Hints and Solutions

1. Ans. B.
Applying conservation of energy

$$\frac{1}{2}m\omega^2\left(\frac{2A}{3}\right)^2+9\times\frac{1}{2}m\omega^2\left[A^2-\left(\frac{2A}{3}\right)^2\right]=\frac{1}{2}m\omega^2A_1^2$$

$$\Rightarrow A_1=\frac{7A}{3}$$

Hence, option B is correct.

2. Ans. A.

$$\frac{1}{\alpha}=\frac{1}{\beta}+1$$

Hence, option A is correct.

3. Ans. C.
Mean value of time

$$=\frac{90+91+95+92}{4}=92s$$

Also absolute errors are

$$92\sim90=\pm2$$
$$92\sim91=\pm1$$
$$92\sim95=\pm3$$
$$92\sim92=0$$

$\therefore$ mean absolute error = $\pm\frac{2+1+3+0}{4}=\pm1.5$ s but since the least count of the clock is 1s so it can't count 1.5s.
Hence it will count 2 s as error. (92 ± 2) s.
Hence, option C is correct.

4. Ans. A.
In OR gate, if all the inputs are low (zero), only. Then the output is low (zero).
Hence, option A is correct.

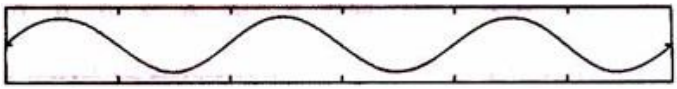
5. Ans. B.

$$\vec{L}=\vec{r}\times\vec{p}$$

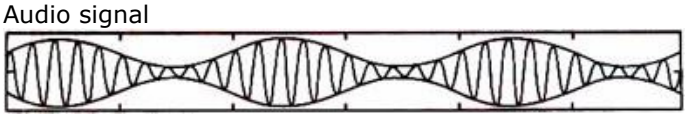
Hence option B is correct.

6. Ans. C.

Carrier wave



Audio signal



Modulated wave

In amplitude modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of audio signal.
Hence option C is correct.

7. Ans. C.

$$\frac{hc}{\lambda}=w_o+\frac{1}{2}mv^2 \quad ...(1)$$

$$\frac{4}{3}\frac{hc}{\lambda}=w_o+\frac{1}{2}mv_1^2 \quad ...(ii)$$

From (i) & (ii)

$$\frac{4}{3}\left(w_o+\frac{1}{2}mv^2\right)=w_o+\frac{1}{2}mv_1^2$$

$$\frac{1}{2}mv_1^2=\frac{w_o}{3}+\frac{4}{3}\frac{1}{2}mv^2$$

$\therefore$ $v_1>v\sqrt{\frac{4}{3}}$.

Hence option C is correct.

8. Ans. B.

$$B_A=\frac{\mu_oI}{2.\frac{\ell}{2\pi}}=\frac{\mu_o\pi I}{\ell}$$

$$B_B=4\times\frac{\mu_o}{4\pi}\frac{I(\cos45+\cos45)}{\frac{\ell}{8}}$$

$$=\frac{\mu_oI\sqrt{2}.8}{\pi l}$$

$$=\frac{B_A}{B_B}=\frac{\pi^2}{8\sqrt{2}}.$$

Hence option B is correct.

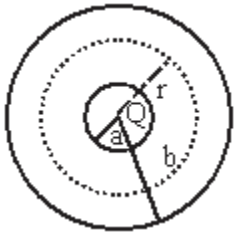
9. Ans. B.

$$f_1=\frac{v}{4\frac{\ell}{2}}=\frac{v}{2\ell}=f$$

Hence option B is correct.

10. Ans. C.
Using gauss theorem for dotted Gaussian surface

$$E.4\pi r^2 = \frac{1}{\epsilon_o}\left[Q + \int_a^r \frac{A}{r}.4\pi r^2 dr\right]$$



$$E = \frac{Q}{4\pi\epsilon_o r^2} + \frac{1}{4\pi r^2 \epsilon_o}.2\pi A\left[r^2 - a^2\right]$$

$$= \frac{Q}{4\pi\epsilon_o r^2} + \frac{A}{2\epsilon_o} - \frac{Aa^2}{2r^2\epsilon_o}$$

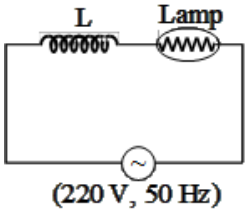
For E to be independent of r

$$\frac{Q}{4\pi\epsilon_o r^2} - \frac{Aa^2}{2\rho^2\epsilon_o} = 0 \Rightarrow A = \frac{Q}{2\pi a^2}$$

Hence option C is correct.

11. Ans. B.

Resistance of the lamp = $\frac{80}{10} = 8\Omega$



Now V_R = 80 V

Here $V = \sqrt{V_L^2 + V_R^2}$ $\Rightarrow$ $220 = \sqrt{V_L^2 + 80^2}$

$\therefore V_L = 10\sqrt{420}\,V$

Now $V_R = IR = 80$ ... (1)

$V_L = IX_L = 10\sqrt{420}$ ... (2)

On dividing (1) and (2),

$$\frac{R}{X_L} = \frac{80}{10\sqrt{420}}$$

$$\frac{8}{2\pi fL} = \frac{8}{\sqrt{420}}$$

On solving, L = 0.065 H.
Hence option B is correct.

