levels remain below the preconsolidation pressure approximate estimates of long-term settlement of shallow foundations can be obtained from

$$S = \mu_0 \mu_1 q \frac{B}{E} \quad \text{........Eq.2.21}$$

where, q = net applied foundation pressure

B = width of foundation

E = Young's modulus

μ_1 = parameter which provides the influence of the shape of the loaded area and depth of the elastic material.

μ_0 = parameter to indicate the influence of the depth of embedment of the foundation load.

by choosing a value of E appropriate to long term, drained conditions. This value of E, denoted by E_s , is the inverse of the coefficient of volume compressibility, m_v. As suggested by Canadian Foundation Engineering Manual the values of E_s may be approximated from the empirical relationship,

$$E_s = m_s \sigma_p'$$

where, $\sigma_p' =$ the effective preconsolidation pressure

m_s , varies with soil type as follows

Stiff clays,	$m_s = 80$
Firm sensitive clays,	$m_s = 60$
Soft clays,	$m_s = 40$

Some relatively minor amount of settlement, not associated with the dissipation of excess pore water pressure, can occur as soon as the foundation is loaded due to lateral deformations of the foundation soil. This 'immediate' settlement occurs at constant volume of the total soil mass and can be computed using Eq. 2.21 provided again, that appropriate values of 'ν' and '*E*' are selected.

For an elastic material to be deformed at constant volume it can be shown that the value of 'ν' must be equal to 0.5. Consequently to calculate the immediate settlement of saturated clay using a total stress analysis this value of ν must be used. In addition the value of '*E*' must be either obtained from undrained laboratory tests or inferred from actual observations of similar structures in the field. The particular value of '*E*' obtained for undrained loading conditions is usually denoted as E_u. The values E_u are generally expressed in terms of the ratio $\frac{E_u}{C_u}$= constant (or $\frac{E_u}{S_u}$= constant) where C_u (or S_u) is the undrained shear strength of the soil.

The following are the approximate values of this ratio as often quoted in the literature,

$E_u = 500\ C_u$ for soft, sensitive clay

$E_u = 1000\ C_u$ for firm to stiff clays

$E_u = 1500\ C_u$ for very stiff clays.

The immediate settlement of footings on saturated clay can therefore be determined using the data shown in Fig 2.26 which has been constructed for ν = 0.5.

Long term creep settlements are usually not significant in some highly compressible or organic soils.

