1. For A = $\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, the determinant of $A^T A^{-1}$ is

A. $\sec^2 x$ B. cos 4x
C. 1 D. 0
Answer ||| A
Solution ||| Correct option is (A).

$$A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$|A| = 1 + \tan^2 x = \sec^2 x$$

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$A^T A^{-1} = \frac{1}{|A|}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$= \frac{1}{\sec^2 x}\begin{bmatrix} 1-\tan^2 x & -2\tan x \\ 2\tan x & 1-\tan^2 x \end{bmatrix}$$

$$|A^T A^{-1}| = \frac{1}{\sec^2 x}\left[(1-\tan^2 x)^2 + 4\tan^2 x\right]$$

$$= \frac{1}{\sec^2 x}\left[(1+\tan^4 x - 2\tan^2 + 4\tan^2 x\right]$$

$$= \frac{1}{\sec^2 x}\left[(1-\tan^4 x + 2\tan^2 x\right]$$

$$= \frac{1}{\sec^2 x}\left[(1-\tan^2 x\right]^2$$

$$= \frac{[\sec^2 x]^2}{\sec^2 x} = \sec^2 x$$

2. The contour on the x-y plane, where the partial derivative of $x^2 + y^2$ with respect to y is equal to the partial derivative of 6y + 4x with respect to x, is
A. y = 2 B. x = 2
C. x + y = 4 D. x − y = 0
Answer ||| A
Solution ||| Correct option is (A).
Given
Partial derivative of (x^2+y^2) with respect to y
= partial derivative of $(6y+4x)$ with respect to x

$$0 + 2y = 0 + 4$$

So, $2y = 4$

$$y = 2$$

3. If C is a circle of radius r with centre z_0, in the complex z-plane and if n is a non-zero integer, then

$$\oint_C \frac{dz}{(z-z_0)^{n+1}}$$

A. $2\pi nj$ B. 0

C. $\frac{nj}{2\pi}$ D. $2\pi n$

Answer ||| B
Solution ||| Correct option is (B).

By the residue theorem,

$$\oint_C \frac{f(z)}{(z-z_0)^{n+1}}dz = 2\pi j \times \text{(residue at that point)}$$

where $\text{Residue} = \lim_{z\to z_0} \frac{1}{n!}\frac{d^n f(z)}{dz^n}$

From given question f (z) = 1
So, residue = 0

Hence, $\oint_C \frac{d_z}{(z-z_0)^{n+1}} = 0$

4. Consider the function g(g) = e^{-t} sin (2πt) u (t) where u (t) is the unit step function. The area under g(t) is_______
A. 2.5 B. 2 C. 0.155 D. 1.45
Answer ||| C
Solution ||| Correct answer is 0.155
Given $g(t) = e^{-t}\sin(2\pi t)\, u\,(t)$
Taking the Laplace transform

$$G(s) = \frac{2\pi}{(3+1)^2 + (2\pi)^2}$$

From definition of Laplace transform

$$G(s) = \int_{-\infty}^{\infty} g(t)\, e^{-st} dt$$

So, $G(0) = \int_{-\infty}^{\infty} g(t)\, dt =$ area under the curve $g(t)$

(taking s = 0)

$$G(0) = \frac{2\pi}{1+(2\pi)^2} = 0.155$$

5. The value of $\sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n$ is__________.

A. 2 B. 6 C. 8 D. 4
Answer ||| A
Solution |||

Let $\sum_{n\ 0}^{\infty} n\left(\frac{1}{2}\right)^n$

$$= 0 + 1 . \left(\frac{1}{2}\right)^1 + 2 . \left(\frac{1}{2}\right)^2 + 3 . \left(\frac{1}{2}\right)^3 ...(i)$$

Multiply equation (1) by 1/2

$$\frac{S}{2} = \left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^4 ...(ii)$$

Subtracting equation (2) from (1)

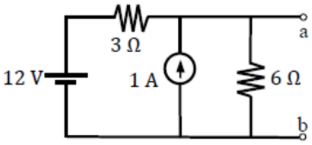
$$S - \frac{S}{2} = \frac{1}{2} + \left(\frac{1}{2}\right)^2 (2-1) + \left(\frac{1}{2}\right)^2 (3-2)$$

$$\frac{S}{2} = \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 \quad (sum\ of\ GP = \frac{a}{1-r})$$

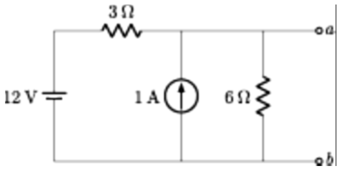
$$\frac{S}{2} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$$

Hence, $S = 2$

6. For the circuit shown in the figure, the Thevenin equivalent voltage (in Volts) across terminals a-b is______



A. 45 B. 50 C. 10 D. 30
Answer ||| C
Solution ||| Correct answer is 10.
Given circuit is



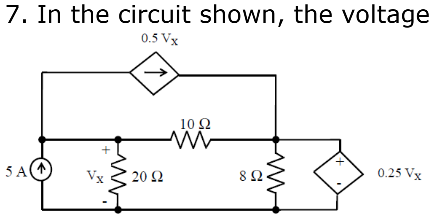
Applyi KCL at node A,

$$1\,A = \frac{12 - V_A}{3} + \frac{V_A}{6}$$

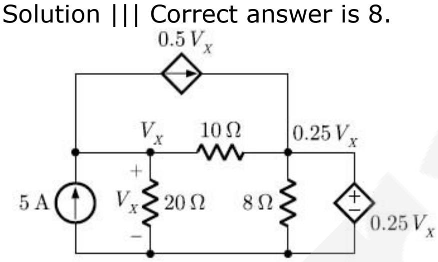
$$1 = \frac{12 - V_A}{3} + \frac{V_A}{6}$$

$$V_A = 10V$$

7. In the circuit shown, the voltage V_x (in Volts) is_______



A. 4 B. 8 C. 10 D. 20
Answer ||| B
Solution ||| Correct answer is 8.



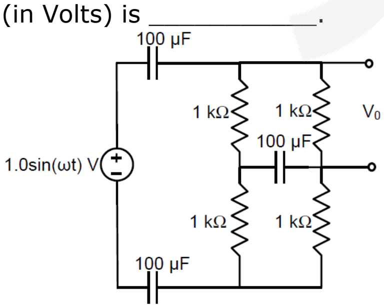
Applying KCL at node V_x,

$$\frac{V_x}{20} + \frac{V_x - 0.25V_x}{10} + 0.5V_x = 5$$

$$V_x\left(\frac{1}{20} + \frac{0.75}{10} + 0.5\right) = 5$$

$$V_x = 8V$$

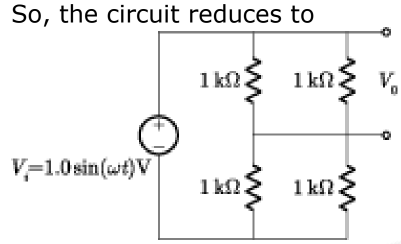
8. At very high frequencies, the peak output voltage V_0 (in Volts) is _____________.



A. 0.5 B. 0.333 C. 1.5 D. 5
Answer ||| A
Solution ||| Correct answer is 0.5

At very high frequencies capacitors acts short circuit because $Z_C = \dfrac{1}{jwC}$ and at $\omega \to \infty,\ \sigma_c \to 0$

So, the circuit reduces to



By voltage division rule,

$$V_0 = \left(\frac{1\,K}{1K + 1K}\right)V_i$$

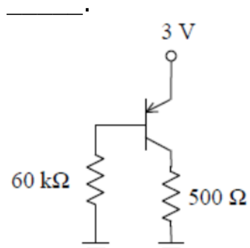
$$V_0 = \frac{V_i}{2} = \frac{1.0\sin\omega t}{2}$$

At peak voltage, sin □t = 1
Hence, V_0 = 0.5 V

9. Which one of the following process is preferred to form the gate dielectric (SiO_2) of MOSFETs?
A. Sputtering
B. Molecular beam epitaxy
C. Wet oxidation
D. Dry oxidation
Answer ||| D
Solution ||| Correct option is (D).
Dry oxidation is better than wet oxidation, so dry oxidation is always preferred.