12. Ans. C.
The equation of the process is y = mx + c

$$P = -\left(\frac{P_o}{V_o}\right)V + 3P_o$$

Now, PV = nRT

$\therefore P = \frac{nRT}{V}$

$$\frac{nRT}{V} = -\left(\frac{P_o}{V_o}\right)V + 3P_o$$

$$nRT = -\left(\frac{P_o}{V_o}\right)V^2 + 3P_oV$$

$$T = -\left(\frac{P_o}{nRV_o}\right)V^2 + \left(\frac{3P_o}{nR}\right)V \quad \text{... (1)}$$

Now for maximum T,

$$\frac{dT}{dV} = 0$$

$$\frac{dT}{dV} = -\left(\frac{2P_o}{nRV_o}\right)V + \left(\frac{3P_o}{nR}\right) = 0$$

$\therefore V = \frac{3}{2}V_o$

Putting in (1), we get

$$T = \left(-\frac{P_o}{nRV_o}\right)\left(\frac{9}{4}V_o^2\right) + \frac{3P_o}{nR}\left(\frac{3}{2}V_o\right)$$

$$= \frac{9}{2}\left(\frac{P_oV_o}{nR}\right) - \frac{9}{4}\left(\frac{P_oV_o}{nR}\right)$$

$$T_{max} = \frac{9}{4}\left(\frac{P_oV_o}{nR}\right)$$

Hence option C is correct.

13. Ans. B.
Total potential energy
U = mgh × 1000 = 10 × 9.8 × 1 × 1000
Since 20% efficiency rate, hence

$3.8 \times 10^7 \times \frac{20}{100}m = 9.8 \times 10^4$

$m = \frac{9.8\times10^4}{3.8\times2\times10^6} = 12.89 \times 10^{-3}$ kg

Hence option B is correct.

14. Ans. A.

$$PQ = \frac{h}{\sin 30^\circ} = 2h = 4m$$

Given that work done against friction along track PQ and QR are equal.
$\therefore \mu$ mg cos 30° × 4 = μmg x

$\frac{4\sqrt{3}}{2} = x$

$\Rightarrow x = 2\sqrt{3} = 2 \times 1.732 = 3.46 \cong 3.5$ m

Loss in PE = work done against friction along PQ and QR

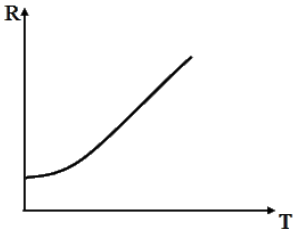
$\therefore mg\ 2 = \mu mg \frac{\sqrt{3}}{2} \times 4 + \mu mg\ (3.5)$

On solving $\mu = 0.29$

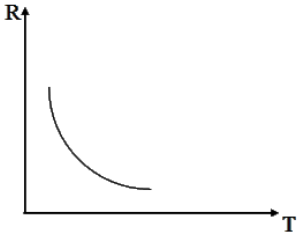
Hence option A is correct.

15. Ans. A.

For metals like Cu, resistance increases linearly with temperature.



For intrinsic semiconductors like Si, resistance decreases exponentially with temperature increases.



Hence option A is correct.

16. Ans. C.

Radio waves have a lower energy that visible. Hence D has least energy. X rays have higher energy than visible hence C has highest. In the visible range, the sequence of decreasing energy is given by VIBGYOR. Hence yellow has lesser energy than blue.

Hence option C is correct.

17. Ans. C.

G = 100 Ω

$I_g = 10^{-3}$ A

The shunt resistance (S) required to convert the galvanometer to an ammeter of full scale deflection current of 10 A is

$S = \frac{I_g G}{I - I_g} = \frac{10^{-3} \times 100}{10 - 10^{-3}} \cong \frac{10^{-1}}{10} = \frac{1}{100} \cong 0.01\ \Omega$

Hence, option C is correct.

18. Ans. B.

No. of atoms left after n half lives is $\frac{N_0}{2^n}$

(where n is the number of half lives)

Number of atoms of A left = $\frac{N_o}{2^4} = \frac{N_o}{16}$

(4 half lives ∵ t = 80 min, $t_1/2$ = 20 min, n =80/20 =4)

Number of atoms of B left = $\frac{N_o}{2^2} = \frac{N_0}{4}$

(Number of half lives = 2)

Hence ratio of decayed number of A and B will be

$\frac{N_o - \frac{N_o}{16}}{N_o - \frac{N_o}{4}} = \frac{\frac{15N_o}{16}}{\frac{3N_o}{4}} - = \frac{5}{4}$

Hence option B is correct.

19. Ans. C.

A **diode** is an electronic component with two electrodes (connectors). It allows electricity to go through it only in one direction.

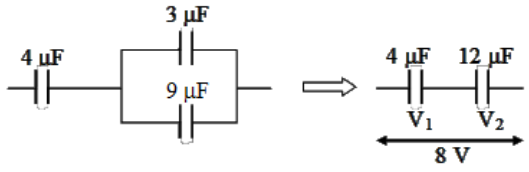
A **zener diode** is like a normal diode, but instead of being destroyed by a big reverse voltage, it lets electricity through. The voltage needed for this is called the breakdown voltage or Zener voltage. Because it is built with a known breakdown voltage it can be used supply a known voltage.

A **solar cell**, or **photovoltaic cell**, is an electrical device that converts the energy of light directly into electricity by the photovoltaic effect, which is a physical and chemical phenomenon.

Light dependent resistance is a light-controlled variable resistor. The resistance of a photoresistor decreases with increasing incident light intensity; in other words, it exhibits photoconductivity. A photoresistor can be applied in light-sensitive detector circuits, and light- and dark-activated switching circuits.

Hence option C is correct.

20. Ans. A.



Now $V_1 + V_2 = 8V$ and $4V_1 = 12V_2$

On solving, $V_1 = 6$ V, $V_2 = 2$ V

Hence $Q_1 = 24$ μC, $Q_2 = 18$ μc

Total Q = 42 μC

Hence, $E = \frac{1}{4\pi\varepsilon_o}\left(\frac{Q}{r^2}\right) = \frac{9 \times 10^9 \times 42 \times 10^{-6}}{30^2}$ = 420 N/C

Hence option A is correct.