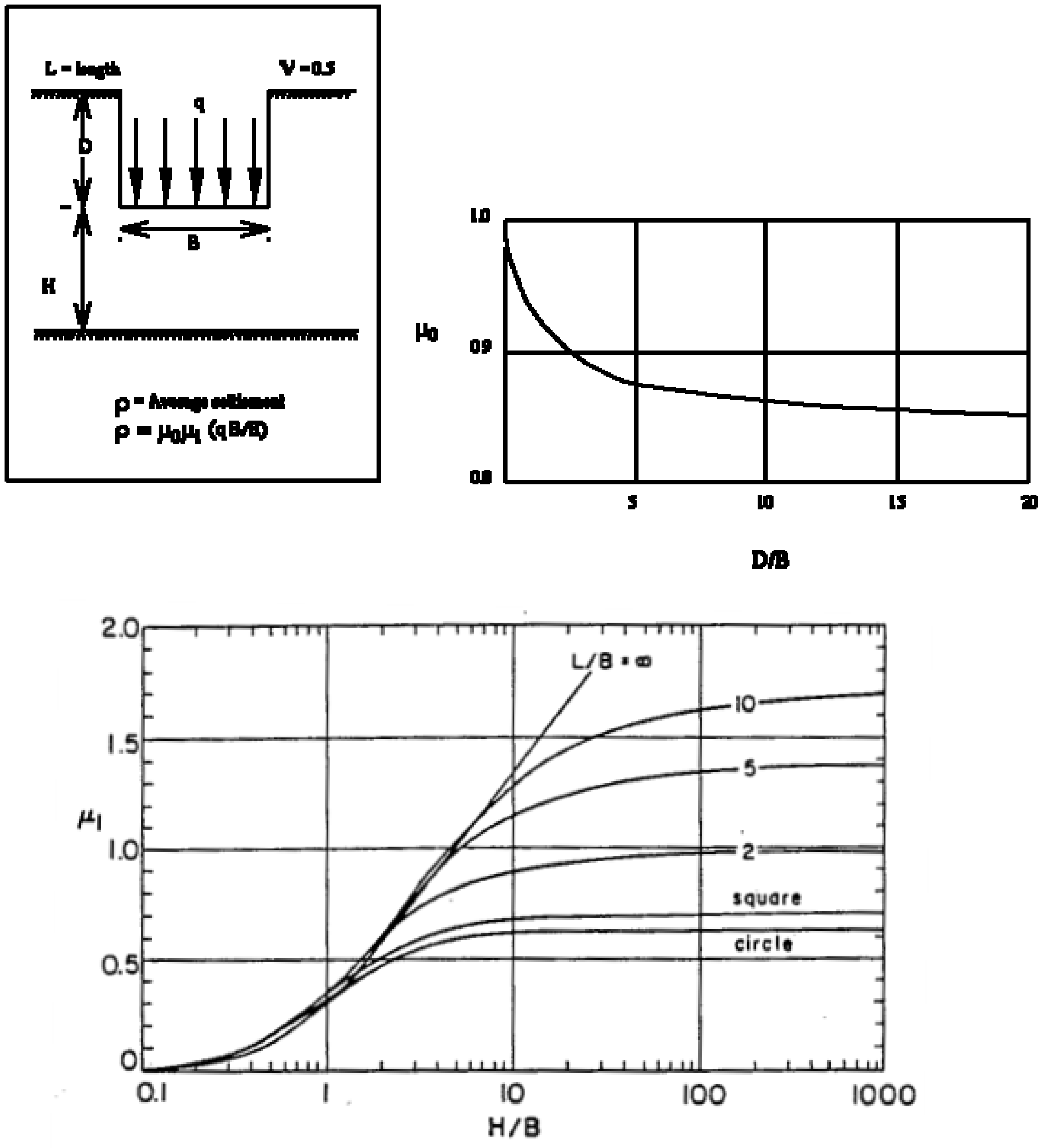


Fig 2.26: Values of μ_0 and μ_1 for settlement calculations

2.8 Design of Axially Loaded Shallow Foundations on Sand

For cohesionless soils the bearing capacity equation expressed in terms of net ultimate bearing capacity becomes,

Continuous Footing: $q_{net\,ult} = \gamma D_f(N_q - 1) + 0.5\gamma B N_\gamma$ …….Eq.2.22

Square Footing: $q_{net\,ult} = \gamma D_f(N_q - 1) + 0.4\gamma B N_\gamma$…….Eq.2.23

Circular Footing: $q_{net\,ult} = \gamma D_f (N_q - 1) + 0.6\gamma R N_\gamma$ …….Eq.2.24

Also,

$$q_{net} = \frac{q_{net\,ult}}{Factor\ of\ Safety}$$

where, Factor of safety

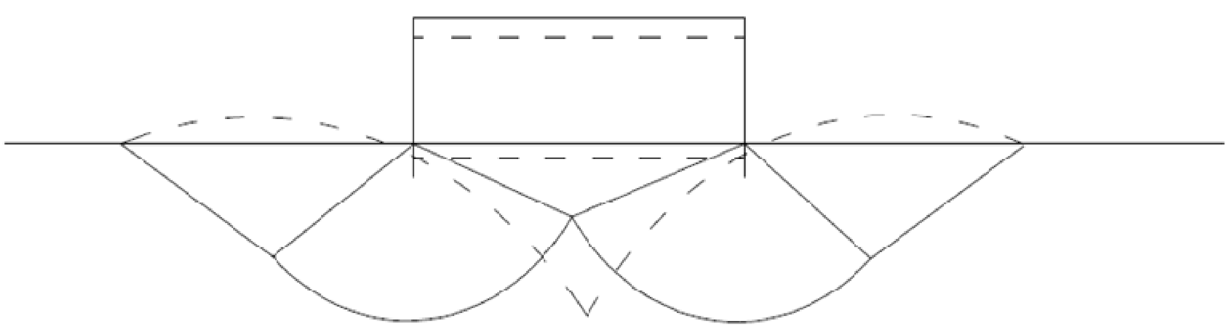
2.9 Types of Bearing Capacity Failure

Experimental investigations have led to recognition of three modes of bearing capacity failure, viz., general shear failure, local shear failure and punching shear failure.

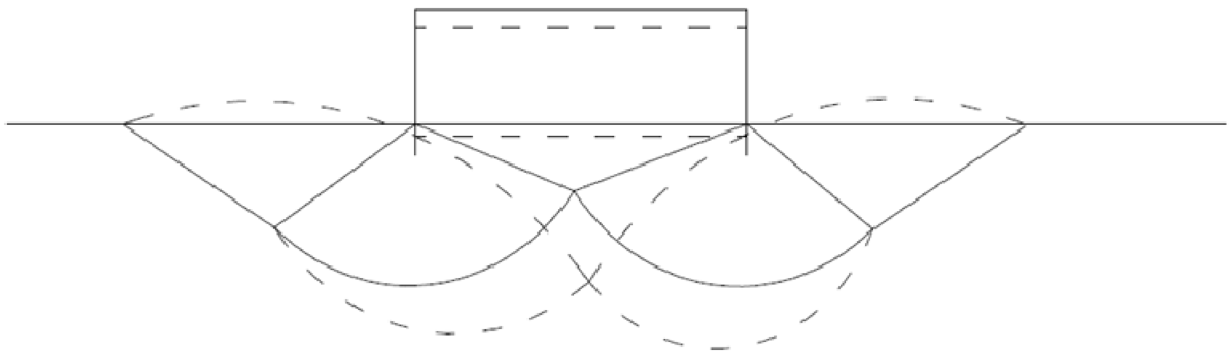
General shear failure can be expected in the case of cohesionless soils and stiff cohesive soils. In this type of failure slow downward movement of footing takes place, just before failure, with sufficient time for plastic zones to develop fully. In tests on model footings considerable bulging of soil on sides of footing has been noticed as shown in Fig 2.27(a).