10. If the base width in a bipolar junction transistor is doubled, which one of the following statements will be TURE?
A. Current gain will increase
B. Unity gain frequency will increase
C. Emitter-base junction capacitance will increase
D. Early voltage will increase
Answer ||| D
Solution ||| Correct option is (D).
On increasing the base width slope of the I_C vs V_{CE} curve decreases due to less (I_C). Hence, decrease in slope means increase in early effect. So, early voltage will increase.

11. In the circuit shown in the figure, the BJT has a current gain (β) of 50. For an emitter-base voltage V_{EB} = 600 mV, the emitter-collector voltage V_{EC} (in Volts) is ______.



A. 2.5 B. 2 C. 3 D. 5
Answer ||| B
Solution ||| Correct answer is 2.

$V_{EB} = 0.6V\ (= 50) \qquad (\beta = 50)$

$V_E - V_B = 0.6V$

$V_E = 3V$ (given)

So, $3 - V_B = 0.6\ V$

$V_B = 2.44$

$$I_B = \frac{V_B}{60k}$$

$$= \frac{2.4}{60}\ mA$$

$$I_C = \beta I_B$$

$$= 50 \times \left(\frac{2.4}{60}\right) mA$$

$$= \left(\frac{2.4}{60}\right) 5\ mA$$

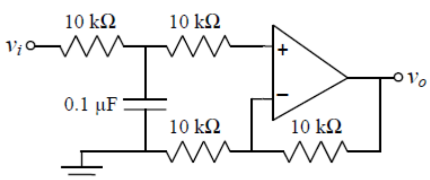
$$V_C = (500\Omega)\left(\frac{2.4}{6}\right)(5) \times 10^{-3}\ V$$

$$= 1V$$

Hence, $V_{EC} = V_E - V_C = 3 - 1 = 2V$

= 2V

12. In the circuit using an ideal opamp, the 3-dB cut-off frequency (in Hz) is ___________.



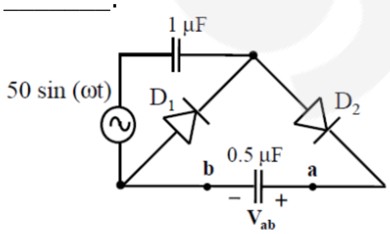
A. 159.23 B. 158.25 C. 165.55 D. 151.22
Answer ||| A
Solution ||| Correct answer is 159.23

$$f_{3dB} = \frac{1}{2\pi RC} \quad (R = 10\text{ k and } C = 0.1\ \mu\text{ F})$$

$$|= \frac{1}{2\pi\ (10) \times 10^3 \times 0.1 \times 10^{-6}}$$

$$= 159.23$$

13. In the circuit shown, assume that diodes D_1 and D_2 are ideal. In the steady state condition, the average voltage V_{ab} (in Volts) across the 0.5 μF capacitor is ________.



A. 200 B. 300 C. 100 D. 400
Answer ||| C
Solution ||| Correct answer is 100.
For positive half cycle, diode D_2 will be ON. So, peak voltage at point *a* is

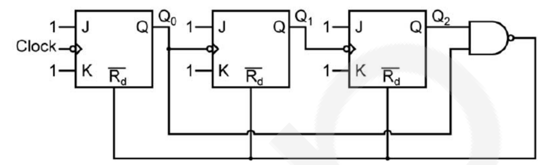
$V_a = 50\ V$

For negative half cycle, diode *D*1 will be ON. So, peak voltage at point *b* is

$V_b = -50\ V$

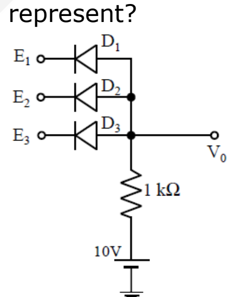
Hence $V_{ab} = V_a - V_b$
$= 50 - (-50) = 100$ V

14. The circuit shown consists of J-K flip-flops, each with an active low asynchronous reset $\left(\overline{R_d}\ input\right)$. The counter corresponding to this circuit is



A. a modulo-5 binary up counter
B. a modulo-6 binary down counter
C. a modulo-5 binary down counter
D. a modulo-6 binary up counter
Answer ||| A
Solution ||| Correct option is (A).
From the figure, it can be seen it is basic modulo UP counter configuration because clock is negative edge triggering.
At Modulo-5
1 0 1
For this state, all the 3 flip flops will be set to initial condition.

15. In the circuit shown diodes D_1, D_2 and D_3 are ideal, an the inputs E_1, E_2 and E_3 are "0 V" for logic '0' and "10 V" for logic '1'. What logic gate does the circuit represent?



A. 3-input OR gate B. 3-input NOR gate
C. 3-input AND gate D. 3-input XOR gate
Answer ||| C
Solution ||| Correct option is (C).
If any of the inputs from E_1, E_2, E_3 is logic 0 (means 0V) then the corresponding diode will be "ON" resulting in 0V at the output and only when all the inputs are logic 1 (means V_{DD}) then V_0 (output voltage) will be high, hence, resulting into 3 input AND-gate. Truth table for the logic circuit is shown below.

E_1	E_2	E_3	V_0
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

16. Which one of the following 8085 microprocessor programs correctly calculates the product of two 8-bit numbers stored in register B and C?

A.
```
MVI A, 00H
JNZ LOOP
CMP C
LOOP DCR B
HLT
```

B.
```
MVI A, 00H
CMP C
LOOP DCR B
JNZ LOOP
HLT
```

C.
```
MVI A, 00H
LOOP ADD C
DCR B
JNZ LOOP
HLT
```

D.
```
MVI A, 00H
ADD C
JNZ LOOP
LOOP INR B
HLT
```

Answer ||| C
Solution ||| Correct option is (C).
We check the given options.
The codes given in option (C), executes the following instructions
MVI A 00H (loading the accumulator with OOH)
LOOP ADD C (adding the contents of C to accumulator and store it to accumulator)
DCR B (Decrementing the content of registers B)
JNZ LOOP
HLT
Hence, decreasing the number in B as many-time as adding the another number C will result in product of two numbers till value in registers B is zero.

17. The impulse response of an LTI system can be obtained by
A. differentiating the unit ramp response
B. differentiating the unit step response
C. integrating the unit ramp response
D. integrating the unit step response
Answer ||| B
Solution ||| Correct option is (B).
By property from LTI systems

$$h(t) = \frac{d(s(t))}{dt}$$

$h(t)$ = impulse response
$s(t)$ = step response

18. Consider a four-point moving average filter defined by the equation y[n] = $\sum_{i=0}^{3}\alpha_1 x[n-i]$. The condition on the filter coefficients that results in a null at zero frequency is
A. $\alpha_1 = \alpha_2 = 0;\ \alpha_0 = -\alpha_3$
B. $\alpha_1 = \alpha_2 = 1;\ \alpha_0 = -\alpha_3$
C. $\alpha_0 = \alpha_3 = 0;\ \alpha_1 = \alpha_2$
D. $\alpha_1 = \alpha_2 = 0;\ \alpha_0 = \alpha_3$
Answer ||| A
Solution ||| Correct option is (A).

Given $y[n] = \sum_{i=0}^{3} \alpha_i x[n-i]$

Taking the DTFT (Discrete time fourier transform),

$$Y(e^{j\Omega}) = \alpha_0 X(e^{j\Omega}) + \alpha_1 (e^{j\Omega})^{-1} X(e^{j\Omega}) + \alpha_2 (e^{j\Omega})^{-2} X(e^{j\Omega}) + \alpha_3 (e^{j\Omega})^{-3} X$$

or $\dfrac{Y(e^{j\Omega})}{X(e^{j\Omega})} = H(e^{j\Omega}) = \alpha_0 + \alpha_1(e^{j\Omega})^{-1} + \alpha_2(e^{j\Omega})^{-2} + \alpha_3(e^{j\Omega})^{-3}$

At zero frequency, put $\Omega = 0$

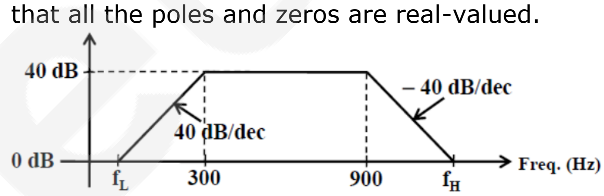
$H(e^{j0}) = \alpha_0 + \alpha_1 + \alpha_2 + \alpha_3$

It should be null.