21. Ans. B.

$V_o = \sqrt{\frac{GM}{R}}$

$V_e = \sqrt{\frac{2GM}{R}}$

Increase $= V_e - V_o$

$= \left(\sqrt{2} - 1\right)\sqrt{\frac{GM}{R}} = \left(\sqrt{2} - 1\right)\sqrt{\frac{gR^2}{(R+h)}}$

$\approx\left(\sqrt{2}-1\right)\sqrt{gR}$

Hence option B is correct.

22. Ans. D.

$=\frac{0.5}{50}=0.01\,mm$

L.C.

error is negative

error = 5 × 0.01 = 0.05

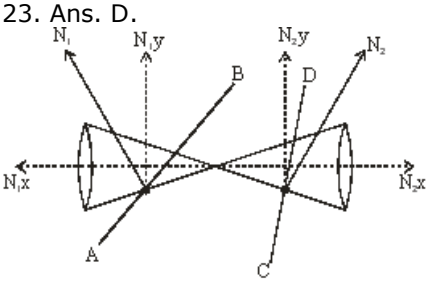
Reading = 0.5 + 25 × 0.01 + 0.05

= 0.5 + 0.25 + 0.05

= 0.80 mm

Hence option D is correct.

23. Ans. D.



Initially C.M. will move straight, hence its distances from left rail will decrease.

Normal reaction at left contact N_1 will increase. Horizontal component of left N_1x will also increase and continue to increase. Hence the roller will tend to move towards left direction.

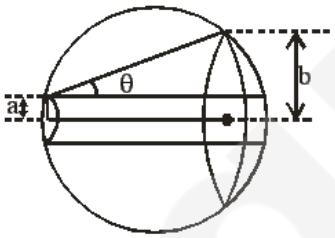
Hence option D is correct.

24. Ans. B.

Hysteresis loss for electromagnets and transformers are less because it can magnetize or demagnetize easily

Hence option B is correct.

25. Ans. B.



$b\sin\theta=n\lambda$

$\frac{b-a}{L}=\tan\theta\approx\sin\theta$

$\frac{a(b-a)}{L}=\lambda$

$ab-a^2=\lambda L$

$a^2-ab+\lambda L=0$

$b^2\geq 4\lambda L$

$b_{\min}=\sqrt{4\lambda L}$

$a\left(\sqrt{4\lambda L}\right)-a^2=\lambda L$

$a=\frac{\lambda^2}{L}$

Hence option B is correct.

26. Ans. A.

$v=\sqrt{\frac{\mu xg}{\mu}}=\sqrt{xg}$

$\frac{dx}{dt}=\sqrt{xg}$

$\int_0^l\frac{dx}{\sqrt{x}}=\sqrt{g}\,t$

$\int_0^l x^{-1/2}dx=\sqrt{g}\,t$

$2x^{1/2}=\sqrt{g}\;\;t$

$\frac{2}{\sqrt{g}}\sqrt{20}=t$

$2\sqrt{2}=t$

Hence option A is correct.

27. Ans. D.

$PV^n=$ constant

Then $C_{process}$ for the above polytropic process is known

$C_{process}=C=C_V+\frac{R}{(1-n)}$

$C-C_V=\frac{R}{(1-n)}$

$(1-n)=\frac{R}{(C-C_V)}$

Finally after solving

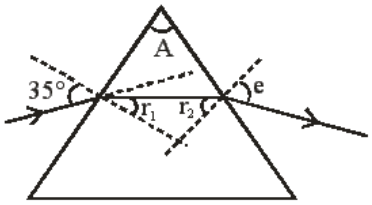
$n=\frac{(C-C_V-R)}{(C-C_V)}$ [as $C_P-C_V=R$]

$n=\left(\frac{C-C_P}{C-C_V}\right)$

Hence option D is correct.

28. Ans. B.
In case of telescopes, magnifying power = 20 means that the object appears 20 times nearer.
Hence option B is correct.

29. Ans. D.



$\angle i = 35°$

$\delta = 40°$

$e = 79°$

i = 35°
e = 79°

$\delta = 40°$

$\delta + 4 = i + e \Rightarrow A = 74°$

with above set of data, as refractive index increases emerging ray will have tendency to suffer TIR.
For i = 35°

$$\mu = \frac{\sin 35°}{\sin r_1} = \frac{\sin 90°}{\sin(74° - r_1)}$$

Using reversibility principle
For i = 79°

$$\mu = \frac{\sin 79°}{\sin r_2} = \frac{\sin 90°}{\sin(74° - r_2)}$$

From 2nd case

$r_2 = 37°$

$\Rightarrow \mu = 1.63$

Hence option D is correct.

30. Ans. C.

$$\left(\frac{\Delta T}{T}\right) = \frac{1}{2}\left(\frac{\Delta l}{l}\right) = \frac{1}{2}(\alpha \Delta T)$$

$$\Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\left[\alpha(\Delta T_1)\right] \Rightarrow \frac{1}{2}\alpha(\Delta T_1) \times 24 \times 3600$$

$$\Rightarrow 12 = \alpha(\Delta T_1) \times 12 \times 3600 \quad \text{.....(i)}$$

$$\Rightarrow 4 = \alpha(\Delta T_2) \times 12 \times 3600 \quad \text{.....(ii)}$$

$$3 = \left(\frac{\Delta T_1}{\Delta T_2}\right) = \frac{40 - T}{T - 20}$$

$3T - 60 = 40 - T$

$T = 25°C$

$\alpha = 1.85 \times 10^{-5} / °C$

Hence option C is correct.