Local shear failure can be expected in the case of loose cohesionless soils and soft cohesive soils. If soil is more compressible, large deformation occurs below the footing before the plastic zones are fully developed. In tests on model footings slight bulging of soil on sides of footing has been noticed as shown in Fig 2.27(b).

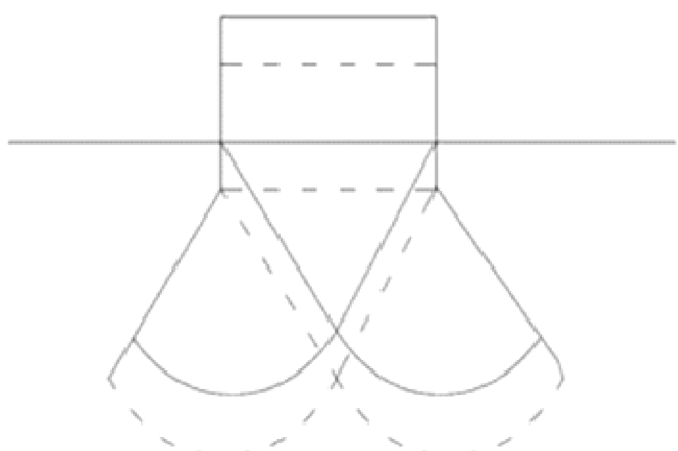
Both general shear failure and local shear failure were recognized by Terzaghi (1943). Vesic (1973) in his study on model footings on sand observed that foundations on loose sand with density index less than 35 per cent fail by deep penetration into soil without any bulging of soil on sides of footing. The plastic zones will be partially developed as shown in Fig 2.27(c). He designated this type of failure as punching shear failure.



(a)General shear failure



(b)Local shear failure



(c)Punching shear failure

Fig 2.27: Types of bearing capacity failure

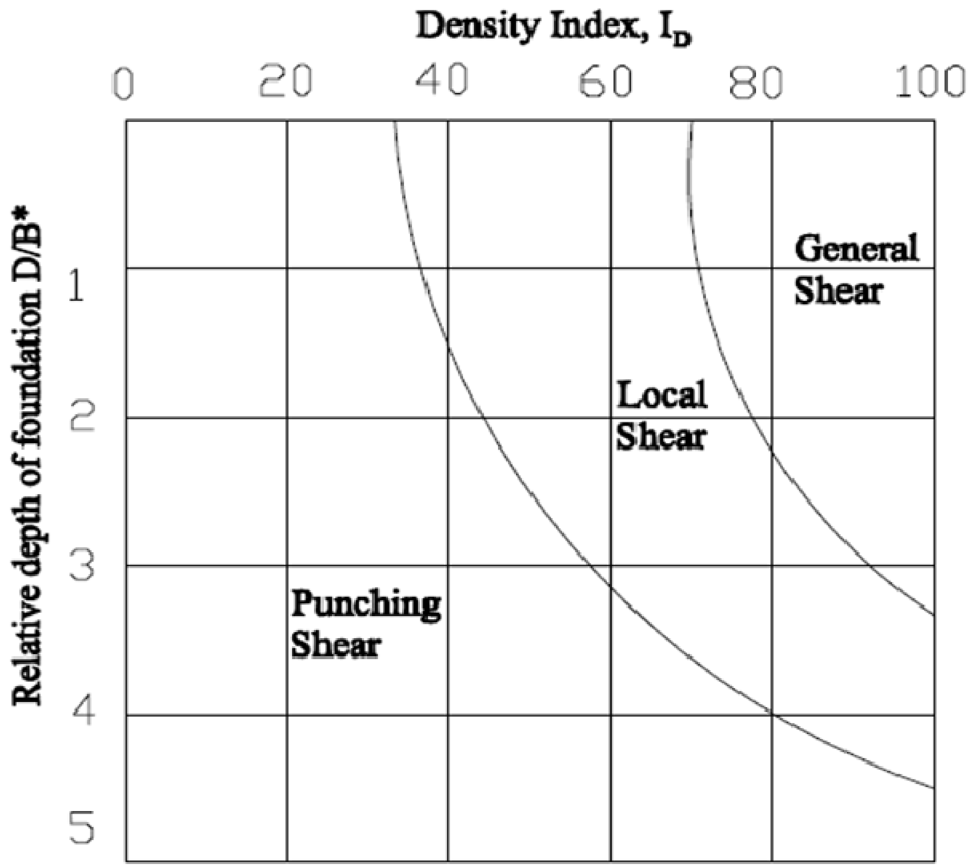


Fig 2.28: Modes of failure from tests on model footings [Vesic (1973]

Note: B^*= B for square or circular footing

$B^* = \frac{2BL}{B+L}$ for rectangular footing.