For $H(e^{j0}) = 0$, only condition required is

$\alpha_1 = \alpha_2 = 0;\ \alpha_0 = -\alpha_3$

19. Consider the Bode plot shown in the figure. Assume that all the poles and zeros are real-valued.



The value of $f_H - f_L$ (in H'z) is______________.
A. 8050 B. 7060
C. 8060 D. 8970
Answer ||| D
Solution ||| Correct answer is 8970.
For f_L, equating the slope,

$$40 = \frac{40 - 0}{\log_{30}(300) - \log_{10}(f_L)}$$

$$= \log_{10}\left(\frac{300}{f_L}\right) = 1$$

$$= \frac{300}{f_L} = 10$$

$$f_L = 30\ Hz$$

For f_H, equating the slope

$$-40 = \frac{0 - 40}{\log_{10} f_H - \left(\dfrac{\log}{900}\right)}$$

$$\log_{10}\left(\frac{f_H}{900}\right) = 1$$

$$f_H = 900 \times 10 = 9000$$

Hence, $f_H - f_L = 9000 - 30 = 8970\ Hz$

20. The phase margin (in degrees) of the system G(s) = $\dfrac{10}{s(s+10)}$ is___________.
A. 25° B. 84.28° C. 82.84° D. 90°
Answer ||| B
Solution ||| Correct answer is 84.28°.

To find PM, we need to find gain cross over frequency ω_g

At $\omega = \omega_g$,

$$|G(s)| = 1$$

So, $$\left| \frac{10}{\omega_g \sqrt{\omega_g^2 + 100}} \right| = 1$$

$$10 = \omega_g \sqrt{\omega_g^2 + 100}$$

Squaring both side,

$$100 = \omega_g^2(\omega_g^2 + 100)$$

$$100 = \omega_g^4 + 100\omega_g^2$$

$$\omega_g^4 + 100\omega_g^2 - 100 = 0$$

Accepted value, $\omega_g^2 = 3.98$

$$\omega_g \approx 2 \; red/\sec$$

Hence, $PM = 180^o + \langle G(jw)|$

$$= 180^o - 90^o - \tan^{-1}\left(\frac{w_g}{10}\right)$$

$$= 90^o - \tan^{-1}\left(\frac{2}{10}\right) = 84.28^o$$

21. The transfer function of a first-order controller is given as

$$G_C(s) = \frac{K(s+a)}{s+b}$$

Where K, a and b are positive real numbers. The condition for this controller to act as a phase lead compensator is

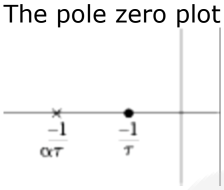
A. a < b B. a > b

C. K < ab D. K > ab

Answer ||| A

Solution ||| Correct option is (A).

The pole zero plot of lead compensator is shown below.



By comparing it to the given problem, we get

$$\frac{1}{\tau} = a$$

$$\frac{1}{\alpha\tau} = b$$

So, $\frac{a}{b} = \alpha$

For lead compensator,

$$\alpha < 1$$

$$\frac{a}{b} < 1$$

$$a < b$$

22. The modulation scheme commonly used for transmission from GSM mobile terminals is

A. 4-QAM

B. 16-PSK

C. Walsh-Hadamard orthogonal codes

D. Gaussian Minimum Shift Keying (GMSK)

Answer ||| D

Solution ||| Correct option is (D).

Gaussian Minimum Shift Keying (GMSK) is used for GSM mobile terminals.

23. A message signal m(t) = $A_m \sin(2\pi f_m t)$ is used to modulate the phase of a carrier $A_c \cos(2\pi f_c t)$ to get the modulated signal y(t) = $A_c \cos(2\pi f_c t + m(t))$. The bandwidth of y(t)

A. depends on A_m but no on f_m

B. depends on f_m but not on A_m

C. depends on both A_m and f_m

D. does not depend on A_m or f_m

Answer ||| C

Solution ||| Correct option is (C).

Bandwidth of *PM* signal is given by

$$B = 2(\Delta f + f_m)$$

So, it depends upon f_m and

$$\Delta f = \frac{k_f m_p}{2\pi}$$

($m_p = A_m$ = message signal amplitude)

24. The directly of an antenna array can be increased by adding more antenna elements, as a larger number of elements

A. improves the radiation efficiency

B. increases the effective area of the antenna

C. results in a better impedance matching

D. allows more power to be transmitted by the antenna

Answer ||| B

Solution ||| As per antenna array concept,

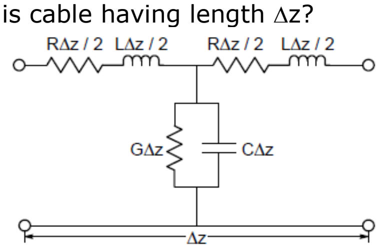
$$D_\omega \frac{4\pi}{\lambda^9} Ae$$

$$D \uparrow > Ae \uparrow$$

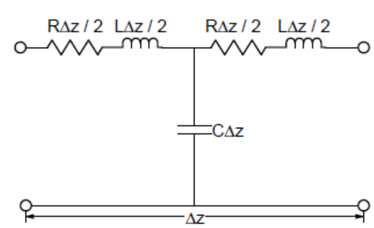
Therefore, the directivity of an antenna array can be increased by increasing the effective area of the antenna.

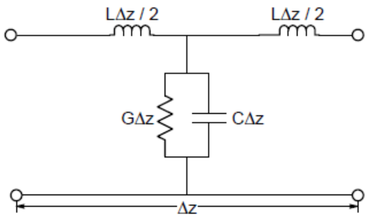
25. A coaxial cable is made of two brass conductors. The spacing between the conductors is filled with Teflon (ε_r = 2.1, tan δ = 0). Which one of the following circuits can represent the lumped element model of a small piece of this cable having length Δz?

A.

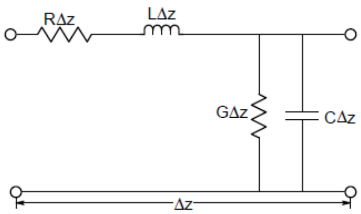


B.





C.



D.

Answer ||| B

Solution ||| Correct option is (B).

Given that

$\tan\delta = 0$

$\tan\delta = loss\ \tan gent = \dfrac{\sigma}{w\varepsilon} = 0$

So, $\sigma = 0$

Hence, conductivity is

G = 0

26. The Newton-Raphson method used to solve the equation f(x) = $x^3 - 5x^2 + 6x - 8 = 0$. Taking the initial guess as x = 5, is_________.

A. 4.3 B. 2 C. 15 D. 15

Answer ||| A

Solution ||| Correct answer is 4.29

$f(x) = x^3 - 5x^2 + 6x - 8$

$x_0 = 5$ (initial point)

$f'(x) = 3x^2 - 10x + 6$

By Newton-Raphson method,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{f(5)}{f'(5)}$$

$$So,\ f(5) = (5)^3 - 5(5)^2 + 6(5) - 8 = 22$$

$$f'(5) = 3(5)^2 - 10(5) + 6 = 31$$

$$So,\ x_1 = 5 - \frac{22}{31} = 5 - 0.7097$$

$$= 4.2903$$

27. A fair die with faces {1, 2, 3, 4, 5, 6} is thrown repeatedly till '3' is observed for the first time. Let X denote the number of times the die is thrown. The expected value of X is ______.

A. 4 B. 2X C. 6 D. 12

Answer ||| C

Solution ||| Correct answer is 6.