31. Ans. A.
Initially there were equal moles of ideal gas in both bulbs.
Now when temperature of second bulb is raised to T_2 and both are still connected it means momentarily (until both the bulbs comes to thermal equilibrium) some moles of gas must have been got shifted to first bulb to equalize pressure of both bulbs. Suppose n_1 and n_2 are the moles of gases present in first and second bulbs then

$$p_f V = n_1 R T_1 \qquad p_f V = n_2 R T_2 \qquad p_i V = \frac{(n_1 + n_2)}{2} R T_1$$

$$\frac{2p_i V}{RT_1} = \frac{p_f V}{RT_1} + \frac{p_f V}{RT_2}$$

Or $$p_f = 2p_i\left(\frac{T_2}{T_1 + T_2}\right)$$

Hence option A is correct.

32. Ans. A.
Water molecules have extensive intermolecular hydrogen bonding and no intramolecular hydrogen bonding.
Hence option A is correct.

33. Ans. B.

$$R - CONH_2 + Br_2 + 4NaOH \longrightarrow RNH_2 + 2NaBr + NaCO_3 + 2H_2O$$

Hence option B is correct.

34. Ans. B.
The first ionization energy of d-block elements are greater than the first ionization energy of alkali metals.
Therefore, The order is

IE_1 : Na : = 495.8 kJ/mol
IE_1 : Sc : = 631.0 kJ/mol
So, Scandium has highest ionization energy.
Hence option B is correct.

35. Ans. A.
The permissible limit for Nitrate to be present in drinking water is 50 parts per million.
Hence option A is correct.

36. Ans. B.
Equation

I $C_{(s)} + O_{2(g)} \longrightarrow CO_{2(g)} \qquad \Delta H = -393.5$

Equation II $CO_{(g)} + \frac{1}{2}O_{2(g)} \longrightarrow CO_{2(g)} \qquad \Delta H = -283.5$

Subtracting equation II from equation I we get

$$C_{(s)} + \frac{1}{2}O_{2(g)} \longrightarrow CO_{(g)}$$

$$\therefore \Delta H = -393.5 - (-283.5) \approx -110.0\ kJ/mol$$

Hence option B is correct.

37. Ans. A.

$$A + B \longrightarrow C + D$$

Initial 1 1 1 1
Equilibrium 1-x 1-x 1+x 1+x

$$K_c = \frac{[C][D]}{[A][B]} = 100$$

$$\Rightarrow \frac{(1+x)(1+x)}{(1-x)(1-x)} = 100$$

$$\frac{1+x}{1-x} = \pm 10$$

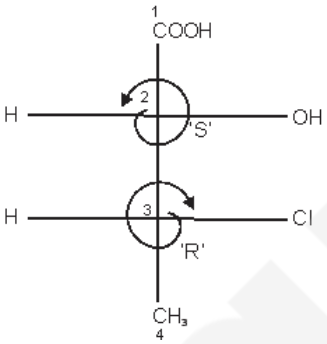
$$\Rightarrow 1+x = 10-10x$$

$$\Rightarrow 11x = 9$$

$$x = \frac{9}{11} = 0.818$$

$\therefore$ (D) at equilibrium = 1 + x = 1 + 0.818 = 1.818
Hence option A is correct.

38. Ans. D.



Hence option D is correct.

39. Ans. A.
Freundlich adsorption 1/n isotherm equation is

$\frac{x}{m} = kp^{1/n}$ where k & n are constants.

So, $\log\frac{x}{m} = \log k + \frac{1}{n}\log p$ (y = mx+c form)
So slope = 1/n and intercept = log k
Hence option A is correct.

40. Ans. B.
Glycerol boils at pretty high temperature of 290°C. To prevent it from decomposition it is made to distill at lower temperature by reducing external pressure during its isolation from spent-lye.
Hence option B is correct.

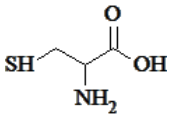
41. Ans. D.

Anionic detergents mainly contains $-SO_4^-$ as polar group.
Hence option D is correct.

42. Ans. C.

In NO_2^+, Number of valence electrons = 16
No lone pairs on central atom
So, it is sp hybridized.
Hence option C is correct.

43. Ans. A.



Cysteine

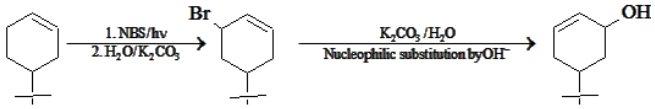
Hence option A is correct.

44. Ans. A.
Galena (PbS) is a sulphide ore hence it is concentrated by froth flotation process.
Hence option A is correct.

45. Ans. B.
High density polythene is used for manufacturing buckets and dustbins.
Hence option B is correct.

46. Ans. D.
CrO_2 is attracted very strongly by magnetic field.
Hence option D is correct.

47. Ans. D.



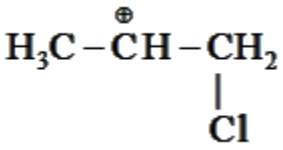
Hence option D is correct.

48. Ans. D.
Hottest part of the flame is at the tip part of the inner curve (in blue flame).
Hence option D is correct.

49. Ans. D.
Every mole of alkane, requires (3n+1)/2 moles of oxygen for complete combustion, where n is the number of C atoms.
Similarly, every mole of alkene requires 3n/2 moles of oxygen for complete combustion.
The condition is n should be an integer.
Here, 15ml of hydrocarbon requires 75ml of oxygen.
Which is 5 times.
Therefore, (3n+1)/2 = 5
gives, n=3
Also, 3n/2 = 5
gives n as a rational number. Therefore, the hydrocarbon is an alkane and has 3 carbon atoms. Hence Option D is correct.

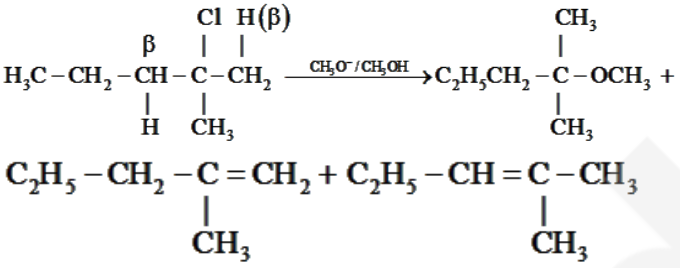
50. Ans. C.
Orthophosphorous acid H_3PO_3
and pyrophosphorous acids $H_4P_2O_5$
Hence option C is correct.

51. Ans. D.
Cl_2 $\left[Cl^+\right]$ is added first, hence possible intermediate among the given
option will be



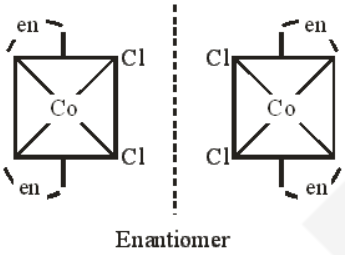
Hence option D is correct.

52. Ans. C.
Under given condition both S_N and elimination is possible.



Hence option C is correct.

53. Ans. D.



Enantiomer
Hence option D is correct.

54. Ans. B.

$$4Li + O_2 \rightarrow 2Li_2O\,(\text{oxide})$$

$$2Na + O_2 \rightarrow Na_2O_2\,(\text{peroxide})$$

$$K + O_2 \rightarrow KO_2\,(\text{superoxide})$$

Hence option B is correct.

55. Ans. A.
We assume that the vapour pressure has to be found out at 100°C.
Vapour pressure of pure water at 100°C is 760 Torr.
w = 18 gm; W = 178.2 gm

$$\frac{P_O - P_S}{P_S} = \frac{n}{N} = \frac{w}{\Sigma n} = \frac{\frac{18}{180}}{\frac{18}{180} + \frac{178.2}{18}} = 0.01$$

$$\frac{P_O}{P_S} - 1 = 0.01$$

$$\frac{P_O}{P_S} = 1.01$$

$$P_S = \frac{760}{1.01} = 752.47$$

Hence option A is correct.

56. Ans. C.

$$4Zn(s) + 10HNO_3(\text{dil}) \rightarrow 4Zn(NO_3)_2 + 5H_2O + N_2O$$

$$Zn(s) + 4HNO_3(\text{conc.}) \rightarrow Zn(NO_3)_2 + 2H_2O + 2NO_2$$

Hence option C is correct.

57. Ans. D.

$$H_2O_2 \rightarrow H_2O + \frac{1}{2}O_2$$

$t_{1/2}$ = 25 min

$$\text{rate} = K\left[H_2O\right]$$

$$= \frac{0.693}{t_{1/2}}\left[0.05\right] = \frac{0.693}{25} \times 0.05 = 1.386 \times 10^{-3}\ \text{mole lit min}^{-1}$$

Expression for rate will be,

$$\text{rate of reaction} = \frac{-d\left[H_2O_2\right]}{dt} = 2 \times \frac{d\left[O_2\right]}{dt}$$

$$\Rightarrow + \frac{d\left[O_2\right]}{dt} = - \frac{d\left[H_2O\right]}{dt} \times \frac{1}{2}$$

$$= \frac{1.386 \times 10^{-3}}{2}\ \text{mol lit}^{-1}\ \text{min}$$

$$= 6.93 \times 10^{-4}\ \text{mol lit}^{-1}\ \text{min}$$

Hence option D is correct.

58. Ans. D.
$[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$ has same no. of unpaired electron (i.e. four) in Cr^{2+} and Fe^{2+} in respectively. Thus they have same magnetic moment i.e. $\sqrt{24}$ B.M.
Hence option D is correct.

59. Ans. B.
Galvanization is the process of applying a protective 'Zn' coating to steel or iron to prevent rusting.
Hence option B is correct.

60. Ans. B.
de Broglie equation

$$\lambda = \frac{h}{mv} \quad ...(1)$$

$$\left[K.E. = \frac{1}{2}mv^2\right] \times m \quad ...(2)$$

$$(mv)^2 = 2K.E.m$$

$$Mv = \sqrt{2K.E.m} \quad ...(3)$$

For a charged particle moving under acceleration by applying potential 'V' the E_K (kinetic energy) of charged particle will be

$$E_K = eV \quad ...(4)$$

$$mv = \sqrt{2meV} \quad ...(5)$$

$$\lambda = \frac{h}{\sqrt{2meV}} \quad ...(6)$$

$$\boxed{\frac{h}{\lambda} = \sqrt{2meV}}$$

Hence option B is correct.

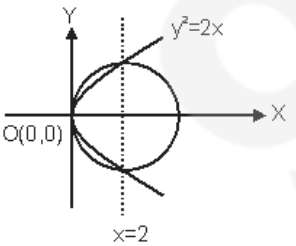
61. Ans. D.

$$y^2 = 2x, \quad x^2 + y^2 = 4x \Rightarrow x^2 = 2x \Rightarrow x = = 0, 2$$

Required area = $\int_0^2 \left(\sqrt{4x - x^2} - \sqrt{2x}\right) dx$

$$= \int_0^2 \left(\sqrt{4 - (x-2)^2} - \sqrt{2}\sqrt{x}\right) dx$$

$$\left[\frac{x-2}{2}\sqrt{4x - x^2} + 2\sin^{-1}\left(\frac{x-2}{2}\right) - \frac{2\sqrt{2}}{3}x^{3/2}\right]_0^2$$



$$= 0 + 0 - \frac{8}{3} - \left[0 + 2\left(-\frac{\pi}{2}\right)\right]$$

$$= \pi - \frac{8}{3}$$

Hence option D is correct.