We have

Probability of getting $3 = \dfrac{1}{6}$

Probability of not getting $3 = 1 - \dfrac{1}{6} = \dfrac{5}{6}$

Now, the random variable *X* represents the number of throws required for getting 3. So,

$$X = 1,\ P(x = 1) = \frac{1}{6}$$

$$X = 2,\ P(x = 2) = \frac{5}{6} \times \frac{1}{6}$$

$$X = 3,\ P(x = 3) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$$

$$X = 4,\ P(x = 4) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$$

$$E[X] = \sum xP(x)$$

$$= (1)\left(\frac{1}{6}\right) + (2)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + 3\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) + 4\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right).....$$

$$= \frac{1}{6}\left[1 + 2\left(\frac{5}{6}\right) + 3\left(\frac{5}{6}\right)^2 + 4\left(\frac{5}{6}\right)^3 ....\right]$$

Let

$$S - 1 + 2\left(\frac{5}{6}\right) + 3\left(\frac{5}{6}\right)^2 + 4\left(\frac{5}{6}\right)^3$$

$$\left(\frac{5}{6}\right)S = \frac{5}{6} + 2\left(\frac{5}{6}\right)^2 + 3\left(\frac{5}{6}\right)^3 + 4\left(\frac{5}{6}\right)^4$$

Equation (1) – (2),

$$S - \frac{5}{6}S = 1 + \left(\frac{5}{6}\right)(2-1) + \left(\frac{5}{6}\right)^2 + (3-2) + \left(\frac{5}{6}\right)^3 (4-3)...$$

$$\frac{5}{6} = 1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^3 ....$$

$$\frac{5}{6} = \frac{1}{1 - \frac{5}{6}} = 6\ or\ S = 36$$

Hence, the expected value of *X* is

$$E[x] = \frac{36}{6} = 6$$

28. Consider the differential equation

$$\frac{d^2x(t)}{dt^2} + 3\frac{dx(t)}{dt} + 2x(t) = 0.$$

Given x(0) = 20 and x(1) = 10/e, where e = 2.718, the value of x(2) is __________.

A. 0 B. 2.55 C. 1.2 D. 0.856

Answer ||| D

Solution ||| Correct answer is 0.856

Given, $\dfrac{d^2x(t)}{dt^2} + 3\dfrac{dx(t)}{dt} + 2x(t) = 0$

$$where\quad x(0) = 20, x(1) = \frac{10}{e}, e = 2.718$$

This is homogeneous equation. So particular solution is zero. We obtain auxiliary equation as

$$m^2 + 3m + 2 = 0$$

$$m = -1, -2$$

$$x(t) = Ae^{-t} + Be^{-2t}$$

$$x(0) = A + B = 20 \qquad ......(1)$$

$$x(1) = Ae^{-1} + Be^{-2} = \frac{10}{e} \ ....(2)$$

Solving equations (1) and (2),

$$A=\left(\frac{10e-20}{e-1}\right),\ B=\frac{10e}{e-1}$$

$$x(t)=\left(\frac{10e-20}{e-1}\right)e^{-t}+\left(\frac{10e}{e-1}\right)e^{-2t}$$

Put t = 2

$$x(2)=\left(\frac{10e-20}{e-1}\right)e^{-2}+\left(\frac{10e}{e-1}\right)e^{-4}$$

$$x(2)=0.8556$$

29. A vector field D = $2\rho^2 a_\rho + z a_z$ exists inside a cylindrical region enclosed by the surfaces ρ = 1, z = 0 and z = 5. Let S be the surface bounding this cylindrical region. The surface integral of this field on S $\left(\oiint_S \mathbf{D.ds}\right)$ is ______.

A. 56.33 B. 80.5 C. 78.00 D. 78.52

Answer ||| D

Solution ||| Correct answer is 78.52

Given vector field,

$$D=2p^2a_p+za_s$$

By using divergence theorem

$$\oint_V(\nabla . D)\,dV=\oint_S D\,.\,dS$$

$$\nabla . D=\frac{1}{p}\frac{\partial}{\partial p}(pDp)+\frac{1}{p}\frac{\partial D_\phi}{\partial \phi}+\frac{\partial D_z}{\partial z}$$

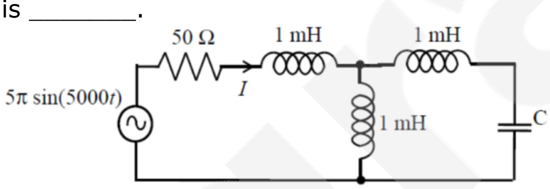
$$=\frac{1}{p}\frac{\partial}{\partial p}(p2p^2)+0+1$$

$$=\frac{1}{p}(2)(3)\,p^2+1=6p+1$$

$$\int_V(\nabla D)\,dV=\int_{p=0}^{1}\int_{\phi=0}^{2\pi}\int_{z=0}^{5}(6p+1)(p)\,dp\,d\phi\,dz$$

= 78.52

30. In the circuit shown, the current I flowing through the 50 Ω resistor will be zero if the value of capacitor C(in μF) is ________.

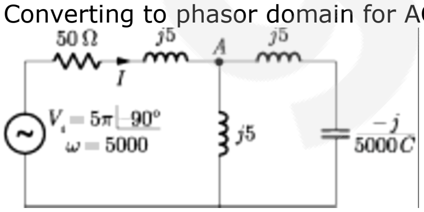


A. 10 B. 20 C. 30 D. 50

Answer ||| B

Solution ||| Correct answer is 20.

Converting to phasor domain for AC analysis



If current I = 0,

Voltage at node A = input voltage

Applying KCL at node A,

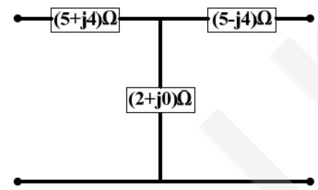
$$\frac{V_i}{j5}+\frac{V_i}{j\left(5-\frac{1}{5000C}\right)}=0$$

or $$\frac{1}{j5}+\frac{-1}{j\left(5-\frac{1}{5000C}\right)}$$

For value of C, we solve above equation. Hence,

$C = 20\ \mu F$

31. The ABCD parameters of the following 2-port network are



A. $\begin{bmatrix}3.5+j2 & 20.5\\ 20.5 & 3.5-j2\end{bmatrix}$ B. $\begin{bmatrix}3.5+j2 & 30.5\\ 0.5 & 3.5-j2\end{bmatrix}$

C. $\begin{bmatrix}10 & 2+j0\\ 2+j0 & 10\end{bmatrix}$ D. $\begin{bmatrix}7+j4 & 0.5\\ 30.5 & 7-j4\end{bmatrix}$

Answer ||| B

Solution ||| Correct option is (B).

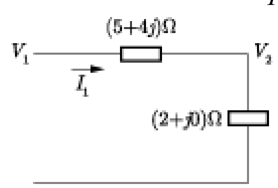
ABCD parameters are defined as

$$\begin{bmatrix}V_1\\ I_1\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}V_2\\ -I_2\end{bmatrix}$$

$$\begin{bmatrix}3.5+j2 & 30.5\\ 0.5 & 3.5-j2\end{bmatrix}$$

$$V_1 = AV_2 - BI_2$$

$$I_1 = CY_2 - DI_2$$



From the circuit,

$$A=\frac{V_1}{V_2}$$

$$V_2=\frac{2}{7+4j}V_1$$

$$\frac{V_1}{V_2}=A=\frac{7+4j}{2}=3.5+2j$$

$$\frac{I_1}{V_2}=C$$

$$I_1=\frac{V_1}{7+4j}$$

$$V_2=\left(\frac{2}{7+4j}\right)V_1$$

$$V_2=\left(\frac{2}{7+4j}\right)(I_1)(7+4j)$$

$$\frac{I_1}{V_2}=0.5=C$$

Similarly, we may obtain other parameters. The value of parameter C satisfies the option B. only. So, the ABCD parameters are

$$\begin{bmatrix} 3.5+j2 & 30.5 \\ 0.5 & 3.5-j2 \end{bmatrix}$$

32. A network is described by the state model as

$$x_1 = 2x_1 - x_2 + 3u$$
$$x_2 = -4x_2 - u$$
$$y = 3x_1 - 2x_2$$

The transfer function H(s) $\left(= \frac{Y(s)}{U(s)}\right)$ is.