62. Ans. A.

$$f(x) + 2f\left(\frac{1}{x}\right) = 3x \quad ... (1)$$

Replace $x \to \frac{1}{x}$

$$\Rightarrow f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$$

$$\Rightarrow 2f\left(\frac{1}{x}\right) + 4f(x) = \frac{6}{x} \quad ... (2)$$

$$\Rightarrow 3f(x) = \frac{6}{x} - 3x \quad \text{From (1) and (2)}$$

$$\Rightarrow f(x) = \frac{2}{x} - x$$

$$\Rightarrow f(-x) = -\frac{2}{x} + x$$

$$\Rightarrow f(x) = f(-x) \Rightarrow \frac{2}{x} - x = \frac{-2}{x} + x$$

$$\Rightarrow \frac{4}{x} = 2x \quad \Rightarrow \quad x^2 = 2 \quad \Rightarrow \quad x = \pm\sqrt{2}$$

Hence option A is correct.

63. Ans. D.

$$\int \frac{\left(2x^{12} + 5x^9\right)}{\left(x^5 + x^3 + 1\right)^3} dx$$

$$= \int \frac{\left(\frac{2}{x^3} + \frac{5}{x^6}\right) dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3}$$

Let $1 + \frac{1}{x^2} + \frac{1}{x^5} = t \Rightarrow \left(\frac{-2}{x^3} - \frac{5}{x^6}\right) dx = dt$

$$I = -\int \frac{dt}{t^3} = \frac{1}{2t^2} + C$$

$$= \frac{1}{2\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^2} + C = \frac{x^{10}}{2\left(x^5 + x^3 + 1\right)^2} + C$$

Hence option D is correct.

64. Ans. D.

$f(x)= |\log 2-\sin x|$

$g(x)=f(f(x))$

$g(x)=|\log 2-\sin(|\log 2-\sin x|)|$

$g(x)=\log 2-\sin(\log 2-\sin x)(\text{in neighbourhood of } x=0)$

$g'(0)=-\cos(\log 2-\sin 0).(-\cos 0)$

$=\cos(\log 2)$

Hence option D is correct.

65. Ans. B.

Circles touch externally and required circle touches x-axis. Centre (h, k) and radius = | k |

$c_1c_2=r_1+r_2$

$(h-4)^2+(k-4)^2=(6+|k|)^2$

$h^2-2.4h+16+k^2-8k+16=36+k^2+12|k|$

$h^2-8h-8k-12|k|=4$

$x^2-8x-8y-12|y|=4$

It represents two-semi parabola

Hence option B is correct.

66. Ans. C.

$(x^2-5x+5)^{(x^2+4x-60)}=1$

$a^b=1$, when a is 1 $b\in R$

a is not zero b = 0

a is -1 $b\in$ even integer

$x^2-5x+5=1$

$x=1,4$

$x^2+4x-60=0$; $x=-10,6$; $a\neq 0$

$x^2-5x+5=-1$, $x^2+4x-60=$ even integer

$x=2,3$ at x = 2, $x^2+4x-60=-48$

(even integer)

at x = 3, $9+12-60=$ odd

solutions are $x=1,4,-10,6,2$

$\therefore$ sum = 3

Hence option C is correct.

67. Ans. D.

Given terms of A.P. are

$T_2=a+d$; $T_5=a+4d$; $T_9=a+8d$

If T_2, T_5 & T_9 are in G.P.

$(a+4d)^2=(a+d)(a+8d)$

$a^2+16d^2+8ad=a^2+ad+8ad+8d^2$

$\Rightarrow 8d^2=ad \Rightarrow d(8d-a)=0 \Rightarrow 8d=a \ (\because d\neq 0)$

Now common ratio is

$\frac{a+4d}{a+d}=\frac{12d}{9d}=\frac{4}{3}$

Hence option D is correct.

68. Ans. A.

Given

$\frac{2b^2}{a}=8$ & $2b=ae$

$\Rightarrow 4b^2=a^2e^2$;

$\Rightarrow 4a^2(e^2-1)=a^2e^2 \Rightarrow e=\frac{2}{\sqrt{3}}$

Hence option A is correct.

69. Ans. A.

Here total number of terms will be 2n + 1 which cannot be equal to 28.

Hence option A is correct.

70. Ans. A.

p	q	~ p	~ q	p ∧ ~ q	~ p ∧ q	(p ∧ ~ q) ∨ q	(p ∧ ~ q) ∨ q ∨ (~ p ∧ q)	p ∨ q
T	T	F	F	F	F	T	T	T
T	F	F	T	T	F	T	T	T
F	T	T	F	F	T	T	T	T
F	F	T	T	F	F	F	F	F

So $(p\wedge\sim q)\vee q\vee(\sim p\wedge q)$ is equivalent to $p\vee q$

Hence option A is correct.

71. Ans. D.

y = $\tan^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right)$

$\Rightarrow$ y = $\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$

$\Rightarrow y = \frac{\pi}{4}+\frac{x}{2}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}$

Hence, slope of normal $\Rightarrow -2$

$\therefore$ Equation of normal becomes

$y-\left(\frac{\pi}{4}+\frac{\pi}{12}\right) = -2\left(x-\frac{\pi}{6}\right)$

The point $\left(0, \frac{2\pi}{3}\right)$ satisfying the equation.

Hence option D is correct.

72. Ans. D.

Let, $y = \left(\frac{(n+1)(n+2)....3n}{n^{2n}}\right)^{1/n}$

$\log y = \frac{1}{n}\left[\log\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)....\left(1+\frac{2n}{n}\right)\right\}\right]$

$\log y = \sum_{r=1}^{2n}\frac{1}{n}\log\left(1+\frac{r}{n}\right)$

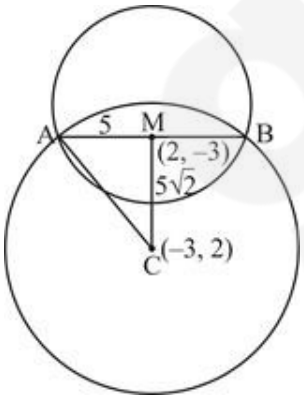
$\lim_{n\to\infty}\log y = \lim_{n\to\infty}\sum_{r=1}^{2n}\frac{1}{n}\log\left(1+\frac{r}{n}\right)$

$\lim_{n\to\infty}\log y = \int_0^2 \log(1+x)dx = 3\log 3-2$

$y = e^{3\log 3-2} = \frac{27}{e^2}$

Hence option D is correct.

73. Ans. D.



$\because$ AC = $\sqrt{50+25} = \sqrt{75}$

Hence option D is correct.

74. Ans. B.

$P(E_1) = \frac{6}{36} = \frac{1}{6}$

$P(E_2) = \frac{6}{36} = \frac{1}{6}$

$P(E_3) = \frac{18}{36} = \frac{1}{2}$

$P(E_1 \cap E_2) = \frac{1}{36}$

$P(E_2 \cap E_3) = \frac{1}{12}$

$P(E_1 \cap E_3) = \frac{1}{12}$ and $P(E_1 \cap E_2 \cap E_3) = 0$

Hence, E_1, E_2 and E_3 are not independent.

Hence option B is correct.

75. Ans. B.

$Z = \frac{2+3i\sin\theta}{1-2i\sin\theta}$

For Z to be purely imaginary, $\overline{Z} = -Z$

$\Rightarrow \frac{2-3i\sin\theta}{1+2i\sin\theta} = \frac{-2-3i\sin\theta}{1-2i\sin\theta}$

$\Rightarrow \sin\theta = \frac{1}{\sqrt{3}}$

$\theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Hence option B is correct.

76. Ans. D.

Here, $T_n = \left(\frac{8+(n-1)4}{5}\right)^2$

$T_n = \frac{16}{25}(n+1)^2$

$S_n = \frac{16}{25}\left[\frac{(n+1)(n+2)(2n+3)}{6}-1\right]$

$S_{10} = \frac{16}{25}\left[\frac{11\times12\times23}{6}-1\right]$

$= \frac{16}{25}[506-1] = \frac{16}{25}\times505 = \frac{16}{5}\times101 \Rightarrow m = 101$

Hence option D is correct.

77. Ans. B.
D = 0

$$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$1(\lambda+1)-\lambda(-\lambda^2+1)-1(\lambda+1)$ = 0

$\lambda+1+\lambda^3-\lambda-\lambda-1$ = 0

$\lambda^3-\lambda$ = 0

$\lambda(\lambda^2-1)$ = 0

λ = 0, 1, –1

(2) exactly three value of λ.
Hence option B is correct.

78. Ans. B.
As per given condition,

$2\ell-m$ = 3 … (i)

and $3\ell-2m$ = 5 … (ii)

$\Rightarrow$ ℓ = 1 and m = –1

Hence, ℓ^2 + m^2 = 2
Hence option B is correct.

79. Ans. B.
A, L, L, M, S

No. of words starting with A = $\frac{4!}{2!}$ = 12
No. of words starting with L = 4! = 24

No. of words starting with M = $\frac{4!}{2!}$ = 12

No. of words starting with S A = $\frac{3!}{2!}$ = 3
No. of words starting with S L = 3! = 6
and next word formed will be SMALL.
Hence, rank of the word SMALL = 12 + 24 + 12 + 3 + 6 + 1 = 58
Hence option B is correct.

80. Ans. D.

$\bar{x} = \frac{16+a}{4}$

Variance = $(3.5)^2$ = 12.25

Variance = $\frac{\sum x_i^2}{n}-(\bar{x})^2$

$\sum x_i^2$ = 134 + a^2

12.25 = $\frac{134+a^2}{4}-\left(\frac{16+a}{4}\right)^2$

$\frac{49}{4}$ = $\frac{536+4a^2-256-a^2-32a}{16}$

196 = 280 + $3a^2$ – 32a
$3a^2$ – 32a + 84 = 0
Hence option D is correct.

81. Ans. A.

$4x+2\pi r=2$

$2x+\pi r=1$ …(i)

$A=x^2+\pi r^2$

$A=x^2+\pi\left(\frac{1-2x}{\pi}\right)^2=x^2+\frac{1}{\pi}(1-2x)^2$

For maximum / minimum

$\frac{dA}{dx}=0$

$2x-\frac{1}{\pi}4(1-2x)=0$

$x=\left(\frac{2}{\pi+4}\right)$

$\frac{d^2A}{dx^2}=2+\frac{8}{\pi}>0$ For $x=\frac{2}{\pi+4}$

Area is minimum

$r=\frac{1-2\left(\frac{2}{\pi+4}\right)}{\pi}=\frac{1}{(\pi+4)}$

$\frac{x}{r}=2$

x = 2r
Hence, option A is correct.

82. Ans. A.

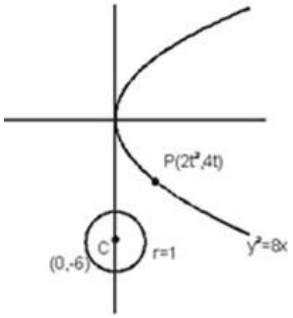
$P=\lim_{x\to0^+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}=e^{\lim_{x\to0^+}\frac{\tan^2\sqrt{x}}{2x}}$

$=e^{\lim_{x\to0^+}\frac{1}{2}\left(\frac{\tan\sqrt{x}}{\sqrt{x}}\right)^2}=e^{\frac{1}{2}}$

$$\log p = \frac{1}{2}$$

Hence option A is correct.