A. $\frac{11s+35}{(s-2)(s+4)}$ B. $\frac{11s-35}{(s-2)(s+4)}$

C. $\frac{11s+38}{(s-2)(s+4)}$ D. $\frac{11s-38}{(s-2)(s+4)}$

Answer ||| A

Solution |||

Correct option is (A).

$$x_1 = 2x_1 - x_2 + 3u$$
$$x_2 = -4x_2 - u$$
$$y = 3x_1 - 2x_2$$

Where $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $U =$ Input vector

In matrix form,

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & -1 \end{bmatrix} X + \begin{bmatrix} 3 \\ -1 \end{bmatrix} U$$

$$Y = [3\ -2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

So, $A = \begin{bmatrix} 2 & -1 \\ 0 & -4 \end{bmatrix}, B = \begin{bmatrix} 3 \\ -1 \end{bmatrix} C = [3\ -2], D = 0$

$$H(s) = C(sI - A)^{-1}\ B + D$$

Here, D = 0. So,

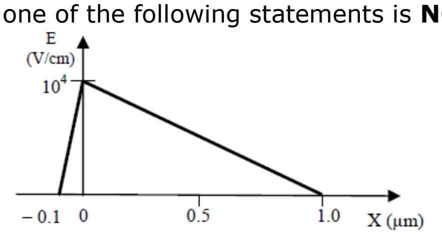
$$H(s) = C(sI - A)^{-1}\ B$$

$$= [3\ -2]\begin{bmatrix} s-2 & 1 \\ 0 & s+4 \end{bmatrix}^{-1}\begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$= [3\ -2]\frac{1}{(s^2+2s-8)}\begin{bmatrix} s+4 & -1 \\ 0 & s-2 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$= \frac{11s+35}{(s-2)(s+4)}$$

33. The electric field profile in the depletion region of a p-n junction in equilibrium is shown in the figure. Which one of the following statements is **NOT TRUE**?



A. The left side of the junction in n-type and the right side is p-type

B. Both the n-type and p-type depletion regions are uniformly doped

C. The potential difference across the depletion region is 700 mV

D. If the p-type region has a doping concentration of 10^{15} cm^{-3}, then the doping concentration in the n-type region will be 10^{16} cm^{-3}.

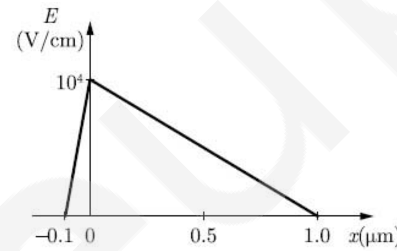
Answer ||| C

Solution ||| Correct option is (C).

We know that the electric field is the negative gradient of electric potential, i.e.

$$E = \frac{-d\ V}{dx}$$

Or $V = -\int Edx$



Area under the curve given in figure will give the built in potential as

$$U_{bi} = \frac{1}{2}(0.1)\ xm \times 10^4\ V/cm + \frac{1}{2}\times 1\ cm \times 10^4\ V/cm$$

$$= \frac{1}{2}(0.1)\times 10^{-6}\times 10^4 \times 100 + \frac{1}{2}\times 10^{-4}\times 10^4\ V/cm$$

$$= 0.55\ V$$

In option C. it is given as 700 mV, so it is not true.

34. The current in an enhancement mode NMOS transistor biased in saturation mode was measured to be 1 mA at a drain-source voltage of 5 V. When the drain-source voltage was increased to 6V while keeping gate-source voltage same, the drain current increased to 1.02 mA. Assume that drain to source saturation voltage is much smaller than the applied drain-source voltage. The channel length modulation parameter λ In V^{-1}) is _____.

A. 0.002 B. 0.022 C. 0.4 D. 0.033

Answer ||| B

Solution ||| Correct answer is 0.022

Given transistor is in saturation region, and

Current = 1 mA at V_{DS} = 5 V

Current = 1.02 mA at V_{DS} = 6 V

Assuming V_G constant, current in saturation region is

$$I_D = \frac{k}{2}(V_{GS} - V_{TN})^2(1+\lambda V_{DS})$$

$$I_{D_1} = (1+\lambda V_{DS_1})$$

$$I_D = (1+\lambda V_{DS_2})$$

So, $\frac{1.02\ mA}{1\ mA} = \frac{(1+\lambda V_{DS_2})}{(1+\lambda V_{DS1})}$

$$1.02 = \frac{1+6\lambda}{1+5\lambda}$$

$$\lambda = 0.022\ V^{-1}$$

35. An npn BJT having reverse saturation current $I_s = 10^{-15}$ A is biased in the forward active region with V_{BE} = 700

mV. The thermal voltage (V_T) is 25 mV and the current gain (β) may vary from 50 to 150 due to manufacturing variations. The maximum emitter current (in μA) is____.
A. 655 B. 1555 C. 1475.5 D. 100
Answer ||| C
Solution ||| Correct answer is 1475.5
Collector current is given by

$$I_C = I_S \exp\left[\frac{V_{BE}}{V_T}\right] = 10^{15}\ A \exp\left[\frac{700\ mV}{25\ m\ V}\right]$$

$$= 10^{-15}\ A \exp\ [28]$$

Also, we may define

$$I_C = \beta I_B$$

or $I_B = \dfrac{I_C}{\beta}$

and $I_E = (\beta + 1) I_B$

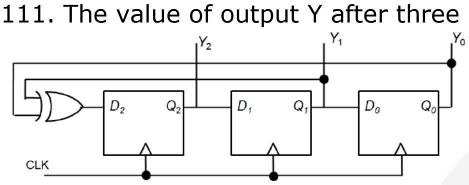
So, for maximum I_E , I_B and β should be maximum.
Therefore, we obtain

$$I_E = (\beta + 1)\ I_B = \frac{(\beta + 1)\ I_C}{\beta}$$

$$I_E = \frac{\beta + 1}{\beta}\ (I_C) = \frac{(\beta + 1)}{\beta}\ (10^{-15} A)\ \exp\ (28)$$

Here, we will take β = 50 for maximum IE. Hence,
I_E = 1475.51A

36. A three bit pseudo random number generator is shown. Initially the value of output Y ≡ $Y_2\ Y_1 Y_0$ is set to 111. The value of output Y after three clock cycles is



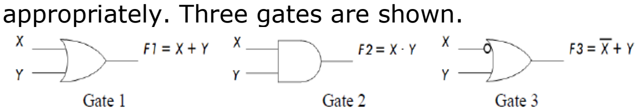
A. 000 B. 001 C. 010 D. 100
Answer ||| D
Solution ||| Correct option is (D).
For *D*-flip flop
Q(*t* + 1) = *D*(*t*)
or next state = input
So, we may write

$$Q_2(t+1) = Q_1(t) + Q_0(t)$$
$$Q_1(t+1) = Q_2(t)$$
$$Q_0(t+1) = Q_1(t)$$

The resulting output is illustrated in the truth table below.

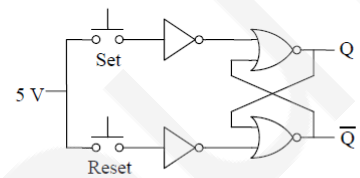
	Q_2	Q_1	Q_0
Given initial	1	1	1
1st clock	0	1	1
2nd clock	0	0	1
3rd clock	1	0	0

37. A universal logic gate can implement nay Boolean function by connecting sufficient number of them appropriately. Three gates are shown.



Which one of the following statements is TRUE?
A. Gate 1 is a universal gate
B. Gate 2 is a universal gate
C. Gate 3 is a universal gate
D. None of the gates shown is a universal gate
Answer ||| C
Solution ||| Correct option is (C).
In general, the only universal gates are NAND and NOR gates but none of the given question is NAND/NOR gate. However, we must observe the given Gate 3. All the Boolean function can be implemented by using the gate. Hence it is a universal gate.

38. An SR latch is implemented using TTL gates as shown in the figure. The set and rest pulse inputs are provided using the push-button switches. It is observed that the circuit fails to work as desired. The SR latch can be made functional by changing

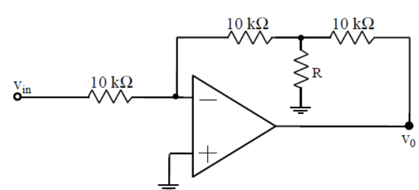


A. NOR gates to NAND gates
B. inverters to buffers
C. NOR gates no NAND gates and inverters to buffers
D. 5 V to ground
Answer ||| D
Solution ||| Correct option is (D).
SR latch truth table is shown below.

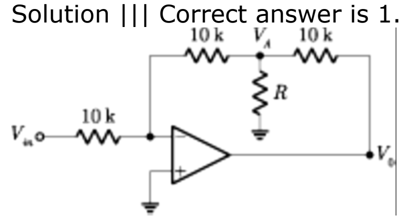
S	*R*	*Q*
0	0	hold
0	1	reset
1	0	Set
1	1	Not Defined

The above truth table can be obtained from the given circuit, if we change 5 V to ground.

39. In the circuit shown, assume that the opamp is ideal. If the gain (v_0 / v_{in}) is −12, the value of R (in kΩ) is ______.