83. Ans. C.



Equation of normal at P
$y + tx = 4t + 2t^3$
As it passes through (0, -6)
$\Rightarrow$ $-6 = 4t + 2t^3$
or $2t^3 + 4t + 6 = 0$
or $t^3 + 2t + 3 = 0$
or $(t + 1)(t^2 - t + 3) = 0$
$t = -1$

$$P \equiv (2, -4)$$

Equation of required circle is
$(x - 2)^2 + (y + 4)^2 = 4 + 4$
or $x^2 + y^2 - 4x + 8y + 12 = 0$
Hence option C is correct.

84. Ans. B.
$y(1 + xy)\,dx = x\,dy$
$xy^2\,dx = -(ydx - x\,dy)$

$$x dx = -\left(\frac{ydx - xdy}{y^2}\right) = -d\left(\frac{x}{y}\right)$$

Integrating both sides

$$\frac{x^2}{2} + \frac{x}{y} = C$$

As curve passes through the point (1, -1)

$$\Rightarrow \frac{1}{2} - 1 = C$$

$$\therefore C = -\frac{1}{2}$$

$$\therefore \frac{x}{y} = -\frac{x^2}{2} - \frac{1}{2}$$

$$y = \frac{x}{-\left(\frac{1}{2} + \frac{x^2}{2}\right)}$$

$$f(x) = \frac{x}{-\left(\frac{1}{2} + \frac{x^2}{2}\right)}$$

$$f\left(-\frac{1}{2}\right) = \frac{-\frac{1}{2}}{-\left(\frac{1}{2} + \frac{1}{8}\right)}$$

$$= \frac{\frac{1}{2}}{\frac{5}{8}} = \frac{4}{5}$$

Hence option B is correct.

85. Ans. B.

$$(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$$

$$\vec{a}.\vec{b} = -\frac{\sqrt{3}}{2}$$

$$|\vec{a}|.|\vec{b}|\cos\theta = -\frac{\sqrt{3}}{2}$$

$$1.1.\cos\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \frac{5\pi}{6}$$

Hence option B is correct.

86. Ans. D.

$$A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$$

$A(\text{Adj } A) = A A^T$

$$\Rightarrow |A|.I_2 = A A^T$$

$$(10a + 3b)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$$

or

$$\begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix} = \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix}$$

or
$15a - 2b = 0$ ... (i)
$10a + 3b = 13$ ...(ii)
and $10a + 3b = 25a^2 + b^2$ ...(iii)
from (i) and (ii)

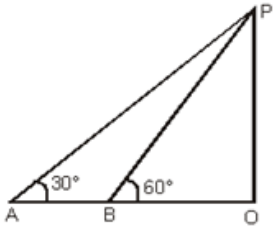
$$a=\frac{2}{5}\quad b=3$$

which satisfies (iii) equation also

$$\therefore\quad 5a+b=5\times\frac{2}{5}+3=5$$

Hence option D is correct.

87. Ans. B.



Let velocity is V,

$$V=\frac{AB}{10}$$

$$t_{BO}=\frac{BO}{V}=\frac{BO}{AB}\times 10$$

$$=\frac{PO\ \cot 60^{o}}{AB}\times 10$$

$$AO=\frac{PO}{\tan 30^{o}}$$

AB + BO = PO Cot 30 °
AB = PO Cot 30 ° - PO Cot 60 °

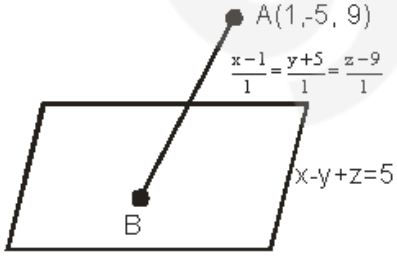
$$\therefore\quad t_{BO}=\frac{PO\cot 60^{o}}{PO\cot 30^{o}-PO\cot 60^{o}}\times 10$$

$$=\frac{\frac{1}{\sqrt{3}}}{\sqrt{3}-\frac{1}{\sqrt{3}}}\times 10$$

= 5 minutes

Hence option B is correct.

88. Ans. D.



Co-ordinates of point B can be taken as (r + 1, r – 5, r + 9)
As this lie on plane x – y + z – 5 = 0

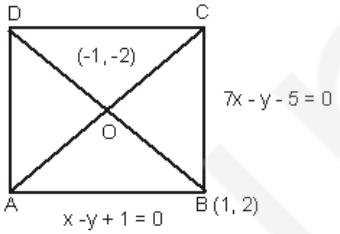
$\Rightarrow$ (r + 1) – (r – 5) + (r + 9) – 5 = 0
or r + 10 = 0
r = - 10
B $\equiv$ (-9, -15, -1)

$$AB=\sqrt{(1+9)^2+(-5+15)^2+(9+1)^2}$$

$$=\sqrt{3\times 100}=10\sqrt{3}$$

Hence option D is correct.

89. Ans. A.



Equation of AC

$$y+2=-\frac{1}{2}(x+1)$$

$\Rightarrow$ x + 2y + 5 = 0
Solving, this equation with 7x – y – 5 = 0 we get

$$C\left(\frac{1}{3},\frac{-8}{3}\right)$$

Hence option A is correct.

90. Ans. A.
cos x + cos 2x + cos 3 x + cos 4x = 0
or 2 cos 2x cos x + 2 cos 3x cos x = 0
or 2 cos x (cos 2x + cos 3x) = 0

$$2\cos x\left(2\cos\frac{5x}{2}\cos\frac{x}{2}\right)=0$$

or

$$4\cos x\cos\frac{5x}{2}\cos\frac{x}{2}=0$$

or

$$x=\frac{\pi}{5},\frac{\pi}{2},\frac{3\pi}{5},\frac{3\pi}{2},\pi,\frac{7\pi}{5},\frac{9\pi}{5}$$

Hence option A is correct.

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