A. 1 B. 16 C. 15 D. 2
Answer ||| A
Solution ||| Correct answer is 1.



Given

$$\frac{V_0}{V_{in}} = -12$$

$$\frac{V_{in} - 0}{10k} = \frac{0 - V_A}{10\ k}$$

At node A

$$\frac{V_A - 0}{10k} + \frac{V_A}{R}| + \frac{V_A - V_0}{10k} = 0$$

$$V_{in} = -V_A$$

Or $V_A = -V_{in}$

Substituting equation (2) in (1), we have

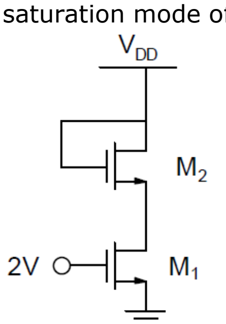
$$\frac{-V_{in}}{10k} - \frac{V_{in}}{R} + \frac{-V_{in} - V_0}{10k} = 0 \text{ and } V_0 = -12\ V_{in}$$

$$\frac{-V_{in}}{10k} - \frac{V_{in}}{R} + \frac{(-V_{in} + 12V_{in})}{10k} = 0$$

Solving above equation, we get

R = 1 k

40. In the circuit shown, both the enhancement mode NMO transistors have the following characteristics: $k_n = \mu_n C_{ox}$ (W/L) = 1 mA/V^2; V_{TN} = 1V. Assume that the channel length modulation parameter λ is zero and body is shorted to source. The minimum supply voltage V_{DD} (in volts) needed to ensure that transistor M_1 operates in saturation mode of operation is ________.



A. 1 B. 2 C. 3 D. 4

Answer ||| C

Solution ||| correct answer is 3.

For M_1 to be in saturation,

$$V_{DS_1} > V_{GS_1} - V_{TN}$$

$$V_{D_1} > 2 - 1$$

$$V_{D_1} > 1$$

For minimum value $V_{D_1} = 1\ V$

So, $I_{D_1} = \frac{1\ mA/V^2}{2}(V_{us} - V_{TN})^2$

$$= \frac{1\ mA/V^2}{2}(2-1)^2 = \frac{1 mA}{2}(1)$$

$$I_{D_1} = 0.5\ mA$$

Now, for M_2

$V_{DS_2} V_{GS_2} - V_{TN}$ will hold

Since, $V_G = V_D$

So, it will always be in saturation. Therefore

$$I_{D_2} = \frac{1}{2}(k)\ (V_{GS} - V_{TN})^2$$

Now, $I_{D1} = I_{D_2}$

(current in series connected components)

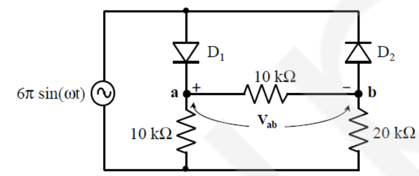
$$0.5\ mA = (1 mA/V^2)\frac{1}{2}\ (V_{DD} - 1 - 1)^2$$

$$(V_{DD}\quad 2)^2 = 1$$

$$V_{DD} = 3\ V$$

This is the minimum required value.

41. In the circuit shown, assume that the diodes D_1 and D_2 are ideal. The average value of voltage V_{ab} (in volts), across terminals 'a' and 'b' is _________.



A. 2 Volts B. 3 Volts C. 6 Volts D. 5 Volts

Answer ||| D

Solution ||| Correct answer is 5.

In positive half-cycle (D_1 = ON, D_2 = OFF), we have

$$V_{ab} = \frac{6\pi}{3}\sin\omega t = 2\pi \sin\omega t$$

Again, in Negative half-cycle (D_1 = OFF, D_2 = ON), we have

$$V_{ab} = \frac{6\pi}{3}\sin\omega t = 3\pi \sin\omega t$$

Hence, the average of V_{ab} is

$$= \frac{2\pi}{\pi} + \frac{3\pi}{\pi} = 5\ volts$$

42. Suppose x[n] is an absolutely summable discrete-time signal. Its z-transform is a rational function with two poles and two zeroes. The poles are at z = ±2j. Which one of the following statements is TRUE for the signal x[n]?

A. It is a finite duration signal

B. It is a causal signal

C. It is a non-causal signal

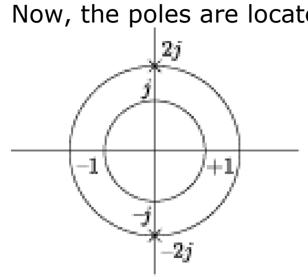
D. It is a periodic signal

Answer ||| C

Solution ||| Correct option is (C).

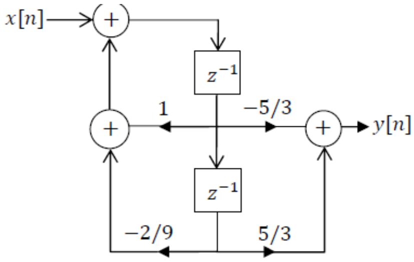
Given sequence is absolutely summable, so its Fourier transform (DTFT) exists.

Now, the poles are located below.



Here, ROC is inside the circle including unit circle. Hence, it is a non-causal system.

43. A realization of a stable discrete time system is shown in the figure. If the system is excited by a unit step sequence input x[n], the response y[n] is



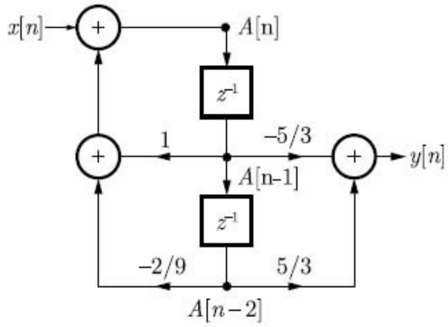
A. $4\left(-\frac{1}{3}\right)^n u[n]-5\left(-\frac{2}{3}\right)^n u[n]$

B. $5\left(-\frac{2}{3}\right)^n u[n]-3\left(-\frac{1}{3}\right)^n u[n]$

C. $5\left(\frac{1}{3}\right)^n u[n]-5\left(-\frac{2}{3}\right)^n u[n]$

D. $5\left(\frac{2}{3}\right)^n u[n]-5\left(-\frac{1}{3}\right)^n u[n]$

Answer ||| C
Solution ||| Correct option is (C).
We redraw the given system as



From the circuit, we have

$$A[n]=x[n]+A[n-1]-\frac{2}{9}A[n-2]$$

$$A(z)\times\left[1-z^{-1}+\frac{2}{9}z^{-2}\right]=X(z) \qquad \text{...(i)}$$

$$\frac{A(z)}{X(z)}=\frac{1}{1-2^{-1}+\frac{2}{9}z^{-2}} \qquad \text{................(1)}$$

Again,

$$Y[n]=-\frac{5}{3}A[n-1]+\frac{5}{3}A[n-2]$$

$$Y[z]=\left[\frac{-5}{3}z^{-1}+\frac{5}{3}z^{-2}\right]A(z)$$

$$\frac{Y(z)}{A(z)}=\frac{-5}{3}z^{-1}+\frac{5}{3}z^{-2} \qquad \text{......(2)}$$

Multiplying equations (1) and (2),

$$\frac{Y(z)}{X(z)}=\frac{\frac{-5}{3}z^{-1}+\frac{5}{3}z^{-2}}{1-z^{-1}+\frac{2}{9}z^{-2}}$$

For unit step response,

$$X(z)=\frac{1}{1-z^{-1}}$$

$$So,\ Y(z)=\frac{\frac{-5}{3}z^{-1}[1-z^{-1}]}{1-z^{-1}+\frac{2}{9}z^{-2}}\times\frac{1}{1-z^{-1}}$$

$$=\frac{5}{1-\frac{1}{3}z^{-1}}-\frac{5}{1-\frac{2}{3}z^{-1}}$$

Hence, $y[n]=5\left(\frac{1}{3}\right)^n u[n]-5\left(\frac{2}{3}\right)^n u[n]$

44. Let

$$\tilde{x}[n]=1+\cos\left(\frac{\pi n}{8}\right)$$

be a periodic signal period 16. Its DFS coefficients are defined by

$$a_k=\frac{1}{16}\sum_{n=0}^{15}\tilde{x}[n]\exp\left(-j\frac{\pi}{8}kn\right)$$

for all k. The value of the coefficient a_{31} is_____________.
A. 0.5 B. 3 C. 2.5 D. 5
Answer ||| A
Solution ||| Correct answer is 0.5
Given

$$x[n]=1+\cos\left(\frac{\pi n}{8}\right),\ with\ N=16$$

$$x[n]=1+\frac{1}{2}e^{3\frac{2\pi n}{16}}+\frac{1}{2}e^{3\frac{2\pi n}{16}}$$

$$a_1=a_{-1}=\frac{1}{2},\ a_0=1$$

For discrete series,

So, $a_k=a_{k+N}$

$a_{31}=0.5$

45. Consider a continuous-time signal defined as

$$x(t)=\left(\frac{\sin(\pi t/2)}{(\pi t/2)}\right)*\sum_{n=-\infty}^{\infty}\delta(t-10n)$$

Where '*' denotes the convolution operation and t is in seconds. The Nyquist sampling rate In samples/sec) for x (t) is__________.
A. 2.3 B. 3.3 C. 5.2 D. 0.4
Answer ||| D
Solution ||| Correct answer is 0.4

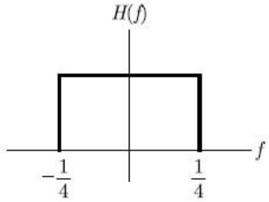
$$x(t)=\left(\frac{\sin\left(\frac{\pi t}{2}\right)}{\left(\frac{\pi t}{2}\right)}\right)*\sum_{n=-\infty}^{\infty}\delta(t-10n)$$

Time domain convolution = frequency domain multiplication so, we obtain

$$\sum_{n=-\infty}^{\infty} \delta(t-10n) \leftrightarrow \frac{1}{10}\sum_{n=-\infty}^{\infty} \delta(f-kf_S)$$

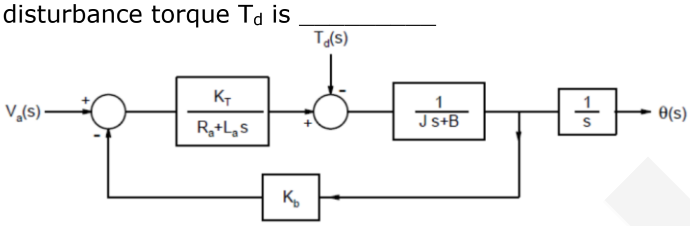
$$f_S = \frac{1}{T_S} = 0.1$$

$$\frac{\sin\left(\frac{\pi t}{2}\right)}{\left(\frac{\pi t}{2}\right)} \longleftrightarrow$$



Thus, the multiplication will result in maximum frequency of 0.2. Hence,
Nyquist rate = 2 f_m = 2(0.2) = 0.4 sample/sec

46. The position control of a DC servo-motor is given in the figure. The values of the parameters are K_T = 1 N-m/A, R_a = 1Ω, L_a = 0.1 H, J = 5 kg-m², B = 1 N-m/(rad/sec) and K_b = 1V/(rad/sec). The steady-state position response (in radians) due to unit impulse disturbance torque T_d is __________



A. 2.5 B. 0.5 C. -5 D. 4.5
Answer ||| B
Solution ||| Correct answer is 0.5
To find the response due to $T_d(s)$, we will use superposition making input V_a (s) = 0 and finding

Transfer function $= \dfrac{Q(s)}{T_d(s)}$

$T_d(s) = 1$ (unit impulse response)

$$So,\quad \theta(s) = \frac{\left(\frac{1}{Js+B}\right)}{\left[1+\left(\frac{1}{Js+B}\right)\left(\frac{k+kb}{Ra+L_a s}\right)\right]} \times \frac{1}{s}$$

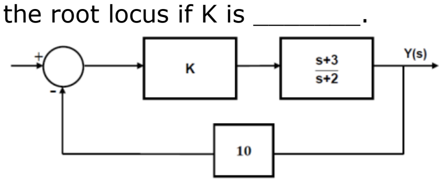
$$= \frac{1}{s\left[(Js+B)\frac{K+K_b}{Ra+L_a s}\right]}$$

For steady state response,

$$\lim_{s\to 0} s\theta(s) = \frac{1}{(Js+B)+\frac{K+K_B}{R_a+L_a s}}$$

$$= \frac{1}{B+\frac{K+K_b}{R_a}} = \frac{1}{1+\frac{1}{1}} = \frac{1}{2} = 0.5$$

47. For the system shown in the figure, s = −2.75 lies on the root locus if K is ________.



A. 0.5 B. -0.5 C. 0.3 D. -0.3
Answer ||| C
Solution ||| Correct answer is 0.3
The open loop transfer function is

$$G(s)\,H(s) = \frac{K\,(s+3)\,(10)}{(s+2)}$$

At point s = - 2.75,
$|G(s)\,H(s)| = 1$

$\left|\dfrac{K\,(s+3)}{(s+2)}\,10\right| = 1$ (point lie on room)

Hence, $K = \dfrac{0.75}{0.25}\left(\dfrac{1}{10}\right) = \dfrac{3}{10} = 0.3$

48. The characteristic equation of an LTI system is given by F(s) = $s^5 + 2s^4 + 3s^3 + 6s^2 - 4s - 8 = 0$. The number of roots that lie strictly in the left half s-plane is __________.
A. 2 B. 5 C. 3 D. 30
Answer ||| A
Solution ||| Correct answer is 2.
Given characteristic equation,

$$F(s) = s^5 + 2s^4 + 3s^3 + 6s^2 - 4s - 8 = 0$$

Applying the Routh stability criterion,
s^5 1 3 - 4
s^4 2 6 - 8
s^3 0 0 0
s^2
s^1
s^0
It contains complete zero row, so we obtain the auxiliary equation as $2s^4 + 6s^2 - 8 = 0$
Put $x = s^2$,
$2x^2 + 6x - 8 = 0$
x = 1, -4
So, s^2 = 1 or s = ±1
and $s^2 = -4$ or $s = \pm 2j$
Hence, one root s =− 1 lies on left side. Taking differential of auxiliary equation, $8s^3 + 12s = 0$
Now, the Routh array is redrawn as
s^5 1 3 -4
s^4 2 6 -8
s^3 8 12 0
s^2 3 -8 0
s^1 -9.33 0
s^0 0
Since, there is only one sign change in the first column of Routh array, so one pole lie in R.H.P and two poles lie on imaginary axis. Hence, the remaining two poles lies in L.H.P.

49. Two sequences $x_1[n]$ and $x_2[n]$ have the same energy. Suppose $x_1[n] = \alpha\, 0.5^n\, u[n]$, where α is a positive real number and u[n] is the unit step sequence. Assume

$$x_2[n] = \begin{cases} \sqrt{1.5} & for\ n = 0,1 \\ 0 & otherwise. \end{cases}$$

Then the value of α is___________.
A. 2 B. 1.5 C. 2.5 D. 14
Answer ||| B

Solution ||| Correct answer is 1.5

$x_1[n] = -(0.5)nu[n]$

Energy of signal $x_1[n]$ is

$$= \sum_{n=0}^{\infty} x_1^2[n]$$

$$= \sum_{n=0}^{\infty} \alpha^2 (0.5)^{2n}$$

$$= \alpha^2 \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n}$$

$$= \alpha^2 \left[1 + \frac{1}{4} + \frac{1}{16} .....\right]$$

$$= \alpha^2 \left[\frac{1}{1-\frac{1}{4}}\right]$$

$$= \frac{4}{3}\alpha^2$$

Again, $x_2[n] = \left\{\sqrt{1.5}, \sqrt{1.5}, 0, 0, ...\right\}$

So, energy of signal $x_2[n]$ is

$$= \left(\sqrt{1.5}\right)^2 + \left(1.5\right)^2$$

$$= 1.5 + 1.5 = 3$$

Given that

Energy of signal $x_1[n]$ = Energy of signal $x_2[n]$

$$\frac{4}{3}\alpha^2 = 3$$

$$\alpha^2 = \frac{9}{4}$$

$$\alpha = \frac{+3}{2} = \pm\ 1.5$$

Since α is a positive real number, so we have

α = 1.5

50. The variance of the random variable X with probability density function $f(x) = \frac{1}{2}|x|e^{-|x|}$ is __________.

A. 6 B. 12 C. 5 D. 15

Answer ||| A

Solution |||

Correct answer is 6.

Given $f(x) = \frac{1}{2}|x|\ e^{-|x|}$

By definition, variance is

$$V(X) = E[x^2] - \{E[x]\}^2$$

$$E[X] = \int_{-\infty}^{\infty} xf\infty(x) = \int_{-\infty}^{\infty} x\frac{1}{2}|x|e^{-|x|}dx$$

Since it is an odd function, we get

$$E[X] = 0$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x) = \int_{-\infty}^{\infty} \frac{1}{2}x^2 f|x|e^{-|z|}dx$$

$$= \int_{-\infty}^{\infty} x^3 e^{-x} dx = 6$$

Hence, $V(x) = 6 - (0)^2$
$= 6$

51. The complex envelope of the bandpass signal , centered about Hz, is

A. $\left(\frac{\sin(\pi t/5)}{\pi t/5}\right)e^{j\frac{\pi}{4}}$ B. $\left(\frac{\sin(\pi t/5)}{\pi t/5}\right)e^{-j\frac{\pi}{4}}$

C. $\sqrt{2}\left(\frac{\sin(\pi t/5)}{\pi t/5}\right)e^{j\frac{\pi}{4}}$ D. $\sqrt{2}\left(\frac{\sin(\pi t/5)}{\pi t/5}\right)e^{j\frac{\pi}{4}}$

Answer ||| C

Solution ||| Correct option is (C).

$$x(t) = -\sqrt{2}\left(\frac{\sin\left(\frac{\pi t}{5}\right)}{\frac{\pi t}{5}}\right)\sin\left[\pi t - \frac{\pi}{4}\right]$$

$$= -\sqrt{2}\left(\frac{\sin\left(\frac{\pi t}{5}\right)}{\left(\frac{\pi t}{5}\right)}\right)\left[\cos\frac{\pi}{4}\sin\pi t - \sin\frac{\pi}{4}\cos\pi t\right]$$

$$= \left(\frac{\sin\left(\frac{\pi t}{5}\right)}{\left(\frac{\pi t}{5}\right)}\right)\cos\pi t - \left(\frac{\sin\left(\frac{\pi t}{5}\right)}{\left(\frac{\pi t}{5}\right)}\right)\sin\pi t$$

Let X_C be complex envelope of above signal. So, we have

$$X_C = \frac{\sin\frac{\pi t}{5}}{\left(\frac{\pi t}{5}\right)} + j\frac{\sin\frac{\pi t}{5}}{\frac{\pi t}{5}}$$

$$= \frac{\sin\frac{\pi t}{5}}{\frac{\pi t}{5}}[1+j] = \sqrt{2}\frac{\sin\left(\frac{\pi t}{5}\right)}{\left(\frac{\pi t}{5}\right)}e^{j\frac{\pi}{4}}$$

52. A random binary wave y(t) is given by

$$y(t)=\sum_{n=-\infty}^{\infty} X_n p(t-nT-\phi)$$

Where p(t) = u(t) – u(t –T), u(t) is the unit function and ϕ is an independent random variable with uniform distribution in [0, T]. The sequence $\{X_n\}$ consists of independent and identically distributed binary valued random variables with $P\{X_n = +1\} = P\{X_n = -1\} = 0.5$ for each n.
The value of autocorrelation

$$R_{yy}\left(\frac{3T}{4}\right)\triangleq E\left[y(t)y\left(t-\frac{3T}{4}\right)\right]$$

equals_______.
A. 0.24 to 0.26 B. 1 to 2.5 C. 0.001 to 0.003 D. 2
Answer ||| A
Solution |||

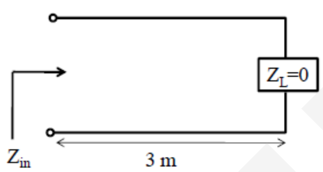
$$y(t)=\sum_{n=-f}^{x} X_n P(t-nT-\phi)$$

$$R_{yy(z)}=\left[1-\frac{|\tau|}{T}\right]$$

Derivation of above autocorrelation function can be found in any book dealing with random process.
[B.P. Lathi, Simon, Haykin, Schaum series]

$$R_{yy}\left(\frac{3T}{4}\right)=\left[1-\frac{3\pi/4}{\pi}\right]=\frac{1}{4}=0.25$$

53. Consider the 3 m long lossless air-filled transmission line shown in the figure. It has a characteristic impedance of $120\pi\Omega$, is terminated by a short circuit, and is excited with a frequency of 37.5 MHz. What is the nature of the input impedance (Z_{in})?



A. Open
B. Short
C. Inductive
D. Capacitive
Answer ||| D
Solution ||| Correct option is (D).
Given

$L = 3m$ (lossless)
$Z_0 = 120\pi\Omega,\ Z_L = 0\Omega$

For lossless line, we have

$$Z_{in}=Z_0\left(\frac{Z_L+jZ_0\tan\beta L}{Z_0+jZ_L\tan\beta L}\right)$$

Now, $\lambda = ?$

$f = 37.5\ MHz$
$C = f\lambda$

So, $\lambda=\frac{3\times10^8}{37.5\times10^6}=8$

$$\beta L=\frac{2\pi}{\lambda}(L)=\frac{2\pi}{8}(3)=\frac{3\pi}{4}$$

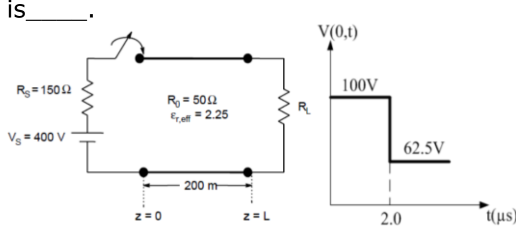
Put $Z_L = 0$ in equation (1),

$$Z_{in}=jZ_0\tan\beta L$$

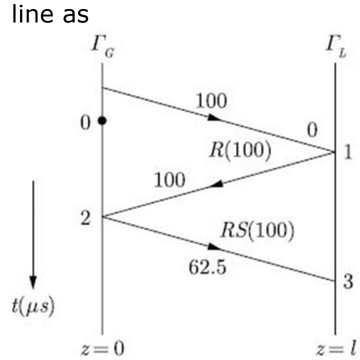
$$Z_{in}=j(120\pi)\tan\frac{3\pi}{4}$$

or $Z_{in}<0$ ($\tan 3\pi/4$ = negative quantity)
Hence, input impedance is capacitive in nature.

54. A 200 m log transmission line having parameters shown in the figure is terminated into a load R_L. The line is connected to a 400 V source having source resistance R_S through a switch, which is closed at t = 0. The transient response of the circuit at the input of the line (z = 0) is also drawn in the figure. The value of R_L (in Ω) is_____.



A. 25 B. 30 C. 45 D. 50
Answer ||| B
Solution ||| Correct answer is 30.
We form the bounce diagram for the given transmission line as



Here, Γ_G and Γ_L are the reflection coefficients at generator and load end, defined as

$$\Gamma_L = \frac{R_L - 50}{R_L + 50} \quad \text{..........(1)}$$

and $\Gamma_G = \dfrac{1}{2}$ ....(2)

Again, we have

$$V(t = 2\mu s, Z = 0) = 62.5$$

So, we can write

$$62.5 = V(t=0, z=0) + V(t=1, z=0) + V(t=2, z=0)$$

$$62.5 = 100 + \Gamma_L(100) + \Gamma_G\Gamma_L(100)$$

Substituting equation (2), we get

$$\Gamma_L = \frac{-1}{4}$$

Hence, substituting the above result in equation (1), we obtain

$$R_L = 30\,\Omega$$

55. A coaxial capacitor of inner radius 1 mm and outer radius 5 mm has a capacitance per unit length of 172 pF/m. If the ratio of outer radius to inner radius is doubled, the capacitance per unit length (in pF/m) is ____.
A. 120.22 B. 220 C. 150 D. 142
Answer ||| A

Solution ||| Correct answer is 120.22
Capacitance of coaxial capacitor is defined as

$$C = \frac{2\pi\varepsilon l}{In\left(\frac{b}{a}\right)}$$

So, $$\frac{C_1}{C_2} = \frac{In\left(\frac{b_2}{b_2}\right)}{In\left(\frac{b_1}{b_1}\right)}$$

$$\frac{172\,\text{pF/m}}{C^2} = \frac{\text{In}\left(\frac{10}{1}\right)}{\text{In }(5)}$$

C_2 = 120.22 pF/m

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